

Iulia & Teodoru Gugoiu

The Book of Fractions

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Iulia & Teodoru Gugoiu

The Book of Fractions

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Preface

"The Book of Fractions" presents one of the primary concepts of middle and high school mathematics: the concept of fractions. This book was developed as a workbook and reference useful to students, teachers, parents, or anyone else who needs to review or improve their understanding of the mathematical concept of fractions.

The structure of this book is very simple: it is organized as a collection of 50 quasi-independent worksheets and an answer key. Each worksheet contains:

a short description of the concepts, notations, and conventions that constitute the topic of the worksheet; step-by-step examples (completely solved) demonstrating the techniques and skills the student should gain by the end of each worksheet; and

an exhaustive test to be completed independently by the students.

The concept of fractions and the relations between fractions and other types of numbers, like many abstract mathematical concepts, is not always easy to understand. Bearing this in mind, the authors of this book introduce each topic gradually, starting with the basic concepts and operations and progressing to the more difficult ones. Geared specifically to help the beginners, the first part of the book contains graphical representations of the fractions.

The techniques for solving both simple and complex equations implying fractions are explained. As well, complete worksheets are provided, starting with very simple and basic equations and progressing to extremely complex equations requiring the application of a full range of operations with fractions.

"The Book of Fractions" also presents the link between fractions and other related mathematical concepts, such as ratios, percentages, proportions, and the application of fractions to real life concepts like time and money.

The importance of the concept of fractions comes both from its link to natural numbers and its link to more complex mathematical concepts, like rational numbers. As such, the concept of fractions is a milestone in the mathematical evolution of a student, being a concept that is simultaneously concrete (as a part of a whole) and abstract (as a set of two numbers and a hidden division operation).

The concept of equivalent fractions is an essential part of understanding fractions, and a full range of techniques is presented, starting with graphical representations (suitable for students in lower grades) and progressing to advanced uses, like the factor tree method of finding the LCD.

The order of operations is also presented, gradually, after each main operation with fractions: addition, subtraction, multiplication, and division; using multi-term expressions; expressions containing grouping symbols of one or more levels; and more complex operations with fractions, like powers with positive and negative exponents.

Single-step questions (requiring a basic knowledge and understanding of the topic presented in the worksheets) and multi-step questions (requiring a complete understanding of all of the concepts presented in the worksheets to that point) are presented throughout the entire book.

Combining more than 15 years of academic studies and 30 years of teaching experience, the authors of this book wrote it with the intention of sharing their knowledge, experience and teaching strategies with all the partners involved in the educational process.

Iulia & Teodoru Gugoiu,

Toronto, 2006

Understanding fractions

1. A fraction represents a part of a whole.
Example 1.



The whole is divided into four equal parts.
Three parts are taken (considered).

2. The corresponding fraction is:

3

—
4



The *numerator* represents how many parts are taken.

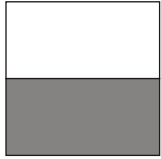


Fraction line or division bar



The *denominator* represents the number of equal parts into which the whole is divided.

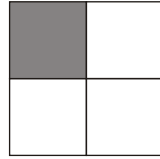
F01. Write the fraction that represents the part of the object that has been shaded:



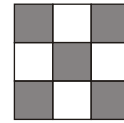
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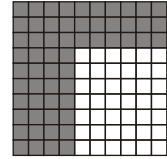
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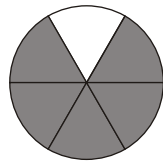
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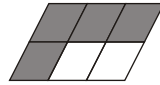
e)



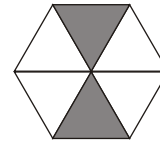
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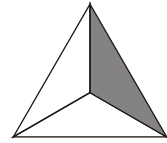
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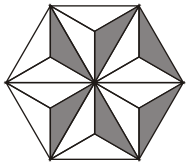
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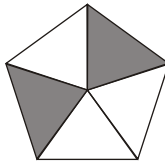
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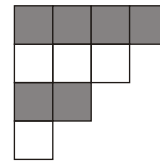
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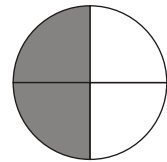
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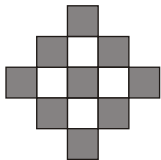
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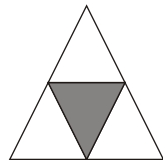
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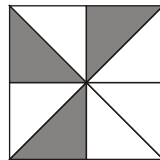
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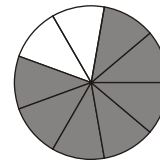
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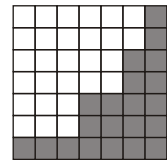
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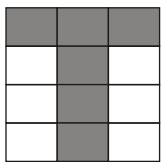
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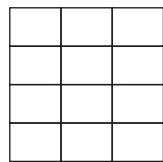
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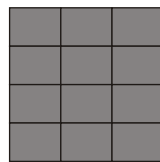
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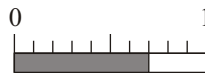
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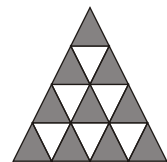
v)



w)



x)



y)

The graphical representation of a fraction

1. A fraction represents a part of a whole.

Example 1.

3 ← The *numerator* represents how many parts are taken.

— ← Fraction line or division bar

4 ← The *denominator* represents the number of equal parts into which the whole is divided.

2. A corresponding graphical representation (diagram) is:



The whole is divided into four equal parts. Three parts are taken (considered).

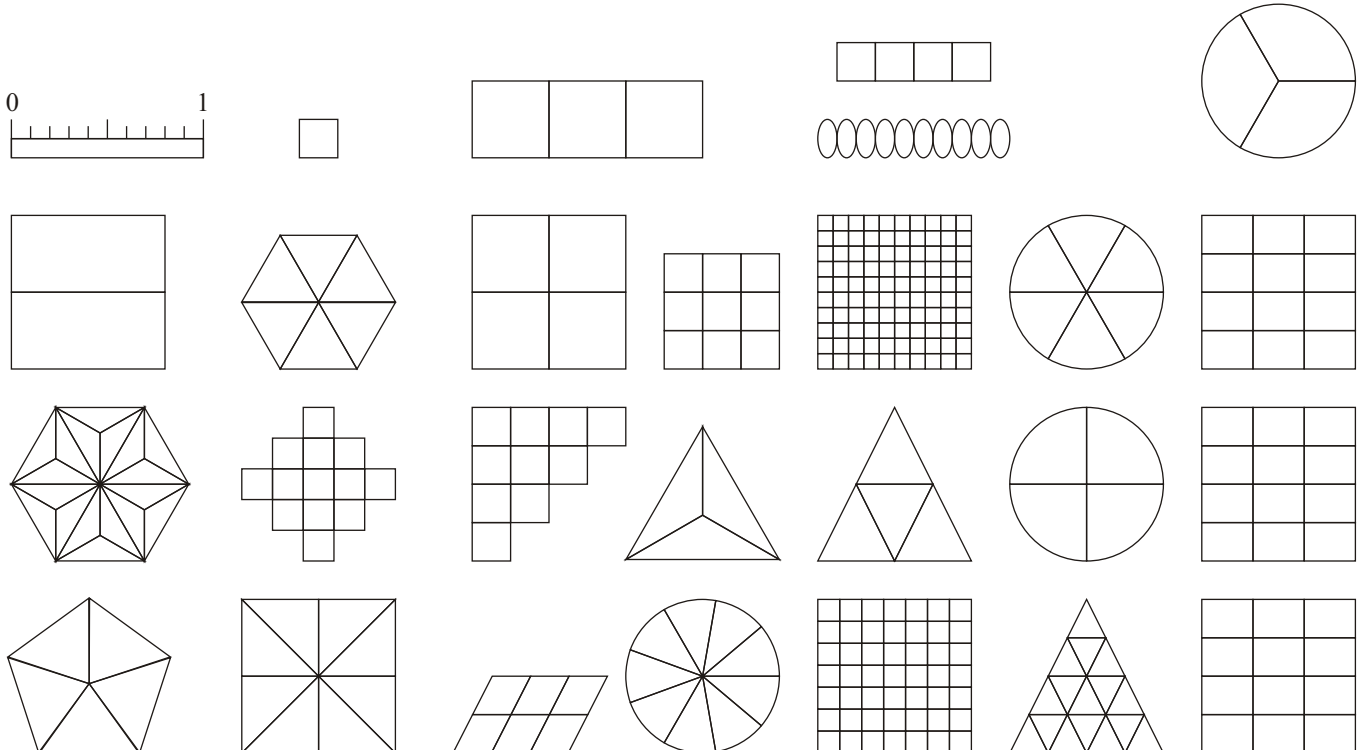
F02. Draw a diagram to show each fraction (use the images on the bottom of this page):

a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{1}{4}$ d) $\frac{2}{5}$ e) $\frac{1}{6}$ f) $\frac{2}{4}$ g) $\frac{0}{3}$

h) $\frac{2}{9}$ i) $\frac{5}{6}$ j) $\frac{2}{12}$ k) $\frac{9}{10}$ l) $\frac{1}{1}$ m) $\frac{3}{3}$ n) $\frac{4}{6}$

o) $\frac{3}{4}$ p) $\frac{4}{12}$ q) $\frac{5}{10}$ r) $\frac{4}{9}$ s) $\frac{2}{4}$ t) $\frac{8}{12}$ u) $\frac{5}{13}$

v) $\frac{5}{16}$ w) $\frac{1}{8}$ x) $\frac{7}{49}$ y) $\frac{37}{100}$ z) $\frac{11}{18}$



Reading or writing fractions in words

1. You can use words to refer to a part of a whole.
So one whole has:

2 halves	7 sevenths	12 twelfths	100 hundredths
3 thirds	8 eighths	13 thirteenths	1000 thousandths
4 quarters	9 ninths	20 twentieths	1000000 millionths
5 fifths	10 tenths	30 thirtieths	1000000000 billionths
6 sixths	11 elevenths	50 fiftieths	

Example 1.

The fraction $\frac{3}{4}$

can be written in words as:

three quarters

F03. Write the following fractions in words:

- a) $\frac{2}{3}$ b) $\frac{3}{100}$ c) $\frac{1}{10}$ d) $\frac{1}{2}$ e) $\frac{3}{7}$ f) $\frac{3}{20}$ g) $\frac{1}{1000}$
- h) $\frac{4}{5}$ i) $\frac{8}{30}$ j) $\frac{8}{13}$ k) $\frac{8}{9}$ l) $\frac{5}{6}$ m) $\frac{5}{8}$ n) $\frac{7}{1000}$
- o) $\frac{3}{50}$ p) $\frac{2}{5}$ q) $\frac{21}{100}$ r) $\frac{6}{12}$ s) $\frac{7}{11}$ t) $\frac{11}{50}$ u) $\frac{11}{1000000}$
- v) $\frac{2}{9}$ w) $\frac{7}{10}$ x) $\frac{11}{12}$ y) $\frac{2}{50}$ z) $\frac{9}{1000000000}$

F04. Find the fraction written in words:

- a) one third b) one half c) one sixth
- d) two fifths e) four sevenths f) seven eighths
- g) eleven fiftieths h) seven twentieths i) five twelfths
- j) eight ninths k) six tenths l) nine thousandths
- m) fifteen millionths n) eight sixths o) three fiftieths
- p) eleven billionths q) twenty-three hundredths r) seven thirteenths
- s) eleven twelfths t) three billionths u) thirteen thirtieths
- v) one fifth w) one eleventh x) eight ninths
- y) six tenths z) six twelfths

Understanding the fraction notation

1. A fraction also represents a quotient of two quantities: $\frac{\text{divident}}{\text{divisor}}$
2. *The dividend* (numerator) represents how many parts are taken.
The divisor (denominator) represents the number of equal parts into which the whole is divided.

Example 1. The dividend (numerator) is 3.
 $\frac{3}{4}$ The divisor (denominator) is 4.
 The fraction in words is *three quarters*.

A possible graphical representation of this fraction is:



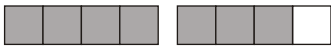
F05. Fill out the following table:

	Fraction	Numerator (Divident)	Denominator (Divisor)	The fraction written in words	Graphical representation
a)	$\frac{2}{3}$	2	3	two thirds	
b)		1	4		
c)				three fifths	
d)					
e)		3			
f)	$\frac{\square}{5}$	2			
g)	$\frac{2}{\square}$		5		
h)	$\frac{3}{\square}$		 quarters	
i)				five	
j)	$\frac{3}{\square}$				
k)		4	 sixths	
l)			5	three	

Understanding the mixed numbers

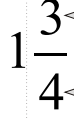
1. A mixed number is an addition of wholes and a part of a whole.

Example 1.



There are one complete whole and three quarters of the second whole

whole-number part
(the number of complete wholes)



fraction part

The *numerator* indicates how many parts are taken from the last whole.
The *denominator* represents the number of equal parts into which the whole is divided.

F06. Find the mixed number that corresponds to the shaded region:

a)

b)

c)

d)

e)

f)

g)

h)

i)

j)

k)

l)

m)

n)

o)

p)

q)

r)

s)

t)

u)

v)

w)

x)

F07. Find a possible graphical representation of each mixed number:

a) $1\frac{2}{3}$ b) $3\frac{1}{2}$ c) $2\frac{5}{6}$ d) $3\frac{4}{5}$ e) $3\frac{3}{7}$ f) $1\frac{5}{8}$ g) $4\frac{5}{11}$

h) $3\frac{1}{6}$ i) $2\frac{2}{9}$ j) $2\frac{2}{5}$ k) $2\frac{2}{10}$ l) $1\frac{3}{20}$ m) $6\frac{6}{10}$ n) $1\frac{3}{6}$

o) $3\frac{5}{8}$ p) $1\frac{7}{16}$ q) $2\frac{7}{10}$ r) $2\frac{4}{5}$ s) $5\frac{1}{1}$ t) $2\frac{5}{6}$ u) $3\frac{7}{10}$

v) $1\frac{1}{6}$ w) $5\frac{1}{2}$ x) $3\frac{7}{9}$ y) $2\frac{3}{8}$ z) $6\frac{3}{4}$

Reading and writing mixed numbers in words

1. You can use words to refer to a part of a whole.
So one whole has:

2 halves	7 sevenths	12 twelfths	100 hundredths
3 thirds	8 eighths	13 thirteenths	1000 thousandths
4 quarters	9 ninths	20 twentieths	1000000 millionths
5 fifths	10 tenths	30 thirtieths	1000000000 billionths
6 sixths	11 elevenths	50 fiftieths	

Example 1.

The fraction $2\frac{3}{4}$

can be written in words as:
two wholes and three quarters or
two and three quarters

F08. Write the following mixed numbers in words:

- a) $1\frac{1}{2}$ b) $2\frac{1}{3}$ c) $1\frac{1}{4}$ d) $2\frac{3}{5}$ e) $1\frac{5}{6}$ f) $2\frac{3}{7}$ g) $3\frac{5}{8}$
- h) $1\frac{5}{9}$ i) $2\frac{3}{10}$ j) $1\frac{2}{11}$ k) $3\frac{5}{12}$ l) $1\frac{2}{15}$ m) $3\frac{7}{20}$ n) $2\frac{9}{30}$
- o) $2\frac{7}{50}$ p) $2\frac{3}{100}$ q) $3\frac{9}{1000}$ r) $2\frac{7}{1000000}$ s) $1\frac{3}{40}$ t) $2\frac{7}{19}$ u) $3\frac{5}{16}$
- v) $2\frac{3}{17}$ w) $4\frac{3}{14}$ x) $2\frac{5}{15}$ y) $2\frac{1}{60}$ z) $2\frac{3}{90}$

F09. Find the mixed numbers written in words:

- a) two and two thirds b) three and one half c) five and five sixths
- d) two and one third e) four and five sevenths f) seven and five fiftieths
- g) two and three quarters h) three and two ninths i) six and seven hundredths
- j) nine and one half k) eight and eleven fiftieths l) one and five billionths
- m) one and two elevenths n) eight and five sixths o) three and two twelfths
- p) five and three millionths q) twenty and three hundredths r) six and four fifteenths
- s) eleven and four thirtieths t) eight and seven tenths u) four and one third
- v) one and two fifths w) three and two elevenths x) eight and six ninths
- y) five and nine tenths z) one and eleven twelfths

Understanding mixed number notation

1. A mixed number is represented by the expression:

$$\text{wholes} \frac{\text{numerator}}{\text{denominator}}$$

This mixed number written in words is *two wholes and three fifths*.

A possible graphical representation of this mixed number is:

Example 1.

$$2\frac{3}{5}$$

The whole-number part is 2
(the number of complete wholes).
The numerator is 3.
The denominator is 5. $\frac{3}{5}$
The fraction part is: $\frac{3}{5}$

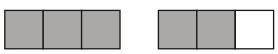


F10. Fill out the following table:



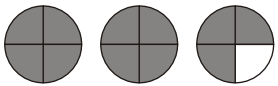


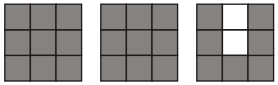
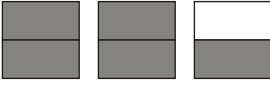
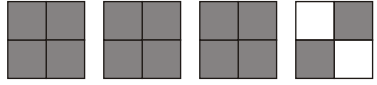


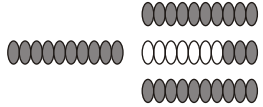


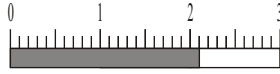

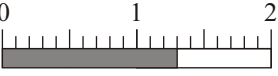
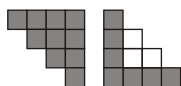

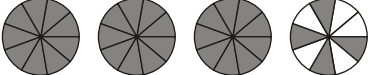
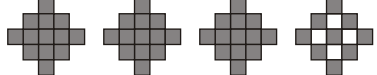

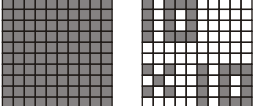

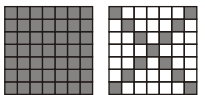
	Mixed Number	Number of wholes	Numerator	Denominator	The mixed number in words	Graphical representation
a)	$2\frac{3}{5}$	2	3	5	two and three fifths	
b)	$2\frac{1}{3}$					
c)		1	3	4		
d)						
e)	$2\frac{\square}{5}$		3			
f)	$3\frac{2}{\square}$			5		
g)	$\square\frac{2}{3}$	2				
h)					three and a half	
i)				6	two and four	
j)		3		 and three fifths	
k)			2		four and thirds	
l)		2			

Understanding improper fractions

1. For an *improper fraction* the number of parts taken (the numerator) is *equal to or greater than* the number of parts the whole is divided into (the denominator).

Example 1. $\frac{5}{3}$ This is a possible graphical representation of this improper fraction: 

F11. Find the improper fraction that corresponds to the shaded region:

a) 	b) 	c) 
d) 	e) 	f) 
g) 	h) 	i) 
j) 	k) 	l) 
m) 	n) 	o) 
p) 	q) 	r) 
s) 	t) 	u) 
v) 	w) 	x) 

F12. Find a possible graphical representation of each improper fraction:

a) $\frac{3}{2}$	b) $\frac{4}{3}$	c) $\frac{5}{2}$	d) $\frac{7}{3}$	e) $\frac{7}{5}$	f) $\frac{11}{4}$	g) $\frac{8}{6}$
h) $\frac{10}{8}$	i) $\frac{20}{9}$	j) $\frac{12}{5}$	k) $\frac{22}{10}$	l) $\frac{25}{12}$	m) $\frac{30}{16}$	n) $\frac{40}{24}$
o) $\frac{21}{6}$	p) $\frac{28}{13}$	q) $\frac{11}{3}$	r) $\frac{40}{18}$	s) $\frac{55}{49}$	t) $\frac{25}{6}$	u) $\frac{17}{9}$
v) $\frac{30}{12}$	w) $\frac{15}{4}$	x) $\frac{17}{5}$	y) $\frac{24}{10}$	z) $\frac{13}{3}$		

Understanding improper fraction notation

1. An *improper fraction* is represented by the expression:

$$\frac{\text{numerator}}{\text{denominator}}$$

where the numerator is equal to or greater than the denominator.

Example 1.

$$\frac{5}{3}$$

The numerator is 5
The denominator is 3

The improper fraction in words is *five thirds*. A possible graphical representation of this improper fraction is:

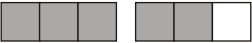


F13. Fill out the following table:

	Fraction	Numerator	Denominator	The fraction in words	Graphical representation
a)	$\frac{5}{4}$	5	4	five quarters	
b)		7	4		
c)				seven fifths	
d)					
e)		16		
f)	$\frac{\square}{5}$	12			
g)	$\frac{7}{\square}$		5		
h)	$\frac{11}{\square}$		 quarters	
i)				seven
j)	$\frac{13}{\square}$				
k)		13	 sixths	
l)			5	thirteen	

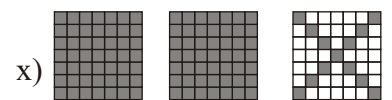
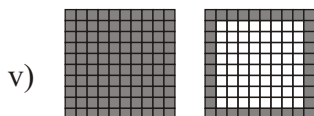
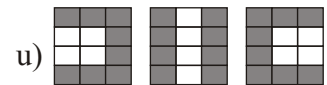
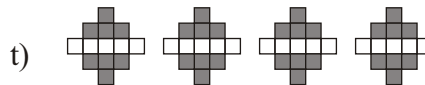
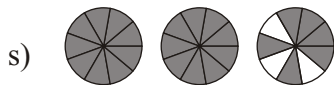
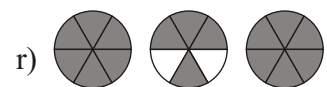
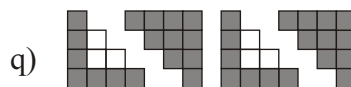
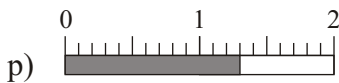
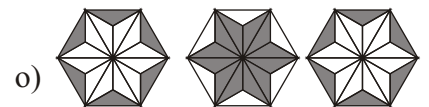
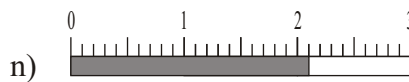
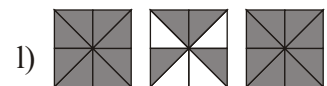
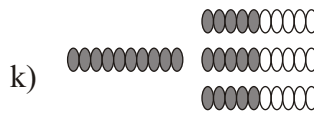
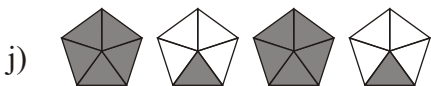
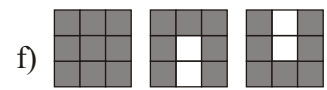
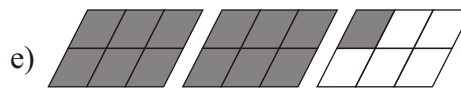
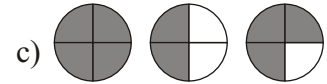
The link between mixed numbers and improper fractions

1. There is a direct link between a mixed number and an improper fraction. A mixed number is a *short way to write the sum of a whole number and a fraction*.

Example 1:  $\frac{5}{3}$ $1\frac{2}{3}$

Example 2:  $\frac{9}{6}$ $1\frac{3}{6}$

F14. Find the mixed number and the improper fraction that correspond to each picture:



Conversion between mixed numbers and improper fractions

1. To convert a mixed number to an improper fraction, use the formula:

$$w \frac{n}{d} = \frac{w \cdot d + n}{d}$$

Example 1. $2 \frac{3}{5} = \frac{2 \cdot 5 + 3}{5} = \frac{10 + 3}{5} = \frac{13}{5}$

2. Fractions that have a denominator of 0 are not defined.

3. To convert an improper fraction to a mixed number, divide the numerator n into the denominator d to obtain the quotient q and the remainder r . Then write:

$$\frac{n}{d} = q \frac{r}{d}$$

Example 2. $\frac{9}{4} = 2 \frac{1}{4}$

F15. Write each mixed number as an improper fraction:

- a) $1 \frac{1}{2}$ b) $2 \frac{2}{3}$ c) $3 \frac{3}{4}$ d) $3 \frac{1}{2}$ e) $2 \frac{3}{7}$ f) $5 \frac{3}{20}$ g) $5 \frac{3}{10}$
- h) $5 \frac{3}{4}$ i) $2 \frac{7}{30}$ j) $1 \frac{5}{13}$ k) $2 \frac{8}{9}$ l) $3 \frac{5}{6}$ m) $2 \frac{5}{8}$ n) $12 \frac{7}{100}$
- o) $2 \frac{3}{50}$ p) $4 \frac{4}{5}$ q) $2 \frac{21}{100}$ r) $2 \frac{1}{12}$ s) $3 \frac{2}{11}$ t) $2 \frac{49}{50}$ u) $2 \frac{11}{100}$
- v) $0 \frac{2}{9}$ w) $2 \frac{0}{10}$ x) $3 \frac{2}{0}$ y) $3 \frac{3}{3}$ z) $2 \frac{15}{10}$

F16. Write each improper fraction as a mixed number:

- a) $\frac{3}{2}$ b) $\frac{4}{3}$ c) $\frac{5}{4}$ d) $\frac{9}{2}$ e) $\frac{13}{7}$ f) $\frac{33}{20}$ g) $\frac{125}{10}$
- h) $\frac{14}{5}$ i) $\frac{8}{3}$ j) $\frac{80}{13}$ k) $\frac{80}{9}$ l) $\frac{50}{6}$ m) $\frac{60}{8}$ n) $\frac{17}{10}$
- o) $\frac{13}{5}$ p) $\frac{22}{5}$ q) $\frac{21}{10}$ r) $\frac{16}{12}$ s) $\frac{70}{11}$ t) $\frac{111}{50}$ u) $\frac{111}{10}$
- v) $\frac{0}{9}$ w) $\frac{7}{0}$ x) $\frac{11}{7}$ y) $\frac{20}{15}$ z) $\frac{70}{9}$

Whole numbers, proper fractions, improper fractions and mixed numbers

1. Although written in fraction notation, some numbers are actually whole numbers.

Example 1: $\frac{10}{2} = 5$ Example 2: $2\frac{3}{3} = 3$

2. A whole number can be converted into a fraction. This conversion is not unique.

Example: $5 = \frac{50}{10} = 4\frac{10}{10} = 4\frac{3}{3} = 2\frac{30}{10} = 3\frac{14}{7}$

3. A *proper fraction* is: $\frac{n}{d}$ where $n < d$ Example: $\frac{2}{3}$

An *improper fraction* is: $\frac{n}{d}$ where $n \geq d$ Example: $\frac{7}{3}$

A *mixed number* is: $w\frac{n}{d}$ Example: $4\frac{5}{3}$

A *mixed number in standard form* is: $w\frac{n}{d}$ where $n < d$ Example: $5\frac{2}{3}$

4. Fractions that have a denominator of 0 are not defined.

F17. Convert fractions to whole numbers. Identify the expressions that are not defined.

a) $\frac{1}{1}$ b) $\frac{2}{2}$ c) $\frac{7}{7}$ d) $\frac{4}{2}$ e) $\frac{9}{3}$ f) $\frac{24}{3}$ g) $\frac{100}{10}$

h) $1\frac{1}{1}$ i) $2\frac{3}{3}$ j) $2\frac{24}{12}$ k) $3\frac{9}{3}$ l) $4\frac{55}{5}$ m) $2\frac{16}{8}$ n) $2\frac{300}{100}$

o) $\frac{0}{1}$ p) $\frac{2}{0}$ q) $\frac{0}{0}$ r) $0\frac{5}{5}$ s) $2\frac{0}{9}$ t) $0\frac{0}{6}$ u) $0\frac{0}{0}$

F18. Convert whole numbers to fractions (the conversion is not unique, so give at least two solutions):

a) 1 b) 3 c) 7 d) 4 e) 2 f) 5 g) 10

h) 0 i) 25 j) 100 k) 11 l) 8 m) 13 n) 17

F19. Identify each of the following expressions as a whole number, a proper fraction, an improper fraction, a mixed number, or a not defined expression:

a) $\frac{3}{2}$ b) 1 c) $\frac{5}{4}$ d) $\frac{2}{9}$ e) $2\frac{1}{7}$ f) $\frac{33}{20}$ g) $2\frac{25}{10}$

h) $\frac{4}{0}$ i) $\frac{0}{3}$ j) 0 k) $\frac{80}{9}$ l) $\frac{5}{16}$ m) $3\frac{5}{8}$ n) $2\frac{17}{10}$

o) $2\frac{3}{3}$ p) $\frac{22}{5}$ q) 30 r) $\frac{16}{2}$ s) $\frac{70}{11}$ t) $1\frac{11}{50}$ u) $\frac{0}{10}$

v) $2\frac{3}{5}$ w) $\frac{22}{5}$ x) $\frac{10}{11}$ y) $\frac{16}{2}$ z) $\frac{7}{1}$

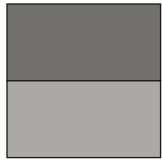
Understanding the addition of like fractions

Two fractions with the same denominators are called *like fractions*. When you add two fractions, you add the parts of the whole they represent.

Example 1.  $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$

So, by adding 1 quarter and 2 quarters you get 3 quarters.

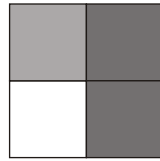
F20. Add the fractions that correspond to the shaded regions:



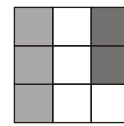
a)



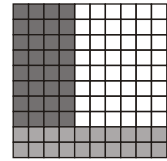
b)



c)



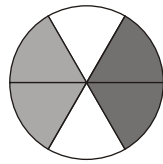
d)



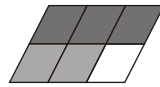
e)



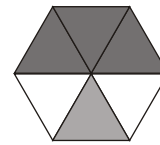
f)



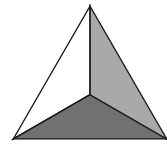
g)



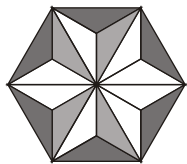
h)



i)



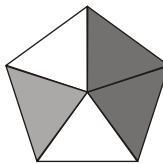
j)



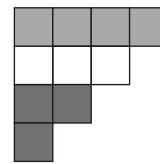
k)



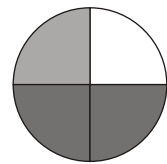
l)



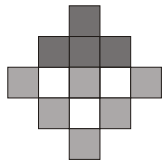
m)



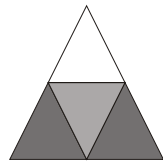
n)



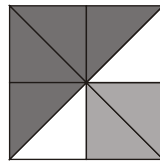
o)



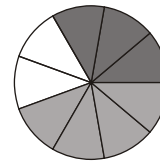
p)



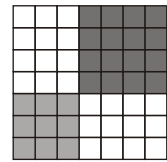
q)



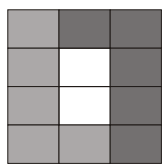
r)



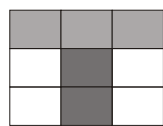
s)



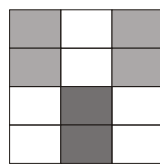
t)



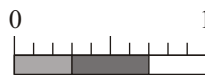
u)



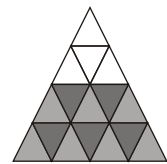
v)



w)



x)



y)

Understanding the addition of like fractions (II)

1. Sometimes when you add two like fractions, the number of parts you add exceeds a whole. The result is an improper fraction or a mixed number.



Or, in mathematical symbols: $\frac{3}{4} + \frac{3}{4} = \frac{6}{4} = 1\frac{2}{4}$

F21. Add the fractions that correspond to the shaded regions. Express the result both as an improper fraction and as a mixed number.

a)

b)

c)

d)

e)

f)

g)

h)

i)

j)

k)

l)

m)

n)

o)

p)

q)

r)

s)

t)

u)

v)

w)

x)

y)

Adding proper and improper fractions with like denominators

1. To add proper or improper fractions with like denominators (called like fractions), add the numerators and keep unchanged the denominator, according to the rule:

$$\frac{n_1}{d} + \frac{n_2}{d} = \frac{n_1 + n_2}{d}$$

Example 1. $\frac{1}{5} + \frac{2}{5} + \frac{3}{5}$

2. If the result is an improper fraction, you can change it to a mixed number.

Example 2. $\frac{3}{4} + \frac{2}{4} + \frac{5}{4} = 1\frac{1}{4}$

F22. Add the fractions:

a) $\frac{2}{4} + \frac{1}{4}$

b) $\frac{1}{3} + \frac{1}{3}$

c) $\frac{2}{5} + \frac{1}{5}$

d) $\frac{3}{11} + \frac{5}{11}$

e) $\frac{1}{6} + \frac{2}{6}$

f) $\frac{2}{7} + \frac{3}{7}$

g) $\frac{5}{19} + \frac{7}{19}$

h) $\frac{20}{100} + \frac{10}{100}$

i) $\frac{6}{35} + \frac{23}{35}$

j) $\frac{2}{41} + \frac{10}{41}$

k) $\frac{0}{13} + \frac{11}{13}$

l) $\frac{21}{54} + \frac{11}{54}$

m) $\frac{2}{10} + \frac{3}{10}$

n) $\frac{5}{12} + \frac{4}{12}$

o) $\frac{3}{17} + \frac{10}{17}$

p) $\frac{0}{4} + \frac{0}{4}$

q) $\frac{3}{13} + \frac{2}{13}$

r) $\frac{7}{20} + \frac{3}{20}$

s) $\frac{7}{25} + \frac{11}{25}$

t) $\frac{5}{9} + \frac{2}{9}$

u) $\frac{2}{19} + \frac{5}{19}$

v) $\frac{2}{10} + \frac{3}{10}$

w) $\frac{7}{30} + \frac{13}{30}$

x) $\frac{3}{13} + \frac{8}{13}$

y) $\frac{15}{100} + \frac{25}{100}$

F23. Add the fractions. Write the result as a mixed number in standard form.

a) $\frac{2}{3} + \frac{2}{3}$

b) $\frac{3}{4} + \frac{2}{4}$

c) $\frac{3}{5} + \frac{4}{5}$

d) $\frac{3}{6} + \frac{5}{6}$

e) $\frac{3}{7} + \frac{5}{7}$

f) $\frac{5}{8} + \frac{7}{8}$

g) $\frac{7}{9} + \frac{8}{9}$

h) $\frac{7}{10} + \frac{7}{10}$

i) $\frac{8}{11} + \frac{10}{11}$

j) $\frac{9}{12} + \frac{5}{12}$

k) $\frac{10}{15} + \frac{11}{15}$

l) $\frac{4}{20} + \frac{19}{20}$

m) $\frac{10}{25} + \frac{20}{25}$

n) $\frac{44}{50} + \frac{44}{50}$

o) $\frac{99}{100} + \frac{11}{100}$

p) $\frac{7}{9} + \frac{6}{9}$

q) $\frac{8}{10} + \frac{9}{10}$

r) $\frac{33}{40} + \frac{19}{40}$

s) $\frac{4}{3} + \frac{9}{3}$

t) $\frac{15}{10} + \frac{9}{10}$

u) $\frac{12}{9} + \frac{5}{9}$

v) $\frac{20}{10} + \frac{12}{10}$

w) $\frac{22}{50} + \frac{77}{50}$

x) $\frac{13}{3} + \frac{12}{3}$

y) $\frac{15}{10} + \frac{34}{10}$

Adding mixed numbers with like denominators

1. To add mixed fractions with like denominators, add separately the wholes and separately the numerators, and keep the denominator unchanged:

$$w_1 \frac{n_1}{d} + w_2 \frac{n_2}{d} = (w_1 + w_2) \frac{n_1 + n_2}{d}$$

Example 1. $2\frac{3}{5} + 3\frac{4}{5} = (2 + 3)\frac{3+4}{5} = 5\frac{7}{5} = 6\frac{2}{5}$

2. In the same way you can add whole and mixed numbers.

Example 2. $2 + 3\frac{1}{4} = 2\frac{0}{4} + 3\frac{1}{4} = (2 + 3)\frac{0+1}{4} = 5\frac{1}{4}$

F24. Add the mixed numbers. Write the result as a mixed number in standard form or as a whole number:

a) $1\frac{1}{2} + 2\frac{1}{2}$

b) $1\frac{1}{3} + 2\frac{1}{3}$

c) $1\frac{1}{4} + 2\frac{2}{4}$

d) $1\frac{1}{5} + 3\frac{3}{5}$

e) $2\frac{3}{6} + 1\frac{2}{6}$

f) $2\frac{3}{7} + 3\frac{2}{7}$

g) $2\frac{2}{8} + 3\frac{3}{8}$

h) $3\frac{2}{9} + 0\frac{1}{9}$

i) $2\frac{5}{10} + \frac{3}{10}$

j) $3\frac{5}{15} + 2\frac{6}{15}$

k) $2\frac{3}{5} + \frac{7}{5}$

l) $4\frac{4}{7} + 2\frac{6}{7}$

m) $2\frac{1}{10} + 3\frac{12}{10}$

n) $2\frac{12}{20} + 5\frac{11}{20}$

o) $3\frac{8}{11} + 3\frac{8}{11}$

p) $5\frac{2}{9} + 2\frac{8}{9}$

q) $2\frac{22}{10} + 5\frac{11}{10}$

r) $3\frac{11}{30} + 2\frac{22}{30}$

s) $3\frac{7}{3} + 2\frac{9}{3}$

t) $2\frac{15}{10} + 3\frac{25}{10}$

u) $2\frac{21}{19} + 1\frac{12}{19}$

v) $2\frac{10}{30} + 5\frac{41}{30}$

w) $3\frac{2}{23} + 1\frac{22}{23}$

x) $4\frac{33}{35} + 2\frac{22}{35}$

y) $1\frac{15}{100} + 2\frac{95}{100}$

F25. Add the wholes and the mixed numbers. Write the result as a mixed number in standard form or as a whole number.

a) $2\frac{1}{2} + 1$

b) $1 + 2\frac{2}{3}$

c) $1 + 3\frac{1}{4}$

d) $2\frac{2}{5} + 5$

e) $5 + 2\frac{5}{6}$

f) $5 + 3\frac{9}{7}$

g) $7\frac{5}{8} + 6$

h) $3 + \frac{10}{9}$

i) $1\frac{1}{10} + 11$

j) $0 + 2\frac{0}{15}$

k) $3\frac{9}{5} + 2$

l) $5 + 1\frac{8}{7}$

m) $1\frac{11}{10} + 3$

n) $2 + 2\frac{22}{20}$

o) $13 + 13\frac{13}{11}$

p) $9\frac{19}{9} + 9$

q) $20 + 10\frac{100}{10}$

r) $11 + 3\frac{11}{30}$

s) $9 + 3\frac{10}{3}$

t) $4\frac{4}{10} + 4$

u) $1 + 9\frac{19}{19}$

v) $4\frac{14}{30} + 5$

w) $2 + 2\frac{40}{23}$

x) $1 + 1\frac{37}{35}$

y) $111 + 1\frac{111}{100}$

Adding more than two like fractions

1. To add more than two fractions or whole numbers, start to add in order, from left to right.

Example 1. $1 \ 3\frac{2}{4} \ \frac{5}{4} \quad 1 \ 3\frac{2}{4} \ \frac{5}{4} \ 4\frac{2}{4} \ \frac{5}{4} \ 4\frac{7}{4} \ 5\frac{3}{4}$

2. Because the addition is a commutative operation, the order in which you add the fractions is not important. So, group them conveniently.

Example 2. $\frac{2}{3} \ 2\frac{3}{5} \ 1\frac{1}{3} \ 2 \ 1\frac{2}{5} \ 2 \ \frac{2}{3} \ 1\frac{1}{3} \ 2\frac{3}{5} \ 1\frac{2}{5} \ 2 \ 2 \ 4 \ 8$

F26. Add the fractions:

- a) $1 \ \frac{3}{2} \ 3\frac{1}{2}$ b) $\frac{1}{4} \ \frac{2}{4} \ \frac{3}{4}$ c) $1\frac{3}{4} \ \frac{5}{4} \ 2\frac{1}{4} \ 2$ d) $3\frac{1}{2} \ 2\frac{1}{2} \ 1\frac{1}{2}$ e) $\frac{1}{5} \ 1\frac{2}{5} \ 2\frac{3}{5} \ 3\frac{4}{5}$
- f) $1 \ \frac{1}{3} \ \frac{2}{3} \ \frac{3}{3}$ g) $2\frac{7}{10} \ \frac{3}{10} \ 2\frac{17}{10}$ h) $2\frac{5}{2} \ 3 \ \frac{3}{2}$ i) $2\frac{3}{50} \ 2\frac{7}{50} \ \frac{3}{50}$ j) $1\frac{1}{6} \ 2\frac{3}{6} \ 3\frac{5}{6} \ 4\frac{7}{6}$
- k) $1 \ \frac{3}{5} \ 1\frac{4}{5}$ l) $1\frac{1}{9} \ \frac{5}{9} \ 1\frac{2}{9}$ m) $1\frac{1}{3} \ 2\frac{2}{3} \ 3\frac{3}{3}$ n) $2\frac{3}{12} \ \frac{5}{12} \ 1\frac{7}{12}$ o) $\frac{1}{11} \ \frac{3}{11} \ \frac{5}{11} \ \frac{7}{11} \ \frac{9}{11}$
- p) $1 \ \frac{1}{9} \ 2\frac{7}{9}$ q) $1\frac{1}{10} \ \frac{3}{10} \ 2\frac{5}{10}$ r) $1\frac{1}{2} \ 2\frac{3}{2} \ 5\frac{5}{2}$ s) $1 \ 2\frac{21}{100} \ 3\frac{31}{100}$ t) $2\frac{5}{10} \ 1\frac{10}{10} \ \frac{15}{10} \ 3\frac{20}{10}$
- u) $0\frac{1}{9} \ \frac{4}{9} \ 1\frac{8}{9}$ v) $3\frac{1}{4} \ 2\frac{3}{4} \ 1\frac{5}{4}$ w) $3\frac{3}{3} \ 2\frac{2}{3} \ 1\frac{1}{3}$ x) $2\frac{1}{10} \ 1\frac{3}{10} \ 0\frac{5}{10}$ y) $\frac{1}{7} \ \frac{2}{7} \ \frac{3}{7} \ \frac{4}{7} \ \frac{5}{7} \ \frac{6}{7}$

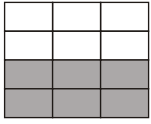
F27. Add the whole and the mixed numbers. Write the result as a mixed number in standard form or as a whole number.

- a) $1 \ \frac{1}{2} \ \frac{3}{2}$ b) $\frac{2}{4} \ 2 \ \frac{1}{4}$ c) $\frac{1}{4} \ 1 \ \frac{3}{4} \ 2$ d) $\frac{1}{2} \ \frac{1}{2} \ 2$ e) $\frac{3}{5} \ 1 \ \frac{3}{5} \ 2$
- f) $1 \ \frac{2}{3} \ 2 \ \frac{4}{3}$ g) $1 \ \frac{7}{10} \ 2$ h) $\frac{7}{2} \ 2 \ \frac{5}{2}$ i) $1\frac{7}{50} \ 3 \ \frac{11}{50}$ j) $1 \ \frac{9}{6} \ \frac{7}{6} \ 2$
- k) $1 \ 1\frac{2}{5} \ 2$ l) $2\frac{2}{9} \ 1\frac{3}{9} \ 3$ m) $2\frac{4}{3} \ 1\frac{2}{3} \ 2$ n) $1\frac{5}{12} \ 1\frac{7}{12} \ 2$ o) $1\frac{2}{11} \ 2 \ 3\frac{4}{11} \ 2 \ 1\frac{8}{11}$
- p) $2 \ 1\frac{7}{9} \ 1\frac{8}{9}$ q) $1 \ 2\frac{5}{10} \ 3\frac{7}{10}$ r) $1\frac{3}{2} \ 2\frac{5}{2} \ 5\frac{7}{2} \ 1$ s) $3\frac{11}{100} \ 2 \ 1\frac{22}{100}$ t) $1\frac{4}{10} \ 3 \ \frac{7}{10} \ 2$
- u) $0\frac{2}{9} \ 2 \ 1\frac{7}{9}$ v) $3\frac{5}{4} \ 1\frac{5}{4} \ 2$ w) $2\frac{4}{3} \ 1\frac{1}{3} \ 2$ x) $1\frac{9}{10} \ 2\frac{7}{10} \ 0$ y) $2\frac{9}{10} \ 1\frac{7}{10} \ 1 \ 3\frac{5}{10}$

Understanding equivalent fractions

1. Two fractions are considered *equivalent* if they represent the same part of the whole.
We'll see later that equivalent fractions *are equal in value* and correspond to the same decimal number.

Example 1.



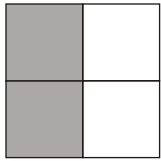
The shaded region can be expressed as:

$$\frac{1}{2} \text{ or } \frac{2}{4} \text{ or } \frac{3}{6} \text{ or } \frac{6}{12}$$

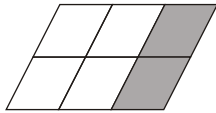
So these fractions are considered equivalent (equal), and you can write:

$$\frac{1}{2} \quad \frac{2}{4} \quad \frac{3}{6} \quad \frac{6}{12}$$

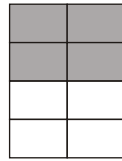
F28. For each image find the equivalent fractions that correspond to the shaded part:



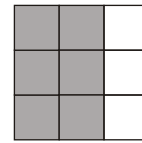
a)



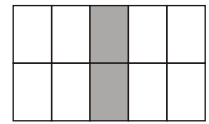
b)



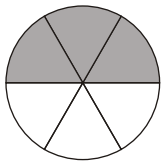
c)



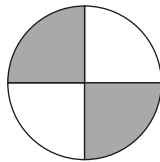
d)



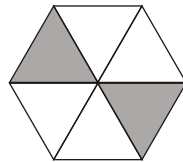
e)



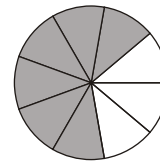
f)



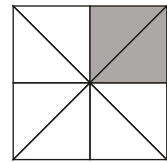
g)



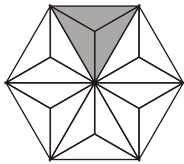
h)



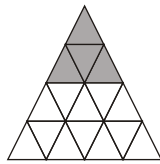
i)



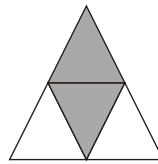
j)



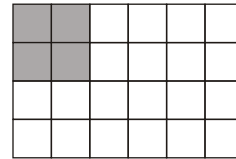
k)



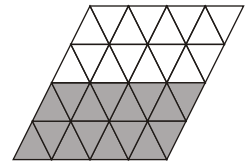
l)



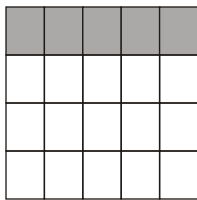
m)



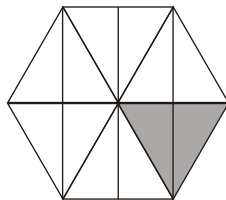
n)



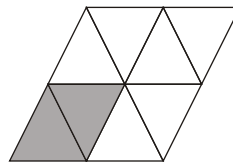
o)



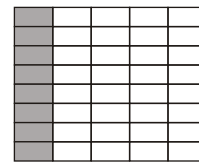
p)



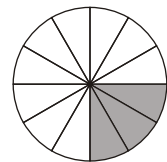
q)



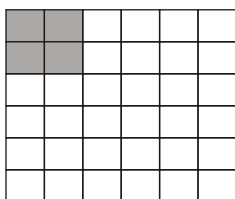
r)



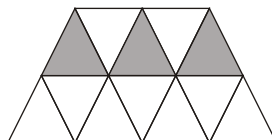
s)



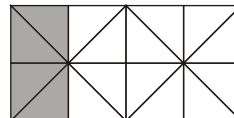
t)



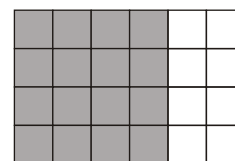
u)



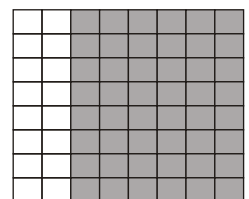
v)



w)



x)



y)

Finding equivalent fractions

There are two methods to find equivalent fractions.
 Method 1. *Multiply* both the numerator and the denominator by the same number, according to the formula:

$$\frac{n}{d} = \frac{n \cdot a}{d \cdot a}$$

Example 1.

$$\frac{1}{2} = \frac{1 \cdot 2}{2 \cdot 2} = \frac{2}{4} \quad \frac{1}{2} = \frac{1 \cdot 3}{2 \cdot 3} = \frac{3}{6} \quad \frac{1}{2} = \frac{1 \cdot 5}{2 \cdot 5} = \frac{5}{10} \quad \frac{1}{2} = \frac{1 \cdot 15}{2 \cdot 15} = \frac{15}{30}$$

So: $\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{15}{30}$

Method 2. *Divide* both the numerator and the denominator by a common factor, according to the formula:

$$\frac{n}{d} = \frac{n \cdot a}{d \cdot a}$$

Example 2.

$$\frac{6}{12} = \frac{6 \cdot 2}{12 \cdot 2} = \frac{3}{6} = \frac{3 \cdot 3}{6 \cdot 3} = \frac{1}{2}$$

So: $\frac{6}{12} = \frac{3}{6} = \frac{1}{2}$

F29. Find at least three equivalent fractions by using the method 1:

- | | | | | |
|------------------|-------------------|--------------------|--------------------|---------------------|
| a) $\frac{1}{2}$ | b) $\frac{1}{3}$ | c) $\frac{2}{3}$ | d) $\frac{1}{4}$ | e) $\frac{3}{4}$ |
| f) $\frac{1}{5}$ | g) $\frac{3}{5}$ | h) $\frac{1}{6}$ | i) $\frac{5}{6}$ | j) $\frac{1}{7}$ |
| k) $\frac{2}{7}$ | l) $\frac{1}{10}$ | m) $\frac{1}{100}$ | n) $\frac{5}{12}$ | o) $\frac{2}{11}$ |
| p) $\frac{2}{9}$ | q) $\frac{3}{7}$ | r) $\frac{5}{11}$ | s) $\frac{2}{15}$ | t) $\frac{4}{5}$ |
| u) $\frac{5}{7}$ | v) $\frac{1}{30}$ | w) $\frac{3}{50}$ | x) $\frac{3}{200}$ | y) $\frac{3}{1000}$ |

F30. Find equivalent fractions by using the method 2:

- | | | | | |
|---------------------|---------------------|---------------------|----------------------|----------------------|
| a) $\frac{2}{4}$ | b) $\frac{3}{9}$ | c) $\frac{4}{16}$ | d) $\frac{5}{25}$ | e) $\frac{6}{36}$ |
| f) $\frac{10}{100}$ | g) $\frac{4}{12}$ | h) $\frac{20}{25}$ | i) $\frac{4}{6}$ | j) $\frac{15}{20}$ |
| k) $\frac{30}{45}$ | l) $\frac{25}{40}$ | m) $\frac{18}{60}$ | n) $\frac{60}{90}$ | o) $\frac{60}{150}$ |
| p) $\frac{20}{35}$ | q) $\frac{75}{120}$ | r) $\frac{66}{154}$ | s) $\frac{56}{420}$ | t) $\frac{32}{128}$ |
| u) $\frac{27}{81}$ | v) $\frac{63}{77}$ | w) $\frac{22}{121}$ | x) $\frac{225}{625}$ | y) $\frac{64}{1024}$ |

Simplifying fractions

To *simplify* (or *reduce*) a fraction means to find the equivalent fraction having the simplest form (in lowest terms).

Method 1. You can simplify a fraction by repetitive division of the numerator and the denominator by a common factor.

$$\frac{n}{d} = \frac{n \ a}{d \ a} \quad \dots \quad \text{where } a \text{ is a common divisor of } n \text{ and } d$$

Example 1.

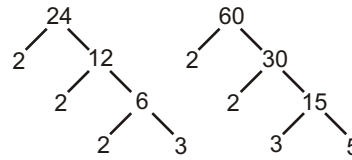
$$\frac{24}{60} = \frac{24 \ 2}{60 \ 2} = \frac{12}{30} = \frac{12 \ 2}{30 \ 2} = \frac{6}{15} = \frac{6 \ 3}{15 \ 3} = \frac{2}{5}$$

Method 2. You can simplify a fraction by division of the numerator and denominator by the Greatest Common Factor (GCF).

$$\frac{n}{d} = \frac{n \ GCF}{d \ GCF} \quad \text{fraction in lowest terms}$$

To find the GCF build the factor trees for the numerator and denominator.

Example 2.



$$24 \ 2^3 \ 3^1$$

$$60 \ 2^2 \ 3^1 \ 5^1$$

$$\text{GCF} \ 2^2 \ 3^1 \ 12$$

$$\frac{24}{60} = \frac{24 \ 12}{60 \ 12} = \frac{2}{5}$$

F31. Write each fraction in lowest terms by using the method 1:

- | | | | | |
|----------------------|----------------------|----------------------|-----------------------|----------------------|
| a) $\frac{6}{72}$ | b) $\frac{18}{42}$ | c) $\frac{50}{75}$ | d) $\frac{32}{128}$ | e) $\frac{60}{80}$ |
| f) $\frac{32}{160}$ | g) $\frac{54}{90}$ | h) $\frac{22}{132}$ | i) $\frac{200}{240}$ | j) $\frac{21}{147}$ |
| k) $\frac{36}{126}$ | l) $\frac{75}{350}$ | m) $\frac{105}{135}$ | n) $\frac{512}{4096}$ | o) $\frac{24}{132}$ |
| p) $\frac{50}{225}$ | q) $\frac{48}{112}$ | r) $\frac{60}{264}$ | s) $\frac{48}{360}$ | t) $\frac{135}{270}$ |
| u) $\frac{180}{252}$ | v) $\frac{126}{270}$ | w) $\frac{264}{600}$ | x) $\frac{45}{300}$ | y) $\frac{336}{480}$ |

F32. Write each fraction in lowest terms by using the method 2:

- | | | | | |
|--------------------|----------------------|---------------------|----------------------|----------------------|
| a) $\frac{18}{24}$ | b) $\frac{48}{60}$ | c) $\frac{54}{81}$ | d) $\frac{90}{120}$ | e) $\frac{56}{72}$ |
| f) $\frac{28}{98}$ | g) $\frac{32}{60}$ | h) $\frac{48}{120}$ | i) $\frac{28}{52}$ | j) $\frac{64}{80}$ |
| k) $\frac{30}{45}$ | l) $\frac{25}{40}$ | m) $\frac{18}{60}$ | n) $\frac{60}{75}$ | o) $\frac{60}{150}$ |
| p) $\frac{20}{45}$ | q) $\frac{75}{120}$ | r) $\frac{66}{154}$ | s) $\frac{56}{420}$ | t) $\frac{32}{128}$ |
| u) $\frac{27}{81}$ | v) $\frac{630}{770}$ | w) $\frac{22}{121}$ | x) $\frac{225}{625}$ | y) $\frac{64}{1024}$ |

Checking fractions for equivalence

Given two or more fractions, you can check whether or not they are equivalent (equal).

1. Express each fraction in lowest terms and then compare them. If you get the same fraction, the original fractions are equivalent.

Example 1. $\frac{24}{32}$ and $\frac{15}{20}$ and $\frac{21}{30}$

In lowest terms they are: $\frac{3}{4}$ and $\frac{3}{4}$ and $\frac{7}{10}$

So: $\frac{24}{32}$ $\frac{15}{20}$ $\frac{24}{32}$ $\frac{21}{30}$ $\frac{15}{20}$ $\frac{21}{30}$

2. For two fractions you can use the cross-multiplication method. If the cross-products you get are equal then the two fractions are equivalent.

$\frac{a}{b}$ and $\frac{c}{d}$ are equivalent if $a d = b c$

Example 2.

$\frac{3}{4}$ and $\frac{21}{30}$ are not equivalent because $3 \cdot 30 \neq 4 \cdot 21$

F33. Check if the fractions are equivalent (use the lowest terms method):

- | | | | | |
|---|--|---|--|---|
| a) $\frac{2}{4}; \frac{3}{6}$ | b) $\frac{2}{6}; \frac{3}{9}$ | c) $\frac{2}{3}; \frac{6}{8}$ | d) $\frac{4}{6}; \frac{8}{12}$ | e) $\frac{5}{10}; \frac{6}{18}$ |
| f) $\frac{30}{75}; \frac{48}{120}$ | g) $\frac{30}{35}; \frac{20}{24}$ | h) $\frac{24}{56}; \frac{15}{35}$ | i) $\frac{9}{25}; \frac{15}{40}$ | j) $\frac{40}{70}; \frac{30}{54}$ |
| k) $\frac{25}{80}; \frac{15}{50}$ | l) $\frac{100}{220}; \frac{30}{66}$ | m) $\frac{14}{24}; \frac{56}{96}$ | n) $\frac{15}{40}; \frac{40}{104}$ | o) $\frac{25}{45}; \frac{40}{75}$ |
| p) $\frac{10}{15}; \frac{16}{24}; \frac{20}{30}$ | q) $\frac{6}{10}; \frac{18}{30}; \frac{25}{40}$ | r) $\frac{25}{30}; \frac{5}{6}; \frac{40}{48}$ | s) $\frac{10}{21}; \frac{6}{15}; \frac{12}{35}$ | t) $\frac{60}{96}; \frac{15}{24}; \frac{25}{40}$ |
| u) $\frac{1}{2}; \frac{4}{8}; \frac{7}{14}; \frac{5}{10}$ | v) $\frac{3}{4}; \frac{8}{10}; \frac{10}{12}; \frac{12}{14}$ | w) $\frac{33}{44}; \frac{21}{28}; \frac{15}{20}; \frac{3}{4}$ | x) $\frac{1}{2}; \frac{2}{4}; \frac{4}{8}; \frac{8}{16}$ | y) $\frac{1}{2}; \frac{2}{3}; \frac{3}{4}; \frac{4}{5}$ |

F34. Check if the fractions are equivalent (use the cross-multiplication method):

- | | | | | |
|---|--|---|--|--|
| a) $\frac{1}{2}; \frac{2}{3}$ | b) $\frac{2}{4}; \frac{4}{8}$ | c) $\frac{6}{12}; \frac{4}{10}$ | d) $\frac{12}{16}; \frac{15}{20}$ | e) $\frac{12}{35}; \frac{4}{12}$ |
| f) $\frac{5}{4}; \frac{6}{5}$ | g) $\frac{9}{12}; \frac{3}{4}$ | h) $\frac{4}{10}; \frac{12}{30}$ | i) $\frac{5}{7}; \frac{7}{5}$ | j) $\frac{30}{35}; \frac{48}{56}$ |
| k) $\frac{40}{45}; \frac{72}{80}$ | l) $\frac{24}{40}; \frac{36}{60}$ | m) $\frac{40}{64}; \frac{49}{81}$ | n) $\frac{5}{20}; \frac{2}{8}$ | o) $\frac{4}{9}; \frac{9}{20}$ |
| p) $1\frac{3}{4}; \frac{9}{5}$ | q) $\frac{125}{15}; \frac{200}{24}$ | r) $\frac{11}{13}; \frac{13}{15}$ | s) $\frac{65}{85}; \frac{40}{51}$ | t) $\frac{105}{133}; \frac{15}{19}$ |
| u) $\frac{3}{4}; \frac{9}{12}; \frac{15}{20}$ | v) $\frac{4}{5}; \frac{5}{6}; \frac{6}{7}$ | w) $\frac{4}{8}; \frac{8}{12}; \frac{12}{16}$ | x) $\frac{12}{20}; \frac{18}{30}; \frac{30}{50}$ | y) $\frac{10}{14}; \frac{25}{35}; \frac{40}{56}$ |

Equations with fractions

Method 1. You can solve simple equations with fractions if you use two properties of equivalent fractions:

$$\frac{n}{d} = \frac{n \cdot a}{d \cdot a} \qquad \frac{n}{d} = \frac{n \cdot a}{d \cdot a}$$

Example 1:

$$\frac{3}{16} = \frac{?}{48}$$

Solution:

$$\frac{3}{16} = \frac{3 \cdot 3}{16 \cdot 3} = \frac{9}{48}$$

Method 2. When applying the method 1, sometimes you need first to express the fraction in the lowest terms:

Example 2:

$$\frac{4}{8} = \frac{?}{10}$$

Solution:

$$\frac{4}{8} = \frac{1}{2} = \frac{1 \cdot 5}{2 \cdot 5} = \frac{5}{10}$$

Method 3. This is the best method for solving simple equations with fractions:

To get a term of a fraction, multiply the adjacent terms and divide by the opposite term.

$$\frac{x}{a} = \frac{b}{c} \qquad x = \frac{a \cdot b}{c}$$

Example 3 (the result is a whole number):

$$\frac{x}{15} = \frac{2}{3} \qquad x = \frac{15 \cdot 2}{3} = 10$$

Example 4 (the result is a fraction):

$$\frac{x}{2} = \frac{3}{7} \qquad x = \frac{2 \cdot 3}{7} = \frac{6}{7}$$

F35. Find the unknown factor of the fraction using the method 1:

a) $\frac{1}{2} = \frac{\quad}{8}$

b) $\frac{\quad}{3} = \frac{6}{9}$

c) $\frac{2}{4} = \frac{\quad}{32}$

d) $\frac{3}{\quad} = \frac{12}{16}$

e) $\frac{3}{5} = \frac{12}{\quad}$

f) $\frac{6}{\quad} = \frac{30}{40}$

g) $\frac{12}{4} = \frac{\quad}{\quad}$

h) $2 = \frac{8}{\quad}$

i) $3 = \frac{\quad}{5}$

j) $\frac{8}{12} = \frac{40}{\quad}$

k) $\frac{5}{\quad} = \frac{25}{75}$

l) $\frac{6}{18} = \frac{18}{\quad}$

m) $\frac{3}{10} = \frac{\quad}{100}$

n) $\frac{9}{16} = \frac{\quad}{128}$

o) $\frac{11}{5} = \frac{12}{\quad}$

F36. Find the unknown factor of the fraction using the method 2:

a) $\frac{2}{4} = \frac{\quad}{6}$

b) $\frac{3}{9} = \frac{7}{\quad}$

c) $\frac{8}{12} = \frac{10}{\quad}$

d) $\frac{12}{28} = \frac{\quad}{21}$

e) $\frac{3}{\quad} = \frac{5}{10}$

f) $\frac{9}{12} = \frac{\quad}{20}$

g) $\frac{12}{15} = \frac{24}{\quad}$

h) $\frac{12}{42} = \frac{16}{\quad}$

i) $\frac{\quad}{72} = \frac{18}{48}$

j) $\frac{60}{\quad} = \frac{40}{56}$

F37. Solve for x using the method 3 (see example 3 above):

a) $\frac{1}{3} = \frac{x}{6}$

b) $\frac{2}{3} = \frac{10}{x}$

c) $\frac{3}{x} = \frac{12}{20}$

d) $\frac{4}{5} = \frac{20}{x}$

e) $\frac{3}{x} = \frac{15}{35}$

f) $5 = \frac{15}{x}$

g) $\frac{4}{x} = \frac{3}{6}$

h) $\frac{x}{16} = \frac{15}{20}$

i) $\frac{20}{x} = \frac{15}{18}$

j) $\frac{16}{x} = \frac{24}{45}$

k) $\frac{45}{72} = \frac{40}{x}$

l) $6 = \frac{x}{6}$

m) $\frac{30}{x} = \frac{80}{48}$

n) $\frac{40}{25} = \frac{x}{30}$

o) $\frac{28}{20} = \frac{x}{25}$

F38. Solve for x using the method 3 (see example 4 above):

a) $\frac{1}{2} = \frac{x}{3}$

b) $\frac{2}{5} = \frac{x}{3}$

c) $\frac{4}{3} = \frac{5}{x}$

d) $1\frac{1}{2} = \frac{5}{x}$

e) $\frac{9}{16} = \frac{x}{3}$

f) $\frac{6}{7} = \frac{x}{8}$

g) $\frac{12}{16} = \frac{x}{5}$

h) $\frac{20}{24} = \frac{3}{x}$

i) $\frac{16}{64} = \frac{x}{2}$

j) $2\frac{20}{35} = \frac{x}{15}$

Adding fractions with unlike denominators

To add fractions with different denominators, you must first replace them with equivalent fractions having the same denominators.

Method 1. This is a general method that works for two fractions and uses cross-multiplication according to the formula:

$$\frac{a}{b} + \frac{c}{d} = \frac{a \cdot d + b \cdot c}{b \cdot d}$$

Example 1:

$$\frac{2}{5} + \frac{3}{4} = \frac{2 \cdot 4 + 5 \cdot 3}{5 \cdot 4} = \frac{8 + 15}{20} = \frac{23}{20} = 1\frac{3}{20}$$

Method 2. First, express the fractions in lowest terms and then write down equivalent fractions by multiplication until you get the lowest common denominator:

Example 2:

$$\frac{2}{4} + \frac{3}{15} = \frac{1}{2} + \frac{1}{5} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$$

So: $\frac{2}{4} + \frac{3}{15} = \frac{5}{10} + \frac{2}{10} = \frac{7}{10}$

F39. Add the fractions using the method 1:

- | | | | | |
|----------------------------------|-----------------------------------|----------------------------------|-------------------------------------|-----------------------------------|
| a) $\frac{1}{2} + \frac{1}{3}$ | b) $\frac{2}{3} + \frac{3}{4}$ | c) $\frac{1}{4} + \frac{2}{5}$ | d) $\frac{1}{5} + \frac{1}{6}$ | e) $\frac{2}{5} + \frac{3}{7}$ |
| f) $\frac{3}{7} + \frac{5}{8}$ | g) $\frac{3}{8} + \frac{5}{9}$ | h) $\frac{5}{9} + \frac{3}{10}$ | i) $\frac{3}{10} + \frac{1}{11}$ | j) $\frac{1}{10} + \frac{2}{11}$ |
| k) $\frac{3}{10} + \frac{4}{15}$ | l) $\frac{10}{3} + \frac{3}{10}$ | m) $\frac{5}{9} + \frac{2}{3}$ | n) $\frac{1}{10} + \frac{2}{15}$ | o) $\frac{3}{15} + \frac{4}{20}$ |
| p) $1\frac{1}{2} + \frac{2}{3}$ | q) $\frac{2}{3} + 2\frac{4}{5}$ | r) $1\frac{5}{3} + 2\frac{3}{6}$ | s) $1\frac{10}{11} + \frac{11}{10}$ | t) $1\frac{2}{5} + 2\frac{1}{4}$ |
| u) $\frac{1}{20} + \frac{4}{16}$ | v) $1\frac{3}{40} + \frac{8}{25}$ | w) $\frac{5}{12} + 1\frac{3}{4}$ | x) $2\frac{1}{6} + 1\frac{3}{8}$ | y) $1\frac{1}{16} + \frac{3}{32}$ |

F40. Add the fractions using the method 2:

- | | | | | |
|-----------------------------------|-----------------------------------|-----------------------------------|----------------------------------|-----------------------------------|
| a) $\frac{1}{2} + \frac{2}{3}$ | b) $\frac{2}{6} + \frac{1}{4}$ | c) $\frac{3}{4} + \frac{2}{10}$ | d) $\frac{2}{5} + \frac{5}{6}$ | e) $\frac{6}{10} + \frac{1}{7}$ |
| f) $\frac{8}{14} + \frac{3}{8}$ | g) $\frac{4}{8} + \frac{3}{9}$ | h) $\frac{5}{9} + \frac{5}{10}$ | i) $\frac{2}{10} + \frac{3}{9}$ | j) $\frac{5}{10} + \frac{2}{12}$ |
| k) $\frac{2}{10} + \frac{5}{15}$ | l) $\frac{10}{3} + \frac{3}{12}$ | m) $\frac{5}{9} + \frac{2}{3}$ | n) $\frac{1}{10} + \frac{2}{15}$ | o) $\frac{3}{15} + \frac{6}{20}$ |
| p) $1\frac{1}{2} + \frac{12}{18}$ | q) $1\frac{2}{3} + 2\frac{3}{5}$ | r) $1\frac{4}{3} + 2\frac{3}{6}$ | s) $1\frac{2}{3} + \frac{8}{10}$ | t) $1\frac{3}{5} + 2\frac{3}{4}$ |
| u) $\frac{4}{20} + \frac{4}{16}$ | v) $1\frac{5}{40} + \frac{5}{25}$ | w) $\frac{4}{12} + 1\frac{4}{16}$ | x) $2\frac{2}{6} + 1\frac{6}{8}$ | y) $1\frac{5}{16} + \frac{5}{48}$ |

Adding fractions with unlike denominators using the LCD method

1. First find the Least (Lowest) Common Denominator (LCD), and then add fractions.

Example 1:

$\frac{5}{24}$	$\frac{2}{30}$?	$\frac{24}{2}$	$\frac{12}{2}$	$\frac{30}{2}$	$\frac{15}{3}$	$\frac{5}{5}$
24	2^3	3^1	2	6	2	3	120
30	2^1	3^1	5^1	120	24	5	120
<i>LCD</i>	2^3	3^1	5^1	120	120	30	4

$$\frac{5}{24} + \frac{2}{30} = \frac{5 \cdot 5}{24 \cdot 5} + \frac{2 \cdot 4}{30 \cdot 4} = \frac{25}{120} + \frac{8}{120} = \frac{33}{120} = \frac{11}{40}$$

2. You can use the LCD method to add more than two fractions.

Example 2:

$\frac{1}{12}$	$\frac{1}{45}$	$\frac{1}{40}$?	$\frac{12}{2}$	$\frac{6}{2}$	$\frac{3}{3}$	$\frac{45}{3}$	$\frac{15}{3}$	$\frac{40}{2}$	$\frac{20}{2}$	$\frac{10}{2}$	$\frac{5}{5}$
12	2^2	3^1	120	45	3^2	5^1	40	2^3	5^1	120	360	360
<i>LCD</i>	2^3	3^2	5^1	360	360	360	360	360	360	360	360	360

$$\frac{1}{12} + \frac{1}{45} + \frac{1}{40} = \frac{1 \cdot 30}{12 \cdot 30} + \frac{1 \cdot 8}{45 \cdot 8} + \frac{1 \cdot 9}{40 \cdot 9} = \frac{30}{360} + \frac{8}{360} + \frac{9}{360} = \frac{47}{360}$$

F41. Add the fractions using the LCD method (see example 1 above):

- | | | | | |
|-----------------------------------|-----------------------------------|------------------------------------|------------------------------------|------------------------------------|
| a) $\frac{1}{4} + \frac{5}{6}$ | b) $\frac{2}{6} + \frac{3}{16}$ | c) $\frac{1}{16} + \frac{3}{20}$ | d) $\frac{4}{15} + \frac{1}{12}$ | e) $\frac{3}{10} + \frac{3}{14}$ |
| f) $\frac{3}{14} + \frac{5}{4}$ | g) $\frac{3}{16} + \frac{5}{18}$ | h) $\frac{5}{12} + \frac{2}{9}$ | i) $\frac{3}{10} + \frac{1}{22}$ | j) $\frac{1}{6} + \frac{2}{33}$ |
| k) $\frac{3}{10} + \frac{4}{15}$ | l) $\frac{7}{12} + \frac{3}{20}$ | m) $\frac{5}{9} + \frac{5}{30}$ | n) $\frac{1}{10} + \frac{2}{25}$ | o) $\frac{2}{15} + \frac{4}{25}$ |
| p) $1\frac{1}{25} + \frac{7}{30}$ | q) $\frac{1}{30} + 2\frac{3}{50}$ | r) $1\frac{5}{36} + 2\frac{3}{16}$ | s) $1\frac{1}{15} + \frac{1}{50}$ | t) $1\frac{3}{50} + 2\frac{1}{45}$ |
| u) $\frac{3}{20} + \frac{3}{16}$ | v) $1\frac{7}{40} + \frac{3}{25}$ | w) $\frac{7}{12} + 1\frac{3}{40}$ | x) $2\frac{1}{14} + 1\frac{3}{84}$ | y) $1\frac{1}{48} + \frac{3}{64}$ |

F42. Add the fractions using the LCD method (see example 2 above):

- | | | | |
|--|--|---|--|
| a) $\frac{1}{2} + \frac{2}{3} + \frac{3}{4}$ | b) $\frac{1}{4} + \frac{5}{6} + \frac{3}{8}$ | c) $\frac{1}{6} + \frac{3}{8} + \frac{7}{10}$ | d) $\frac{1}{4} + \frac{5}{6} + \frac{7}{12} + \frac{3}{16}$ |
| e) $\frac{3}{4} + \frac{3}{10} + \frac{1}{12}$ | f) $\frac{3}{10} + \frac{1}{15} + \frac{1}{30}$ | g) $\frac{5}{12} + \frac{2}{9} + \frac{2}{14}$ | h) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ |
| i) $\frac{3}{10} + \frac{4}{15} + \frac{3}{16}$ | j) $\frac{1}{12} + \frac{3}{20} + \frac{2}{15}$ | k) $\frac{1}{9} + \frac{1}{30} + \frac{1}{36}$ | l) $\frac{1}{4} + \frac{1}{6} + \frac{5}{12} + \frac{2}{15} + \frac{3}{20}$ |
| m) $\frac{1}{3} + \frac{5}{6} + \frac{3}{10}$ | n) $\frac{3}{10} + \frac{1}{25} + \frac{1}{6}$ | o) $\frac{1}{15} + \frac{2}{25} + \frac{2}{35}$ | p) $\frac{1}{15} + \frac{3}{4} + \frac{1}{20} + \frac{5}{32} + \frac{3}{50}$ |
| q) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ | r) $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$ | s) $\frac{1}{5} + \frac{3}{10} + \frac{7}{15} + \frac{3}{20}$ | t) $\frac{1}{5} + \frac{3}{10} + \frac{1}{15} + \frac{3}{20} + \frac{1}{25}$ |

Understanding the subtraction of fractions with like denominators

1. When you subtract two fractions, you subtract the parts of the whole they represent.

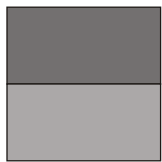
Example 1.



$$\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$$

So, by taking 2 quarters away from 3 quarters you get 1 quarter.

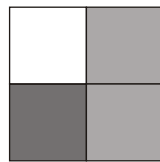
F35. Subtract the fractions that correspond to the shaded regions using the pattern: all shaded parts - all light shaded parts = all dark shaded parts



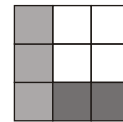
a)



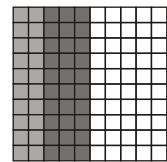
b)



c)



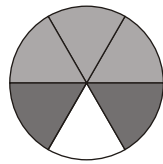
d)



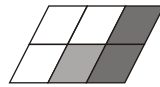
e)



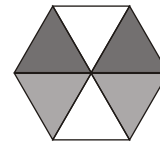
f)



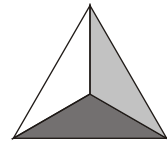
g)



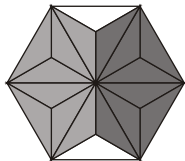
h)



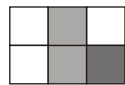
i)



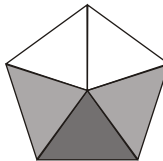
j)



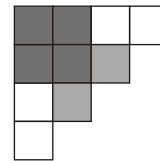
k)



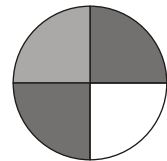
l)



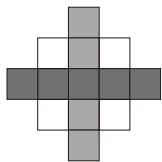
m)



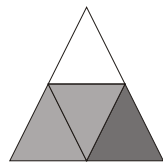
n)



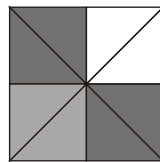
o)



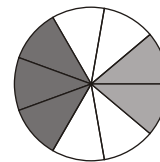
p)



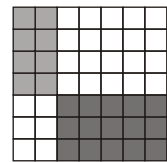
q)



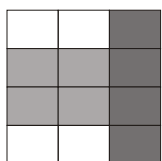
r)



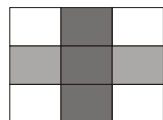
s)



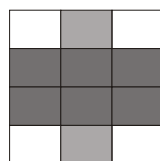
t)



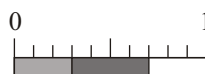
u)



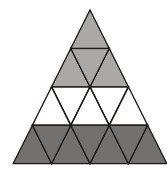
v)



w)



x)



y)

Subtracting fractions with like denominators

1. To subtract proper or improper fractions with like denominators, subtract the numerators and keep the denominators unchanged.

$$\frac{n_1}{d} - \frac{n_2}{d} = \frac{n_1 - n_2}{d}$$

Example 1. $\frac{3}{5} - \frac{2}{5} = \frac{1}{5}$

2. To subtract mixed numbers with like denominators, first change mixed numbers to improper fractions, and then subtract the numerators. Keep the denominators unchanged.

Example 2. $2\frac{1}{3} - 1\frac{2}{3} = \frac{7}{3} - \frac{5}{3} = \frac{2}{3}$

F44. Subtract the fractions (write the results in lowest terms):

- | | | | | |
|-------------------------------------|------------------------------------|------------------------------------|------------------------------------|--------------------------------------|
| a) $\frac{2}{4} - \frac{1}{4}$ | b) $\frac{4}{3} - \frac{2}{3}$ | c) $\frac{4}{5} - \frac{1}{5}$ | d) $\frac{10}{11} - \frac{3}{11}$ | e) $\frac{5}{6} - \frac{2}{6}$ |
| f) $\frac{5}{7} - \frac{2}{7}$ | g) $\frac{13}{19} - \frac{11}{19}$ | h) $\frac{19}{10} - \frac{17}{10}$ | i) $\frac{23}{35} - \frac{13}{35}$ | j) $\frac{7}{40} - \frac{3}{40}$ |
| k) $\frac{5}{12} - \frac{1}{12}$ | l) $\frac{7}{54} - \frac{1}{54}$ | m) $\frac{7}{9} - \frac{1}{9}$ | n) $\frac{11}{32} - \frac{7}{32}$ | o) $\frac{23}{7} - \frac{9}{7}$ |
| p) $\frac{3}{4} - \frac{1}{4}$ | q) $\frac{15}{16} - \frac{3}{16}$ | r) $\frac{13}{24} - \frac{7}{24}$ | s) $\frac{19}{40} - \frac{7}{40}$ | t) $\frac{5}{1000} - \frac{1}{1000}$ |
| u) $\frac{13}{100} - \frac{3}{100}$ | v) $\frac{7}{30} - \frac{1}{30}$ | w) $\frac{17}{50} - \frac{7}{50}$ | x) $\frac{13}{33} - \frac{2}{33}$ | y) $\frac{13}{60} - \frac{7}{60}$ |

F45. Subtract the mixed numbers (write the results in lowest terms):

- | | | | | |
|--|------------------------------------|-------------------------------------|--------------------------------------|--|
| a) $1\frac{1}{3} - \frac{2}{3}$ | b) $2\frac{1}{2} - \frac{3}{2}$ | c) $2\frac{4}{5} - 1\frac{3}{5}$ | d) $3\frac{1}{11} - 2\frac{3}{11}$ | e) $5\frac{5}{6} - 3\frac{2}{6}$ |
| f) $3\frac{5}{7} - 1$ | g) $2 - \frac{11}{19}$ | h) $10\frac{1}{10} - \frac{11}{10}$ | i) $2\frac{3}{35} - \frac{13}{35}$ | j) $4\frac{3}{40} - 2\frac{5}{40}$ |
| k) $2\frac{5}{12} - \frac{7}{12}$ | l) $3\frac{7}{14} - \frac{3}{14}$ | m) $5\frac{1}{9} - 2\frac{5}{9}$ | n) $2\frac{11}{32} - 1\frac{17}{32}$ | o) $3\frac{2}{7} - 2\frac{9}{7}$ |
| p) $2\frac{1}{4} - \frac{3}{4}$ | q) $5\frac{15}{16} - \frac{5}{16}$ | r) $1\frac{3}{24} - \frac{7}{24}$ | s) $3\frac{19}{40} - 1\frac{9}{40}$ | t) $2\frac{5}{1000} - 1\frac{3}{1000}$ |
| u) $3\frac{13}{100} - 2\frac{17}{100}$ | v) $5\frac{7}{30} - 2\frac{1}{30}$ | w) $3\frac{13}{50} - 2\frac{7}{50}$ | x) $3\frac{1}{33} - 2\frac{4}{33}$ | y) $5\frac{13}{60} - 2\frac{7}{60}$ |

Subtracting mixed numbers with like denominators

Method 1. To subtract mixed numbers with like denominators, subtract separately the wholes, and separately the numerators. Keep the denominator unchanged, according to the rule:

$$w_1 \frac{n_1}{d} - w_2 \frac{n_2}{d} = (w_1 - w_2) \frac{n_1 - n_2}{d}$$

Example 1:

$$3\frac{3}{5} - 1\frac{1}{5} = (3 - 1)\frac{3 - 1}{5} = 2\frac{2}{5}$$

Method 2. If the numerator of the first fraction is less than the numerator of the second fraction, you must rewrite the first fraction as an improper fraction by exchanging one whole.

Example 2:

$$3\frac{1}{3} - 1\frac{2}{3} = 2\frac{4}{3} - 1\frac{2}{3} = (2 - 1)\frac{4 - 2}{3} = 1\frac{2}{3}$$

F46. Subtract the mixed numbers using the method 1 (write the results in lowest terms).

- | | | | | |
|--|--------------------------------------|-------------------------------------|--------------------------------------|--|
| a) $2\frac{2}{3} - 1\frac{1}{3}$ | b) $5\frac{3}{2} - 3\frac{1}{2}$ | c) $10\frac{3}{5} - 2\frac{1}{5}$ | d) $11\frac{10}{11} - 5\frac{5}{11}$ | e) $4\frac{4}{6} - 2\frac{1}{6}$ |
| f) $5\frac{5}{7} - 2\frac{2}{7}$ | g) $6\frac{11}{19} - 5\frac{1}{19}$ | h) $5\frac{9}{10} - 2\frac{1}{10}$ | i) $3\frac{17}{35} - 2\frac{12}{35}$ | j) $5\frac{17}{40} - 3\frac{7}{40}$ |
| k) $4\frac{7}{12} - 1\frac{5}{12}$ | l) $5\frac{7}{14} - 3\frac{3}{14}$ | m) $3\frac{7}{9} - 2\frac{1}{9}$ | n) $10\frac{17}{32} - 7\frac{9}{32}$ | o) $4\frac{5}{7} - 1\frac{2}{7}$ |
| p) $5\frac{3}{4} - 2\frac{1}{4}$ | q) $7\frac{15}{16} - 3\frac{7}{16}$ | r) $4\frac{17}{24} - 2\frac{5}{24}$ | s) $7\frac{27}{40} - 5\frac{17}{40}$ | t) $5\frac{37}{1000} - 2\frac{17}{1000}$ |
| u) $5\frac{47}{100} - 3\frac{27}{100}$ | v) $9\frac{17}{30} - 7\frac{11}{30}$ | w) $5\frac{11}{50} - \frac{5}{50}$ | x) $2\frac{11}{33} - \frac{5}{33}$ | y) $7\frac{37}{60} - 5\frac{17}{60}$ |

F47. Subtract the mixed numbers using the method 2 (write the results in lowest terms):

- | | | | | |
|--|------------------------------------|--------------------------------------|--------------------------------------|---|
| a) $2\frac{1}{3} - 1\frac{2}{3}$ | b) $5\frac{1}{7} - 2\frac{3}{7}$ | c) $5\frac{3}{5} - 2\frac{4}{5}$ | d) $4\frac{2}{11} - 1\frac{5}{11}$ | e) $7\frac{2}{6} - 2\frac{5}{6}$ |
| f) $3\frac{1}{9} - 1\frac{5}{9}$ | g) $2 - \frac{11}{19}$ | h) $10\frac{1}{10} - \frac{11}{10}$ | i) $2\frac{3}{35} - \frac{13}{35}$ | j) $4\frac{3}{40} - 2\frac{5}{40}$ |
| k) $2\frac{5}{12} - \frac{7}{12}$ | l) $6\frac{9}{14} - 1\frac{2}{14}$ | m) $7\frac{2}{9} - 5\frac{5}{9}$ | n) $7\frac{11}{32} - 5\frac{27}{32}$ | o) $8\frac{3}{7} - 5\frac{6}{7}$ |
| p) $5 - 1\frac{3}{4}$ | q) $5\frac{7}{16} - \frac{17}{16}$ | r) $2\frac{13}{24} - 1\frac{17}{24}$ | s) $5\frac{39}{40} - 2\frac{29}{40}$ | t) $3\frac{17}{1000} - 1\frac{7}{1000}$ |
| u) $6\frac{13}{100} - 2\frac{17}{100}$ | v) $9\frac{1}{30} - 3\frac{7}{30}$ | w) $5\frac{3}{50} - \frac{17}{50}$ | x) $2\frac{11}{33} - 1\frac{23}{33}$ | y) $4\frac{7}{60} - 1\frac{15}{60}$ |

Subtracting fractions with unlike denominators

To subtract fractions with unlike denominators, first you must replace the fractions with equivalent fractions having like denominators.

Method 1. This is a general method that works for two fractions and uses cross-multiplication according to the formula:

$$\frac{a}{b} - \frac{c}{d} = \frac{a \cdot d - b \cdot c}{b \cdot d}$$

Example 1:

$$\frac{3}{4} - \frac{2}{5} = \frac{3 \cdot 5 - 4 \cdot 2}{4 \cdot 5} = \frac{15 - 8}{20} = \frac{7}{20}$$

Method 2. First, express the fractions in lowest terms and then write down equivalent fractions by multiplication until you get the lowest common denominator:

Example 2: $\frac{2}{4} - \frac{3}{15} = ?$

$$\frac{2}{4} = \frac{1}{2} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10} \dots \quad \frac{3}{15} = \frac{1}{5} = \frac{2}{10} \dots$$

So: $\frac{2}{4} - \frac{3}{15} = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$

F48. Subtract the fractions using the method 1:

a) $\frac{1}{2} - \frac{1}{3}$

b) $\frac{3}{4} - \frac{2}{3}$

c) $\frac{2}{5} - \frac{1}{4}$

d) $\frac{1}{5} - \frac{1}{6}$

e) $\frac{3}{7} - \frac{2}{5}$

f) $\frac{5}{8} - \frac{3}{7}$

g) $\frac{5}{9} - \frac{3}{8}$

h) $\frac{5}{9} - \frac{3}{10}$

i) $\frac{3}{10} - \frac{1}{11}$

j) $\frac{2}{11} - \frac{1}{10}$

k) $\frac{3}{10} - \frac{4}{15}$

l) $\frac{2}{3} - \frac{3}{10}$

m) $\frac{2}{3} - \frac{5}{9}$

n) $\frac{2}{15} - \frac{1}{10}$

o) $\frac{3}{15} - \frac{4}{20}$

p) $1\frac{1}{2} - \frac{2}{3}$

q) $2\frac{4}{5} - \frac{2}{3}$

r) $1\frac{5}{3} - 2\frac{3}{6}$

s) $1\frac{10}{11} - \frac{11}{10}$

t) $2\frac{1}{4} - 1\frac{2}{5}$

u) $\frac{4}{16} - \frac{3}{20}$

v) $1\frac{3}{40} - \frac{8}{25}$

w) $1\frac{3}{4} - \frac{5}{12}$

x) $2\frac{1}{6} - 1\frac{3}{8}$

y) $1\frac{1}{16} - \frac{3}{32}$

F49. Subtract the fractions using the method 2:

a) $\frac{2}{3} - \frac{1}{2}$

b) $\frac{2}{6} - \frac{1}{4}$

c) $\frac{3}{4} - \frac{2}{10}$

d) $\frac{5}{6} - \frac{2}{5}$

e) $\frac{6}{10} - \frac{3}{7}$

f) $\frac{8}{14} - \frac{3}{8}$

g) $\frac{4}{8} - \frac{3}{9}$

h) $\frac{5}{9} - \frac{5}{10}$

i) $\frac{3}{9} - \frac{2}{10}$

j) $\frac{5}{10} - \frac{5}{12}$

k) $\frac{5}{15} - \frac{2}{10}$

l) $\frac{2}{3} - \frac{5}{12}$

m) $\frac{2}{3} - \frac{5}{9}$

n) $\frac{3}{10} - \frac{2}{15}$

o) $\frac{6}{20} - \frac{3}{15}$

p) $1\frac{1}{2} - \frac{12}{18}$

q) $3\frac{2}{3} - 2\frac{3}{5}$

r) $2\frac{4}{3} - 1\frac{3}{6}$

s) $1\frac{2}{3} - \frac{8}{10}$

t) $3\frac{3}{5} - 2\frac{3}{4}$

u) $\frac{7}{20} - \frac{4}{16}$

v) $1\frac{5}{40} - \frac{5}{25}$

w) $2\frac{4}{12} - 1\frac{4}{16}$

x) $2\frac{2}{6} - 1\frac{6}{8}$

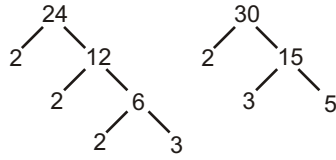
y) $1\frac{5}{16} - \frac{5}{48}$

Subtracting fractions with unlike denominators using the LCD method

1. First find the Least (Lowest) Common Denominator (LCD), then subtract fractions.

Example 1.

$$\frac{5}{24} - \frac{2}{30} = ?$$



$$24 = 2^3 \cdot 3^1$$

$$30 = 2^1 \cdot 3^1 \cdot 5^1$$

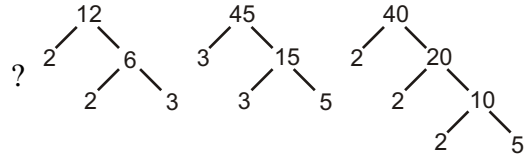
$$LCD = 2^3 \cdot 3^1 \cdot 5^1 = 120$$

$$\frac{5}{24} - \frac{2}{30} = \frac{5 \cdot 5}{24 \cdot 5} - \frac{2 \cdot 4}{30 \cdot 4} = \frac{25}{120} - \frac{8}{120} = \frac{17}{120}$$

2. You can use the LCD method to add or subtract more than two fractions.

Example 2.

$$\frac{1}{12} + \frac{1}{45} + \frac{1}{40} = ?$$



$$12 = 2^2 \cdot 3^1$$

$$LCD = 2^3 \cdot 3^2 \cdot 5^1 = 360$$

$$\frac{1}{12} + \frac{1}{45} + \frac{1}{40} = \frac{1 \cdot 30}{12 \cdot 30} + \frac{1 \cdot 8}{45 \cdot 8} + \frac{1 \cdot 9}{40 \cdot 9} = \frac{29}{360}$$

F50. Subtract the fractions using the LCD method (see the example 1 above):

a) $\frac{5}{6} - \frac{1}{4}$

b) $\frac{2}{6} - \frac{3}{16}$

c) $\frac{3}{20} - \frac{1}{16}$

d) $\frac{4}{15} - \frac{1}{12}$

e) $\frac{3}{10} - \frac{3}{14}$

f) $\frac{5}{4} - \frac{3}{14}$

g) $\frac{5}{18} - \frac{3}{16}$

h) $\frac{5}{12} - \frac{2}{9}$

i) $\frac{3}{10} - \frac{1}{22}$

j) $\frac{1}{6} - \frac{2}{33}$

k) $\frac{3}{10} - \frac{4}{15}$

l) $\frac{5}{12} - \frac{3}{20}$

m) $\frac{4}{9} - \frac{5}{30}$

n) $\frac{1}{10} - \frac{2}{25}$

o) $\frac{4}{25} - \frac{2}{15}$

p) $1\frac{1}{25} - \frac{7}{30}$

q) $2\frac{1}{30} - 1\frac{3}{50}$

r) $1\frac{5}{36} - \frac{1}{16}$

s) $2\frac{1}{15} - 2\frac{1}{50}$

t) $1\frac{3}{50} - 1\frac{2}{45}$

u) $\frac{5}{20} - \frac{3}{16}$

v) $1\frac{3}{40} - \frac{3}{25}$

w) $\frac{1}{12} - \frac{3}{40}$

x) $2\frac{1}{14} - 1\frac{3}{84}$

y) $1\frac{1}{48} - \frac{3}{64}$

F51. Add and subtract the fractions using the LCD method (see the example 2 above):

a) $\frac{1}{2} + \frac{2}{3} + \frac{1}{4}$

b) $\frac{3}{4} + \frac{1}{6} + \frac{3}{8}$

c) $\frac{5}{6} + \frac{3}{8} + \frac{3}{10}$

d) $\frac{3}{4} + \frac{1}{6} + \frac{7}{12} + \frac{3}{16}$

e) $\frac{3}{4} + \frac{3}{10} + \frac{1}{12}$

f) $\frac{3}{10} + \frac{1}{15} + \frac{1}{30}$

g) $\frac{1}{12} + \frac{1}{9} + \frac{1}{14}$

h) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

i) $\frac{3}{10} + \frac{2}{15} + \frac{3}{16}$

j) $\frac{5}{12} + \frac{3}{20} + \frac{2}{15}$

k) $\frac{1}{9} + \frac{1}{30} + \frac{1}{36}$

l) $\frac{1}{4} + \frac{1}{6} + \frac{5}{12} + \frac{2}{15} + \frac{3}{20}$

m) $\frac{2}{3} + \frac{1}{6} + \frac{3}{10}$

n) $\frac{3}{10} + \frac{2}{25} + \frac{1}{6}$

o) $\frac{2}{15} + \frac{3}{25} + \frac{1}{35}$

p) $\frac{13}{15} + \frac{3}{4} + \frac{1}{20} + \frac{1}{32} + \frac{7}{480}$

q) $2\frac{1}{2} + \frac{1}{3} + 1\frac{1}{4} + \frac{4}{5}$

r) $\frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

s) $\frac{2}{5} + \frac{3}{10} + \frac{7}{15} + \frac{3}{20}$

t) $\frac{2}{5} + \frac{3}{10} + \frac{1}{15} + \frac{3}{20} + \frac{2}{25}$

Order of operations (I)

1. Additions and subtractions are operations of the same priority, so the order in which they are done is not important.

By convention, these operations are done one by one from left to right.

Example 1:

$$\frac{3}{2} \frac{1}{3} \frac{1}{4} \frac{3}{2} \frac{1}{3} \frac{1}{4} \frac{7}{6} \frac{1}{4} 1 \frac{5}{12}$$

2. To change the order of operations, use: parentheses (), brackets [] or braces { }. An inner bracket has a greater priority.

Example 2:

$$\frac{3}{2} \frac{1}{3} \frac{1}{4} \frac{3}{2} \frac{7}{12} \frac{11}{12}$$

3. By convention the order in which the brackets appear is $\{ [()] \}$.

Example 3:

$$\frac{3}{2} \frac{4}{3} 1 \frac{1}{3} \frac{1}{4} \frac{3}{2} \frac{4}{3} 1 \frac{1}{12}$$

$$\frac{3}{2} \frac{4}{3} \frac{11}{12} \frac{3}{2} \frac{5}{12} 1 \frac{11}{12}$$

4. One single type of brackets is enough to change the order of operations.

Example 4:

$$\frac{1}{2} 1 \frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{2} 1 \frac{1}{2} \frac{1}{12}$$

$$\frac{1}{2} 1 \frac{5}{12} \frac{1}{2} \frac{7}{12} 1 \frac{1}{12}$$

F52. Solve each exercise by following the proper order of operations:

a) $\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5}$

b) $\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5}$

c) $1 2 \frac{1}{2} 1 \frac{1}{3} 4 \frac{1}{4} 3 \frac{1}{5} 5$

d) $\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{1}{6}$

e) $\frac{7}{5} 1 \frac{3}{5}$

f) $2 \frac{3}{4} 1 \frac{2}{4} \frac{3}{4}$

g) $2 \frac{2}{3} \frac{1}{3} 1 \frac{1}{3}$

h) $\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{1}{6}$

i) $1 \frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5} \frac{1}{6}$

j) $1 \frac{1}{2} \frac{2}{3} \frac{1}{2} \frac{5}{6} \frac{1}{2}$

k) $\frac{1}{2} \frac{1}{3} \frac{1}{4} \frac{1}{5}$

l) $2 1 \frac{1}{2} \frac{1}{3} \frac{1}{4}$

m) $8 \frac{3}{7} 2 \frac{4}{7} 5 3 \frac{3}{4} 1 \frac{5}{7}$

n) $1 2 \frac{1}{2} 1 \frac{1}{3} \frac{1}{4} 1 \frac{1}{5} 1$

o) $\frac{3}{5} \frac{7}{10} (\frac{1}{2} \frac{1}{5}) \frac{1}{2} \frac{1}{10} \frac{4}{5} \frac{3}{4}$

p) $2 \frac{2}{9} \frac{2}{3} \frac{5}{6} \frac{2}{3} \frac{5}{9} \frac{5}{12} \frac{1}{6}$

q) $3 2 1 \frac{3}{8} \frac{1}{2} \frac{7}{10} \frac{3}{5} \frac{1}{10} \frac{3}{4}$

r) $\frac{3}{5} \frac{8}{25} \frac{3}{20} 1 \frac{3}{5} \frac{1}{4} \frac{7}{50}$

s) $1 \frac{1}{2} \frac{4}{3} \frac{3}{4} \frac{4}{5} \frac{3}{4} \frac{2}{3} \frac{1}{2} \frac{1}{5} 1 \frac{1}{2} \frac{2}{3} \frac{1}{4}$

t) $2 \frac{1}{2} 1 \frac{2}{3} \frac{1}{2} 1 \frac{1}{3} 1 \frac{2}{5} \frac{3}{5} \frac{1}{2} 1 1 \frac{2}{5} \frac{3}{10}$

u) $1 \frac{1}{2} \frac{1}{2} \frac{1}{3} \frac{1}{3} \frac{1}{4} \frac{1}{4} \frac{1}{5} \frac{1}{2} \frac{1}{3} \frac{1}{3} \frac{1}{4} \frac{1}{4} \frac{1}{5} \frac{1}{5} \frac{1}{6}$

Multiplying fractions

1. To multiply two proper or improper fractions, you multiply the numerators first and then the denominators, according to the rule:

$$\frac{n_1}{d_1} \cdot \frac{n_2}{d_2} = \frac{n_1 \cdot n_2}{d_1 \cdot d_2}$$

Example 1:

$$\frac{2}{3} \cdot \frac{4}{5} = \frac{2 \cdot 4}{3 \cdot 5} = \frac{8}{15}$$

2. To multiply mixed numbers, first convert them to improper fractions.

Example 2:

$$1\frac{1}{2} \cdot \frac{5}{4} = \frac{3}{2} \cdot \frac{5}{4} = \frac{3 \cdot 5}{2 \cdot 4} = \frac{15}{8} = 1\frac{7}{8}$$

3. To multiply a whole and a fraction, rewrite the whole as a fraction.

Example 3:

$$2 \cdot \frac{4}{5} = \frac{2}{1} \cdot \frac{4}{5} = \frac{2 \cdot 4}{1 \cdot 5} = \frac{8}{5} = 1\frac{3}{5}$$

F53. Multiply the proper or improper fractions (write the results in lowest terms):

a) $\frac{1}{2} \cdot \frac{3}{4}$

b) $\frac{1}{2} \cdot \frac{2}{3}$

c) $\frac{1}{4} \cdot \frac{3}{5}$

d) $\frac{2}{5} \cdot \frac{15}{4}$

e) $\frac{3}{2} \cdot \frac{1}{4}$

f) $\frac{4}{3} \cdot \frac{3}{2}$

g) $\frac{1}{2} \cdot \frac{2}{4}$

h) $\frac{1}{2} \cdot \frac{1}{4}$

i) $\frac{3}{4} \cdot \frac{5}{6}$

j) $\frac{1}{10} \cdot \frac{5}{20}$

k) $\frac{1}{2} \cdot \frac{8}{9}$

l) $\frac{3}{6} \cdot \frac{12}{15}$

m) $\frac{5}{11} \cdot \frac{22}{5}$

n) $\frac{4}{7} \cdot \frac{21}{5}$

o) $\frac{3}{4} \cdot \frac{2}{6}$

F54. Multiply the mixed numbers or fractions (write the results in lowest terms):

a) $1\frac{1}{2} \cdot \frac{1}{2}$

b) $\frac{1}{2} \cdot 2\frac{1}{3}$

c) $1\frac{1}{5} \cdot \frac{5}{3}$

d) $2\frac{2}{5} \cdot \frac{1}{3}$

e) $\frac{2}{3} \cdot 1\frac{3}{4}$

f) $1\frac{1}{5} \cdot 1\frac{3}{12}$

g) $1\frac{1}{2} \cdot 2\frac{1}{2}$

h) $1\frac{2}{3} \cdot 2\frac{2}{5}$

i) $1\frac{1}{4} \cdot 2\frac{3}{2}$

j) $2\frac{2}{11} \cdot 1\frac{1}{12}$

k) $1\frac{1}{2} \cdot 2\frac{2}{3}$

l) $1\frac{5}{6} \cdot 2\frac{2}{5}$

m) $1\frac{1}{10} \cdot \frac{5}{11}$

n) $1\frac{3}{7} \cdot \frac{14}{5}$

o) $2\frac{1}{4} \cdot 1\frac{2}{6}$

p) $1\frac{1}{2} \cdot 2\frac{2}{9}$

q) $2\frac{1}{6} \cdot \frac{3}{13}$

r) $1\frac{4}{11} \cdot 4\frac{2}{5}$

s) $1\frac{3}{7} \cdot \frac{14}{5}$

t) $2\frac{1}{4} \cdot 1\frac{5}{3}$

F55. Multiply the wholes and fractions (write the results in lowest terms):

a) $3 \cdot \frac{1}{4}$

b) $\frac{1}{2} \cdot 2$

c) $2 \cdot \frac{2}{5}$

d) $\frac{2}{5} \cdot 5$

e) $3 \cdot \frac{5}{4}$

f) $5 \cdot \frac{3}{15}$

g) $\frac{1}{2} \cdot 6$

h) $3 \cdot \frac{2}{9}$

i) $\frac{3}{4} \cdot 6$

j) $15 \cdot \frac{4}{20}$

k) $1\frac{1}{2} \cdot 3$

l) $5 \cdot 2\frac{2}{15}$

m) $3 \cdot 2\frac{5}{6}$

n) $1\frac{4}{7} \cdot 14$

o) $2 \cdot 1\frac{1}{6}$

More about multiplying fractions

1. Although the most common multiplication operator is “x”, there are two other acceptable operators: “of” and “ ”

Example 1:

$$\frac{1}{2} \text{ of } \frac{3}{5} \quad \frac{1}{2} \frac{3}{5} \quad \frac{1}{2} \frac{3}{5} \quad \frac{3}{10}$$

Example 2:

$$\frac{1}{2} \frac{3}{5} \quad \frac{1}{2} \frac{3}{5} \quad \frac{3}{10}$$

2. When multiplying fractions, first try to cancel out the common factors between numerators and denominators.

Example 3:

$$\frac{1}{\cancel{2}} \frac{\cancel{2}}{5} \quad \frac{1}{1} \frac{2}{5} \quad \frac{2}{5}$$

$$\frac{\cancel{2}}{\cancel{3}} \frac{\cancel{15}}{\cancel{10}} \quad \frac{2}{3} \frac{1}{1} \quad \frac{2}{3}$$

3. This process of cancellation can be applied more than one time.

Example 4:

F56. Multiply the fractions (write the results in lowest terms):

- | | | | | |
|-----------------------------------|-----------------------------------|-----------------------------------|------------------------------------|-------------------------------------|
| a) $\frac{2}{3}$ of $\frac{1}{2}$ | b) $\frac{2}{3}$ of $\frac{3}{4}$ | c) $\frac{1}{5}$ of $\frac{1}{5}$ | d) $\frac{2}{3}$ of $\frac{4}{5}$ | e) $\frac{1}{5}$ of $\frac{1}{3}$ |
| f) $\frac{5}{2}$ of $\frac{3}{4}$ | g) $\frac{3}{2}$ of $\frac{3}{2}$ | h) $\frac{4}{5}$ of $\frac{5}{4}$ | i) $\frac{1}{2}$ of $\frac{2}{4}$ | j) $\frac{2}{3}$ of $\frac{9}{6}$ |
| k) $\frac{2}{3}$ of 90 | l) 10 of $\frac{1}{2}$ | m) 2 of 3 | n) $1\frac{1}{3}$ of $\frac{1}{4}$ | o) $2\frac{1}{5}$ of $1\frac{1}{2}$ |

F57. Multiply the fractions (write the results in lowest terms):

- | | | | | |
|-------------------------------|-------------------------------|---------------------------------|----------------------------------|--------------------------------|
| a) $\frac{1}{2} \frac{2}{3}$ | b) $\frac{2}{3} \frac{3}{4}$ | c) $\frac{3}{5} \frac{5}{3}$ | d) $1\frac{1}{2} \frac{2}{5}$ | e) $1\frac{1}{2} 2\frac{1}{3}$ |
| f) $\frac{3}{7} \frac{14}{6}$ | g) $\frac{1}{10} \frac{5}{2}$ | h) $\frac{4}{5} \frac{15}{20}$ | i) $\frac{3}{5} \frac{12}{9}$ | j) $\frac{1}{4} \frac{1}{5}$ |
| k) $\frac{1}{2} \frac{2}{1}$ | l) $\frac{1}{2} \frac{1}{2}$ | m) $\frac{5}{12} \frac{18}{15}$ | n) $\frac{10}{12} \frac{22}{30}$ | o) $\frac{6}{21} 30$ |

F58. Cancel out the common factors and then multiply the fractions:

- | | | | | |
|--------------------------------|---------------------------------|--------------------------------|----------------------------------|----------------------------------|
| a) $\frac{1}{5} \frac{15}{4}$ | b) $\frac{2}{3} \frac{3}{2}$ | c) $\frac{2}{5} \frac{10}{4}$ | d) $\frac{3}{7} \frac{14}{9}$ | e) $\frac{5}{3} \frac{6}{10}$ |
| f) $5 \frac{3}{15}$ | g) $\frac{2}{6} 8$ | h) $\frac{3}{4} \frac{12}{9}$ | i) $\frac{33}{4} \frac{40}{44}$ | j) $\frac{4}{22} \frac{121}{20}$ |
| k) $1\frac{1}{2} \frac{2}{3}$ | l) $1\frac{2}{3} 1\frac{6}{15}$ | m) $\frac{15}{7} 2\frac{4}{5}$ | n) $1\frac{1}{7} 21$ | o) $1\frac{1}{2} 2\frac{2}{3}$ |
| p) $1\frac{1}{2} 2\frac{2}{9}$ | q) $2\frac{1}{6} \frac{3}{13}$ | r) $1\frac{4}{10} \frac{5}{8}$ | s) $1\frac{3}{20} \frac{25}{23}$ | t) $2\frac{1}{40} 2\frac{4}{18}$ |

The order of operations (II)

1. Because multiplication is a *commutative operation*, the order in which you multiply them is not important. By convention, the order is from left to right.

Example 1:

$$\frac{2}{3} \cdot \frac{4}{5} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{1}{2} \cdot \frac{8}{15} \cdot \frac{1}{2} \cdot \frac{4}{15}$$

2. Do not forget to cancel out the common factors, before multiplying the fractions.

Example 2:

$$\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{2}{3} \cdot \frac{4}{5} \cdot \frac{1}{2} \cdot \frac{8}{15} \cdot \frac{1}{2} \cdot \frac{4}{15}$$

3. If the expression contains addition, subtraction and multiplication, do the multiplication first.

Example 3:

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{1}{2} \cdot \frac{5}{8} \cdot \frac{9}{8} \cdot 1\frac{1}{8}$$

4. If the expression contains brackets, replace the brackets with the result of the operation(s) inside the brackets.

Example 4:

$$\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{5}{4} \cdot \frac{5}{6} \cdot \frac{25}{24} \cdot 1\frac{1}{24}$$

F59. Multiply the fractions (write the results in lowest terms):

a) $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$ b) $\frac{1}{3} \cdot \frac{2}{5} \cdot \frac{3}{2}$ c) $\frac{4}{9} \cdot \frac{3}{8} \cdot \frac{3}{4}$ d) $\frac{1}{3} \cdot 12 \cdot \frac{3}{4}$ e) $\frac{2}{5} \cdot \frac{1}{3} \cdot \frac{2}{4}$

f) $1\frac{1}{2} \cdot \frac{2}{3} \cdot 2\frac{3}{4}$ g) $2\frac{1}{2} \cdot \frac{2}{5} \cdot 1\frac{3}{4}$ h) $2\frac{1}{2} \cdot 1\frac{2}{3} \cdot \frac{3}{5}$ i) $2\frac{1}{4} \cdot 2 \cdot \frac{2}{9}$ j) $1\frac{1}{5} \cdot \frac{2}{3} \cdot \frac{15}{8}$

k) $\frac{1}{2} \cdot \frac{4}{5} \cdot \frac{10}{16} \cdot \frac{15}{8}$ l) $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{5}{6}$ m) $\frac{1}{2} \cdot \frac{4}{6} \cdot \frac{8}{10} \cdot \frac{10}{12} \cdot \frac{14}{16}$ n) $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{5}{6} \cdot \frac{7}{8} \cdot \frac{9}{10}$

F60. Find the value of each expression (write the results in lowest terms):

a) $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{4}{5}$ b) $\frac{1}{4} \cdot \frac{2}{3} \cdot \frac{5}{6}$ c) $\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3}$ d) $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{6}$ e) $\frac{4}{5} \cdot \frac{1}{3} \cdot \frac{2}{4} \cdot \frac{1}{10}$

f) $\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{2}$ g) $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{1}{3}$ h) $\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{2}{3}$ i) $\frac{3}{5} \cdot \frac{15}{9} \cdot \frac{5}{6} \cdot \frac{2}{3}$ j) $\frac{2}{3} \cdot \frac{3}{4} \cdot \frac{3}{5} \cdot \frac{5}{6}$

k) $\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{3}{8}$ l) $\frac{1}{2} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{2}{3}$ m) $\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{3}{4}$ n) $1\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{6}{5} \cdot 1\frac{2}{3}$ o) $1 \cdot \frac{2}{7} \cdot 1\frac{3}{4} \cdot 1\frac{3}{5} \cdot \frac{15}{8}$

F61. Find the value of each expression (write the results in lowest terms):

a) $\frac{1}{2} \cdot \frac{1}{3} \cdot \frac{4}{5}$ b) $\frac{1}{5} \cdot \frac{2}{3} \cdot \frac{1}{6}$ c) $\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{3}{5}$ d) $\frac{3}{2} \cdot \frac{1}{2} \cdot \frac{1}{3}$ e) $\frac{3}{4} \cdot \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{2}{3}$

f) $\frac{1}{3} \cdot \frac{1}{2} \cdot \frac{3}{5} \cdot \frac{1}{2}$ g) $\frac{1}{2} \cdot \frac{1}{7} \cdot \frac{2}{3} \cdot \frac{1}{2}$ h) $\frac{2}{3} \cdot 1\frac{1}{2} \cdot \frac{1}{6} \cdot \frac{2}{3}$ i) $\frac{4}{7} \cdot \frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5}$ j) $\frac{2}{5} \cdot \frac{1}{2} \cdot \frac{1}{3} \cdot \frac{6}{7}$

k) $1\frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{2}{5}$ l) $\frac{3}{5} \cdot \frac{3}{4} \cdot \frac{2}{3} \cdot \frac{1}{3}$ m) $\frac{2}{3} \cdot \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$ n) $\frac{5}{11} \cdot \frac{2}{3} \cdot \frac{6}{5} \cdot \frac{2}{3}$ o) $1 \cdot \frac{2}{5} \cdot \frac{1}{2} \cdot \frac{3}{3} \cdot 1\frac{1}{9}$

p) $1\frac{1}{3} \cdot \frac{1}{4} \cdot 1\frac{1}{2} \cdot \frac{2}{3} \cdot \frac{3}{4}$ q) $\frac{3}{2} \cdot 1\frac{2}{2} \cdot 1\frac{1}{3} \cdot \frac{1}{4} \cdot \frac{1}{5} \cdot 3\frac{1}{3} \cdot \frac{5}{11} \cdot \frac{3}{5} \cdot \frac{1}{2} \cdot \frac{1}{4} \cdot \frac{2}{7}$

Reciprocal of a fraction

1. The *reciprocal* of a fraction is the fraction obtained by interchanging its numerator and denominator.

So, the reciprocal of

$$\frac{n}{d} \text{ is } \frac{d}{n}$$

Example 1:

The reciprocal of $\frac{3}{4}$ is $\frac{4}{3}$

2. To find the reciprocal of a mixed number, express it as an improper fraction and then interchange the numerator and the denominator (invert the fraction).

Example 2. The reciprocal of $1\frac{3}{4}$ is $\frac{4}{7}$

3. To find the reciprocal of a whole number, express it as an improper fraction and then interchange (invert) the numerator and the denominator.

Example 3. The reciprocal of $3\frac{3}{1}$ is $\frac{1}{3}$

4. If you multiply a fraction by its reciprocal, the product is always 1. Thus, the reciprocal of a fraction is called also the *multiplication inverse* of that fraction.

Example 4:

$$\frac{3}{4} \cdot \frac{4}{3} = 1$$

F62. Find the reciprocal of each fraction:

a) $\frac{1}{2}$

b) $\frac{2}{3}$

c) $\frac{5}{4}$

d) $\frac{7}{3}$

e) $\frac{0}{10}$

f) $\frac{7}{6}$

g) $\frac{13}{3}$

h) $\frac{2}{4}$

i) $\frac{3}{5}$

j) $\frac{3}{0}$

F63. Find the reciprocal of each mixed number:

a) $1\frac{1}{2}$

b) $2\frac{2}{3}$

c) $2\frac{2}{5}$

d) $1\frac{1}{7}$

e) $3\frac{2}{3}$

f) $3\frac{3}{3}$

g) $2\frac{2}{11}$

h) $2\frac{5}{7}$

i) $1\frac{1}{4}$

j) $3\frac{3}{4}$

F64. Find the reciprocal (multiplication inverse) of each whole number:

a) 1

b) 2

c) 5

d) 100

e) 0

F65. Check if the pair of fractions are reciprocal:

a) $\frac{1}{2}$ and $\frac{2}{1}$

b) $1\frac{1}{2}$ and $\frac{2}{4}$

c) $\frac{7}{3}$ and $2\frac{1}{7}$

d) $\frac{1}{3}$ and 3

e) $\frac{3}{5}$ and $2\frac{2}{5}$

f) $3\frac{1}{3}$ and $\frac{3}{11}$

g) 10 and $\frac{1}{10}$

h) $\frac{3}{5}$ and $\frac{5}{2}$

i) $\frac{3}{9}$ and $\frac{6}{2}$

j) $1\frac{3}{5}$ and $\frac{15}{24}$

F66. Find the unknown fraction (f):

a) $f \cdot \frac{1}{4} = 1$

b) $\frac{1}{2} \cdot f = 1$

c) $f \cdot \frac{2}{5} = 1$

d) $f \cdot 5 = 1$

e) $f \cdot 1\frac{5}{4} = 1$

f) $f \cdot \frac{2}{7} = 1$

g) $7 \cdot f = 1$

h) $f \cdot 1\frac{2}{9} = 1$

i) $1\frac{3}{4} \cdot f = 1$

j) $f \cdot 2\frac{4}{5} = 1$

Dividing fractions

1. To divide two proper or improper fractions, change the divisor to its reciprocal and then multiply, according to the rule:

$$\frac{n_1}{d_1} \div \frac{n_2}{d_2} = \frac{n_1}{d_1} \cdot \frac{d_2}{n_2}$$

In short, invert the divisor and multiply.

Example 1:

$$2 \frac{4}{3} \div \frac{2}{5} = 2 \frac{4}{3} \cdot \frac{5}{2} = 2 \frac{5}{3} = 3 \frac{2}{3}$$

2. To divide mixed numbers, first change them to improper fractions.

Example 2:

$$2 \frac{1}{2} \div 1 \frac{2}{3} = \frac{5}{2} \div \frac{5}{3} = \frac{5}{2} \cdot \frac{3}{5} = \frac{3}{2} = 1 \frac{1}{2}$$

3. To divide a whole number and a fraction, rewrite the whole as a fraction.

Example 3:

$$2 \div \frac{4}{5} = \frac{2}{1} \div \frac{4}{5} = \frac{2}{1} \cdot \frac{5}{4} = \frac{5}{2} = 2 \frac{1}{2}$$

F67. Divide the proper or improper fractions (write the results in lowest terms):

a) $\frac{1}{2} \div \frac{3}{4}$

b) $\frac{1}{2} \div \frac{1}{3}$

c) $\frac{2}{3} \div \frac{4}{9}$

d) $\frac{2}{5} \div \frac{8}{15}$

e) $\frac{3}{2} \div \frac{6}{5}$

f) $\frac{4}{3} \div \frac{12}{5}$

g) $\frac{5}{2} \div \frac{3}{4}$

h) $\frac{1}{2} \div \frac{1}{4}$

i) $\frac{10}{4} \div \frac{5}{6}$

j) $\frac{1}{10} \div \frac{3}{20}$

k) $\frac{1}{3} \div \frac{8}{9}$

l) $\frac{3}{6} \div \frac{12}{15}$

m) $\frac{5}{11} \div \frac{25}{22}$

n) $\frac{4}{7} \div \frac{16}{21}$

o) $\frac{3}{4} \div \frac{6}{16}$

F68. Divide the mixed numbers or fractions (write the results in lowest terms):

a) $2 \frac{1}{2} \div \frac{1}{2}$

b) $1 \frac{1}{2} \div 2 \frac{1}{4}$

c) $1 \frac{1}{5} \div 1 \frac{1}{15}$

d) $2 \frac{2}{5} \div \frac{3}{5}$

e) $\frac{2}{3} \div 1 \frac{1}{6}$

f) $1 \frac{1}{5} \div 1 \frac{3}{15}$

g) $1 \frac{1}{2} \div 2 \frac{1}{4}$

h) $1 \frac{2}{3} \div 1 \frac{1}{9}$

i) $1 \frac{1}{4} \div 6 \frac{3}{2}$

j) $2 \frac{3}{11} \div 1 \frac{3}{22}$

k) $1 \frac{1}{2} \div 2 \frac{2}{8}$

l) $1 \frac{5}{6} \div 3 \frac{2}{3}$

m) $1 \frac{2}{10} \div \frac{3}{25}$

n) $1 \frac{3}{7} \div \frac{10}{21}$

o) $2 \frac{1}{4} \div 1 \frac{5}{16}$

p) $1 \frac{1}{3} \div 2 \frac{2}{9}$

q) $2 \frac{1}{6} \div 3 \frac{1}{4}$

r) $1 \frac{2}{13} \div \frac{5}{26}$

s) $1 \frac{3}{7} \div \frac{8}{35}$

t) $2 \frac{1}{4} \div 2 \frac{1}{3}$

F69. Divide the wholes and fractions (write the results in lowest terms):

a) $1 \div \frac{1}{2}$

b) $\frac{1}{3} \div 2$

c) $4 \div \frac{4}{5}$

d) $\frac{5}{2} \div 15$

e) $32 \div 5 \frac{1}{3}$

f) $5 \div 2 \frac{1}{7}$

g) $\frac{1}{3} \div 2$

h) $3 \div 2 \frac{2}{5}$

i) $\frac{3}{4} \div 6$

j) $15 \div 6 \frac{2}{3}$

k) $1 \frac{1}{2} \div 3$

l) $5 \div 2 \frac{3}{11}$

m) $13 \div 3 \frac{5}{7}$

n) $1 \frac{4}{7} \div 44$

o) $5 \div 3$

Division operators

There are four operators used to express the division operation between two numbers or fractions.

1. The operator is

Example 1. $\frac{3}{5} \frac{9}{10} \frac{3}{5} \frac{10}{9} \frac{2}{3}$

2. The operator is :

Example 2. $\frac{3}{7} : \frac{6}{21} \frac{3}{7} \frac{21}{6} \frac{3}{2}$

3. The operator is $-$
(the division line)

Example 3. $\frac{1}{\frac{2}{3}} \frac{1}{2} \frac{4}{3} \frac{2}{3}$

4. The operator is /

Example 4. $\frac{3}{5} / \frac{9}{25} \frac{3}{5} \frac{25}{9} \frac{5}{3} 1 \frac{2}{3}$

F70. Divide the fractions (write the results in lowest terms):

a) $\frac{2}{3} \frac{4}{5}$ b) $1 \frac{1}{2} \frac{2}{3}$ c) $1 \frac{2}{3} 2 \frac{1}{2}$ d) $2 \frac{2}{7}$ e) $4 \frac{5}{5}$

F71. Divide the fractions (write the results in lowest terms):

a) $\frac{1}{2} : \frac{1}{3}$ b) $1 \frac{1}{2} : \frac{1}{4}$ c) $1 \frac{1}{5} : 2 \frac{1}{10}$ d) $\frac{2}{5} : 4$ e) $3 : 7$

F72. Divide the fractions (write the results in lowest terms):

a) $\frac{\frac{2}{3}}{\frac{4}{6}}$ b) $\frac{\frac{4}{5}}{\frac{8}{15}}$ c) $1 \frac{\frac{2}{9}}{\frac{4}{6}}$ d) $\frac{\frac{3}{5}}{2 \frac{1}{10}}$ e) $1 \frac{\frac{1}{2}}{\frac{3}{4}}$

f) $\frac{\frac{2}{4}}{\frac{6}{6}}$ g) $\frac{5}{\frac{10}{3}}$ h) $\frac{3}{2 \frac{1}{4}}$ i) $\frac{3}{\frac{1}{3}}$ j) $\frac{5}{1 \frac{2}{3}}$

k) $\frac{\frac{2}{3}}{\frac{4}{4}}$ l) $\frac{5}{\frac{2}{4}}$ m) $1 \frac{\frac{4}{5}}{\frac{5}{10}}$ n) $\frac{1}{\frac{3}{3}}$ o) $\frac{2 \frac{2}{5}}{4}$

F73. Divide the fractions (write the results in lowest terms):

a) $\frac{1}{2} / \frac{2}{3}$ b) $\frac{2}{3} / \frac{3}{5}$ c) $\frac{5}{3} / \frac{3}{5}$ d) $\frac{1}{4} / \frac{3}{8}$ e) $5 / 3 \frac{1}{3}$

f) $3 / 2 \frac{1}{4}$ g) $\frac{1}{3} / 2$ h) $2 \frac{1}{4} / 3$ i) $2 / 5$ j) $1 \frac{2}{3} / 3 \frac{3}{4}$

k) $\frac{5}{7} / \frac{15}{14}$ l) $4 \frac{2}{3} / 1 \frac{1}{6}$ m) $3 / 1 \frac{1}{3}$ n) $2 \frac{2}{7} / 4$ o) $7 / 2$

Order of operations (III)

1. Division is not a commutative operation, so the order in which you divide numbers (or fractions) is important. If an expression contains more than one division operation, then by convention the divisions must be done, one by one, from left to right.

Example 1:

$$\frac{1}{2} \frac{2}{3} \frac{5}{4} \quad \frac{1}{2} \frac{2}{3} \frac{5}{4} \frac{3}{4} \frac{5}{4} \frac{3}{5}$$

2. Division and multiplication are considered operations of equal priority. If an expression contains both division and multiplication, then by convention the operations must be done, one by one, from left to right.

Example 2:

$$\frac{4}{5} \frac{1}{2} \frac{2}{3} \quad \frac{4}{5} \frac{1}{2} \frac{2}{3} \frac{2}{5} \frac{2}{3} \frac{3}{5}$$

3. Division and multiplication are considered operations of greater priority than addition and subtraction. If an expression contains all these kinds of operations, do the multiplication and division first, then the addition and subtraction.

Example 3:

$$1\frac{3}{5} \frac{2}{3} \frac{6}{10} \frac{3}{5} \frac{6}{1} \quad 1\frac{3}{5} \frac{2}{3} \frac{6}{10} \quad \frac{3}{5} \frac{6}{1} \quad 1\frac{3}{5} \frac{2}{5} \frac{1}{10} \quad 1\frac{1}{10}$$

F74. Use the correct order of operations to find the value of each expression (see example 1):

- a) $\frac{1}{2} \frac{2}{3} \frac{4}{5}$ b) $2 \ 3 \ 5$ c) $1 \ 2 \ 3 \ 4 \ 5$ d) $2\frac{3}{5} \frac{1}{15} \ 4\frac{1}{3}$
- e) $\frac{1}{2} \frac{3}{4} \frac{5}{6} \frac{7}{8}$ f) $1 \ \frac{1}{2} \ 3 \ \frac{2}{3} \ 4$ g) $\frac{1}{2} \ \frac{2}{3} \ \frac{3}{4} \ \frac{4}{5} \ \frac{5}{6}$ h) $1\frac{1}{2} \ 2\frac{2}{3} \ 3\frac{3}{4} \ 4\frac{4}{5}$
- i) $\frac{2}{3} \ \frac{4}{3} \ \frac{2}{5} \ \frac{15}{4}$ j) $1\frac{2}{3} \ 2\frac{1}{4} \ 3\frac{3}{5} \ 4\frac{1}{6}$ k) $1\frac{1}{11} \ \frac{6}{22} \ \frac{4}{9} \ 6$ l) $\frac{3}{7} \ 1\frac{6}{21} \ \frac{5}{6} \ 1\frac{3}{5}$

F75. Use the correct order of operations to find the value of each expression (see example 2):

- a) $\frac{1}{2} \ \frac{3}{4} \ \frac{5}{6}$ b) $2\frac{3}{4} \ \frac{2}{5} \ \frac{4}{15}$ c) $\frac{1}{2} \ \frac{3}{4} \ \frac{5}{6} \ \frac{7}{8} \ \frac{9}{10}$ d) $1 \ 2 \ 3 \ 4 \ 5$
- e) $\frac{1}{5} \ \frac{2}{5} \ \frac{3}{5} \ \frac{4}{5}$ f) $1\frac{1}{2} \ 2\frac{3}{3} \ 3\frac{3}{4} \ 4\frac{4}{5}$ g) $\frac{3}{2} \ \frac{2}{3} \ \frac{3}{16} \ \frac{9}{8}$ h) $1 \ \frac{1}{2} \ \frac{1}{4} \ \frac{1}{8} \ \frac{1}{16} \ \frac{1}{32}$
- i) $\frac{1}{2} \ \frac{3}{4} \ \frac{5}{6} \ \frac{1}{2} \ \frac{3}{4} \ \frac{6}{5}$ j) $\frac{2}{5} \ \frac{3}{5} \ \frac{12}{5} \ \frac{3}{10} \ \frac{3}{4} \ \frac{3}{20}$ k) $\frac{3}{2} \ \frac{6}{4} \ \frac{9}{6} \ \frac{3}{2} \ \frac{15}{10} \ \frac{12}{8} \ \frac{3}{2}$

F76. Use the correct order of operations to find the value of each expression (see example 3):

- a) $\frac{1}{2} \ \frac{3}{4} \ \frac{5}{6} \ \frac{2}{15} \ \frac{1}{9}$ b) $1 \ \frac{2}{3} \ \frac{6}{8} \ \frac{3}{5} \ \frac{6}{10}$ c) $1\frac{2}{15} \ \frac{3}{5} \ \frac{10}{12} \ \frac{3}{2} \ \frac{2}{5} \ \frac{1}{3} \ \frac{1}{2}$
- d) $5 \ 1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7$ e) $1\frac{1}{2} \ \frac{2}{3} \ 2 \ \frac{1}{2} \ \frac{1}{6} \ 1\frac{1}{4}$ f) $1 \ \frac{1}{2} \ \frac{2}{3} \ \frac{3}{4} \ \frac{1}{4} \ \frac{1}{2}$
- g) $1\frac{1}{2} \ \frac{1}{2} \ \frac{1}{3} \ 2 \ \frac{1}{4} \ \frac{1}{2} \ 1 \ \frac{1}{2} \ \frac{1}{6} \ 4$ h) $\frac{1}{12} \ 1\frac{1}{2} \ \frac{3}{4} \ \frac{3}{4} \ \frac{1}{2} \ \frac{3}{4} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ 2\frac{1}{2} \ \frac{6}{2} \ \frac{1}{12}$

Order of operations (IV)

1. You can change the order of operations using brackets.

Example 1:

$$\frac{1}{2} \frac{2}{3} \quad \frac{1}{2} \frac{1}{4} \quad \frac{7}{3} \frac{7}{6} \frac{1}{4} \quad \frac{7}{3} \frac{1}{8}$$

2. When an expression contains fraction (division) lines, the numerators and the denominators must be calculated independently.

Example 2:

$$\frac{\frac{1}{2} \frac{1}{3}}{2} \frac{4}{3} \frac{1}{2} \frac{1}{2} \frac{1}{2} \quad \frac{\frac{1}{6} \frac{2}{1}}{\frac{1}{2} \frac{1}{4}} \frac{1}{3} \frac{1}{2} \frac{2}{3} \frac{4}{1} \frac{1}{3} \frac{8}{9} \frac{25}{9} \quad 2 \frac{7}{9}$$

F77. Solve each exercise by following the proper order of operations (see example 1):

a) $\frac{15}{3} \frac{1}{2} \frac{2}{5} \quad \frac{3}{4} \frac{2}{3} \frac{5}{12}$ b) $\frac{1}{2} \frac{3}{4} \frac{1}{6} \frac{8}{7} \quad \frac{2}{3} \frac{1}{2}$ c) $\frac{3}{5} \frac{10}{9} \frac{5}{27} \quad \frac{2}{25} \frac{5}{6} \quad 1 \quad 3$

d) $\frac{11}{25} \frac{2}{17} \frac{1}{6} \frac{2}{5} \quad \frac{1}{3} \frac{2}{5} \frac{3}{10} \frac{6}{5}$ e) $2 \frac{1}{4} \frac{1}{2} \quad \frac{1}{2} \frac{1}{3} \quad \frac{1}{2} \frac{1}{4} \quad \frac{1}{3} \frac{1}{6} \quad \frac{1}{2} \frac{3}{4} \quad \frac{3}{4} \frac{2}{3} \quad 6$

e) $\frac{1}{2} \quad 1 \quad \frac{1}{2} \frac{3}{4} \quad \frac{5}{4} \quad \frac{3}{4} \frac{1}{2} \quad \frac{3}{4} \frac{1}{2} \frac{2}{6} \quad \frac{1}{2} \frac{1}{3} \quad \frac{6}{5} \frac{1}{12}$

f) $\frac{1}{2} \frac{2}{5} \frac{1}{2} \quad \frac{3}{2} \frac{1}{5} \frac{3}{10} \quad \frac{1}{2} \frac{2}{3} \quad \frac{1}{5} \frac{3}{10} \quad \frac{2}{3} \frac{3}{5} \frac{2}{5} \quad \frac{5}{6} \frac{2}{3} \frac{8}{9}$

F78. Solve each exercise by following the proper order of operations (see example 2):

a) $\frac{\frac{1}{2}}{\frac{3}{4}} \frac{4}{5} \frac{3}{6}$ b) $\frac{\frac{3}{4}}{\frac{5}{2}} \frac{2}{5}$ c) $\frac{\frac{2}{3}}{\frac{3}{5}} \frac{5}{6}$ d) $\frac{\frac{3}{1}}{\frac{2}{3}} \frac{1}{3}$ e) $\frac{\frac{3}{5}}{\frac{2}{3}} \frac{3}{4} \quad 8 \quad \frac{3}{\frac{2}{3 \cdot 1/2}}$

f) $\frac{\frac{2}{3} \frac{3}{4} \frac{2}{9}}{3} \frac{3}{4} \frac{5}{3}$ g) $\frac{2 \cdot 3 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 2}$ h) $\frac{\frac{2}{3} \frac{3}{2} \frac{9}{8} \frac{1}{3}}{\frac{5}{3} \frac{15}{6} \frac{2}{3} \frac{1}{2}}$ i) $\frac{\frac{2}{5} \frac{15}{8} \frac{1}{2} \frac{2}{3} \frac{2}{3}}{\frac{1}{2} \frac{3}{2} \frac{9}{4} \frac{3}{4} \frac{1}{3}}$

j) $1 \frac{\frac{3}{4} \frac{1}{2} \frac{8}{3}}{\frac{1}{2} \frac{5}{12}} \quad \frac{4 \cdot 5 \cdot 1}{3 \cdot 2 \cdot 9} \frac{9}{15}$ k) $\frac{\frac{3}{5} \frac{1}{2} \frac{5}{4}}{\frac{1}{2} \frac{5}{4} \frac{1}{5}} \frac{2}{5} \frac{10}{6} \frac{2}{3} \quad 1 \quad \frac{1}{2} \frac{3}{4} \frac{1}{3} \quad 1$

Raising fractions to a power

1. If you multiply a fraction by itself k times, you can use *exponential notation* to make the expression more compact (simpler):

$$\underbrace{n \ n \ \dots \ n}_{k \text{ times}} \quad n^k \qquad \underbrace{\frac{n}{d} \ \frac{n}{d} \ \dots \ \frac{n}{d}}_{k \text{ times}} \quad \frac{n}{d}^k$$

The left side is the expanded notation and the right side is the exponential notation.

Example 1: $3 \ 3 \ 3 \ 3 \ 3^4$

Example 2: $\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}^3$

2. To calculate the value an expression written in exponential notation, rewrite it in expanded notation and do the multiplication.

Example 3. $\frac{2}{3}^4 \quad \frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \quad \frac{16}{81}$

3. You can use also the following rule to calculate the value of a fraction raised to a power:

$$\frac{\underbrace{n \ n \ \dots \ n}_{k \text{ times}}}{\underbrace{d \ d \ \dots \ d}_{k \text{ times}}} \quad \frac{n}{d}^k \quad \frac{n^k}{d^k}$$

Example 4: $\frac{3}{4}^2 \quad \frac{3^2}{4^2} \quad \frac{3 \ 3}{4 \ 4} \quad \frac{9}{16}$

4. If the exponent is negative, replace the fraction with its reciprocal raised to a positive exponent according to the rule:

$$\frac{n}{d}^{-k} \quad \frac{d}{n}^k$$

Example 5: $\frac{2}{5}^{-3} \quad \frac{5}{2}^3 \quad \frac{5^3}{2^3} \quad \frac{125}{8} \quad 15\frac{5}{8}$

5. By convention, any number (including a fraction) raised to the power of 0 equals 1.

Example 6: $\frac{4}{5}^0 \quad 1$

F79. Write each expression in exponential notation:

a) $\frac{1}{2} \ \frac{1}{2} \ \frac{1}{2} \ \frac{1}{2}$

b) $\frac{3}{4} \ \frac{3}{4}$

c) $\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2}$

d) $\frac{3}{2} \ \frac{3}{2} \ \frac{3}{2} \ \frac{3}{2}$

e) $2 \ 2 \ 2 \ 2$

f) $1\frac{2}{3} \ 1\frac{2}{3} \ 1\frac{2}{3}$

g) $\frac{4}{5} \ \frac{4}{5}$

h) $\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}$

F80. Write each expression in expanded notation:

a) $\frac{3}{4}^2$

b) $\frac{5}{3}^4$

c) $\frac{1}{5}^3$

d) $\frac{2}{3}^2$

e) $\frac{3}{7}^2$

f) $\frac{3}{2}^3$

g) $\frac{5}{4}^1$

h) $\frac{4}{5}^3$

F81. Calculate the value of each expression:

a) $\frac{3}{7}^0$

b) $\frac{3}{2}^3$

c) $\frac{2}{5}^2$

d) $\frac{3}{5}^3$

e) 2^3

f) $\frac{1}{3}^4$

g) $\frac{2}{1}^4$

h) $\frac{3}{2}^4$

F82. Calculate the value of each expression:

a) 3^2

b) $\frac{5}{1}^2$

c) $\frac{1}{2}^4$

d) $\frac{2}{3}^3$

e) $\frac{3}{4}^2$

f) $\frac{5}{7}^1$

g) $\frac{4}{3}^2$

h) $\frac{2}{5}^3$

Order of operations (V)

1. If the base of a power contains other operations, replace the base with the result of that operations, then raise the new base to the exponent.

Example 1. $\frac{1}{2} \frac{1}{3}^2 \frac{5}{6}^2 \frac{25}{36}$

2. If the exponent of a power contains other operations, replace the exponent with the result of that operations, then raise the base to the new exponent.

Example 2. $\frac{2}{3}^{1+2^2} \frac{2}{3}^5 \frac{32}{243}$

3. If an expression contains addition, subtraction, multiplication, division, and powers, do the powers first.

Example 3. $\frac{1}{2} \frac{16}{9} \frac{6}{8} \frac{2}{3}^2 \cdot 1 \frac{1}{2} \frac{16}{9} \frac{6}{8} \frac{4}{9} \cdot 1 \frac{1}{2} \frac{16}{9} \frac{6}{8} \frac{9}{4} \cdot 1 \frac{1}{2} \cdot 3 \cdot 1 \cdot 2 \frac{1}{2}$

4. The order of operations can be changed by brackets.

Example 4. $\frac{1}{2} \cdot 2 \frac{1}{2} \cdot 4 \cdot \frac{3}{4}^2 \cdot 8 \frac{1}{2} \cdot 2 \frac{1}{2} \cdot 4 \cdot \frac{9}{16} \cdot 8 \frac{1}{2} \cdot \frac{1}{4} \cdot 8 \frac{1}{2} \cdot \frac{4}{1} \cdot 8 \frac{1}{2} \cdot \frac{1}{2} \cdot 1$

F75. Calculate the value of each expression (see example 1):

a) $\frac{1}{3} \frac{1}{4}^2$ b) $\frac{1}{5} \frac{15}{4} \frac{8}{7}^2$ c) $\frac{1}{3} \frac{6}{5} \frac{4}{5}^2$ d) $\frac{1}{2} \frac{1}{3}^2 \frac{1}{2} \frac{1}{3}^2$
 e) $\frac{1}{3} \frac{3}{2}^3 \frac{1}{3} \frac{1}{2}^2$ f) $\frac{2}{5} \frac{1}{4} \frac{4}{5}^1$ g) $\frac{2}{3} \frac{5}{6} \frac{1}{2}^2$ h) $\frac{1}{2} \frac{1}{3}^1 \frac{2}{3} \frac{1}{4}^2$

F76. Calculate the value of each expression (see example 2):

a) $\frac{2}{3}^{3+4 \frac{1}{2}}$ b) $\frac{3}{5}^{\frac{1}{2} \frac{1}{6}^1}$ c) $\frac{1}{2}^{\frac{1}{3} \cdot 6 \frac{2}{3} \frac{1}{3}}$ d) $\frac{2}{5}^{(\frac{1}{2} \cdot 3 \frac{3}{2}) \frac{3}{2}}$
 e) $\frac{2}{3} \frac{2}{3}^{\frac{1}{2} \frac{1}{2}}$ f) $\frac{1}{2} \frac{3}{4}^{\frac{1}{3} \frac{1}{6}}$ g) $\frac{1}{3} \frac{3}{2}^{\frac{2}{3} \frac{1}{6}}$ h) $\frac{3}{4} \frac{2}{3}^{\frac{3}{2} (\frac{2}{3} \cdot 2 \frac{2}{3})}$

F77. Calculate the value of each expression (see example 3):

a) $\frac{1}{2}^1 \frac{1}{2}^0 \frac{1}{2}^1$ b) $\frac{2}{3}^2 \frac{2}{3}^0 \frac{2}{3}^2$ c) $\frac{1}{2}^1 \frac{2}{3}^2 \frac{3}{4}^1 \frac{1}{3}$
 d) $\frac{3}{4}^2 \frac{4}{3} \frac{3}{2} \frac{3}{2}^2$ e) $1 \frac{2}{5}^3 \cdot 25 \frac{3}{5} \frac{1}{2}^2 \cdot 2$ f) $1 \frac{3}{4}^3 \frac{1}{2}^5 \frac{1}{3}^2 \frac{1}{2}$
 g) $\frac{\frac{1}{2}^2 \frac{3}{4}^2 \frac{3}{2}^3}{2 \frac{3}{4} \frac{3}{2}^1}$ h) $\frac{\frac{1}{4} \frac{1}{2} \frac{3}{2}^2}{\frac{1}{2}^2 \frac{3}{4}} \frac{\frac{1}{3}^2 \frac{2}{9}}{\frac{1}{6} \frac{2}{9} \frac{4}{3}^2}$ i) $\frac{4 \frac{1}{10}^2}{\frac{3}{5}} \frac{3}{4} \frac{1}{2}^1$
 j) $\frac{\frac{2}{3}^2}{2^2} \frac{3^3}{\frac{3}{4}^2}$ k) $12 \frac{1}{2}^2 \cdot 1 \frac{1}{4} \cdot 2^2 \frac{3}{4} \frac{1}{2} \frac{1}{2}^2 \cdot 5^2$
 l) $\frac{3}{4} \frac{2}{3} \frac{7}{6} \frac{1}{2} \frac{1}{3}^1 \frac{1}{2}^2 \cdot \frac{1}{2}^2 \cdot 1 \frac{1}{2}^3 \frac{1}{2}^2 \cdot 1^3 \frac{3}{2}^2$

Converting fractions to decimals

1. The fraction line is a division operation, so to write a fraction as a decimal, divide the numerator (dividend) by the denominator (divisor).

2. Usually we are dealing with the decimal system (the base is 10). The single prime factors of 10 are 2 and 5. If the denominator of the fraction written in lowest terms has only 2 or 5 as prime factors, then the fraction can be written as a *terminating decimal*.

Example 1:

$$8 \cdot 2 \cdot 2 \cdot 2 \quad \frac{3}{8} \quad 0.375$$

$$\begin{array}{r} 0.375 \\ 8 \overline{)3.000} \\ \underline{24} \\ 60 \\ \underline{56} \\ 40 \\ \underline{40} \\ 00 \end{array}$$

3. If the denominator of the fraction written in lowest terms has factors other than 2 or 5, the fraction can be written as a *non-terminating repeating decimal*.

Example 2:

$$33 \cdot 3 \cdot 11 \quad \frac{40}{33} \quad 1.2121... \quad 1.\overline{21}$$

4. The part of the decimal that repeats is called the *period*. The period is 21 in example 2. The number of repeating digits is the *length of the period*. The length of the period is 2 digits in example 2.

$$\begin{array}{r} 1.212... \\ 33 \overline{)40.000} \\ \underline{33} \\ 70 \\ \underline{66} \\ 40 \\ \underline{33} \\ 70 \\ \underline{66} \\ 4... \end{array}$$

F86. Build the factor tree for the denominator and classify each fraction as a terminating or a non-terminating decimal:

- | | | | | |
|--------------------|------------------|-------------------|---------------------|---------------------|
| a) $\frac{1}{2}$ | b) $\frac{2}{3}$ | c) $\frac{3}{4}$ | d) $\frac{4}{5}$ | e) $\frac{5}{6}$ |
| f) $\frac{2}{7}$ | g) $\frac{7}{9}$ | h) $\frac{3}{10}$ | i) $\frac{8}{11}$ | j) $\frac{2}{15}$ |
| k) $\frac{11}{20}$ | l) $\frac{3}{6}$ | m) $\frac{3}{40}$ | n) $\frac{13}{100}$ | o) $\frac{25}{120}$ |

F87. Write each fraction as a terminating decimal:

- | | | | | |
|--------------------|---------------------|--------------------|--------------------|--------------------|
| a) $\frac{1}{2}$ | b) $\frac{3}{4}$ | c) $\frac{5}{8}$ | d) $\frac{1}{5}$ | e) $\frac{3}{20}$ |
| f) $\frac{11}{80}$ | g) $\frac{13}{100}$ | h) $\frac{1}{125}$ | i) $\frac{3}{500}$ | j) $\frac{1}{625}$ |

F88. Write each fraction as a non-terminating repeating decimal. Use a bar over the repeating decimals:

- | | | | | |
|-------------------|-------------------|--------------------|-------------------|-------------------|
| a) $\frac{1}{3}$ | b) $\frac{1}{6}$ | c) $\frac{11}{36}$ | d) $\frac{7}{9}$ | e) $\frac{2}{11}$ |
| f) $\frac{1}{15}$ | g) $\frac{7}{30}$ | h) $\frac{2}{7}$ | i) $\frac{2}{13}$ | j) $\frac{1}{27}$ |

F89. Write each fraction as either a terminating or non-terminating repeating decimal. Use a bar over the repeating decimals:

- | | | | | |
|--------------------|-------------------|--------------------|-------------------|--------------------|
| a) $1\frac{1}{2}$ | b) $\frac{6}{15}$ | c) $1\frac{7}{21}$ | d) $\frac{1}{75}$ | e) $\frac{7}{3}$ |
| f) $1\frac{5}{4}$ | g) $\frac{5}{64}$ | h) $\frac{1}{32}$ | i) $\frac{3}{64}$ | j) $\frac{1}{128}$ |
| k) $\frac{3}{250}$ | l) $\frac{1}{7}$ | m) $\frac{20}{14}$ | n) $\frac{5}{12}$ | o) $2\frac{3}{13}$ |

Converting decimals to fractions

1. To convert a terminating decimal to a fraction, use the place value of each digit of the decimal part.

Example 1:

$$1.25 = 1 \frac{2}{10} = 1 \frac{5}{50} = 1 \frac{1}{5} \quad 1 \frac{1}{20} = 1 \frac{5}{100} = 1 \frac{1}{20} \quad 1 \frac{1}{4}$$

3. To convert a non-terminating repeating decimal to a fraction, you can use algebra.

Example 3:

$$\begin{array}{r} x \quad 0.666\dots \quad 0.\overline{6} \\ 10x \quad 6.666\dots \\ \hline 10x \quad x \quad 6.666\dots \quad 0.666\dots \\ 9x \quad 6 \\ \hline x \quad \frac{6}{9} \quad \frac{2}{3} \end{array}$$

2. A faster method to convert a terminating decimal to a fraction is:

a) the numerator is the number without the decimal point

b) the denominator is 1 followed by a 0 for each digit of the decimal part

Example 2: $1.625 = \frac{1625}{1000} = \frac{13}{8} = 1\frac{5}{8}$

4. A faster method to convert a non-terminating repeating decimal to a fraction is the following:

1) the numerator is a mixed number:

a) the whole is the decimal written without the decimal point and the period

b) the numerator is the period

c) the denominator is one 9 for each digit of the period

2) the denominator is 1 followed by one 0 for each digit between the decimal point and the period

Example 4:

$$6.\overline{12345} = \frac{6123\frac{45}{99}}{1000} = 6\frac{679}{5500}$$

F90. Write each terminating decimal as a fraction in lowest terms (see the example 1):

- | | | | | |
|----------|----------|-----------|----------|------------|
| a) 0.1 | b) 0.5 | c) 1.4 | d) 1.25 | e) 0.75 |
| f) 0.035 | g) 2.125 | h) 10.125 | i) 5.075 | j) 100.725 |

F91. Write each terminating decimal as a fraction in lowest terms (see the example 2):

- | | | | | |
|----------|---------|----------|---------|-------------|
| a) 0.625 | b) 1.5 | c) 0.125 | d) 0.4 | e) 2.16 |
| f) 0.275 | g) 0.24 | h) 0.35 | i) 2.45 | j) 0.640625 |

F92. Write each non-terminating decimal as a fraction in lowest terms (see the example 3):

- | | | | | |
|----------------------|----------------------|-----------------------|-----------------------|-------------------------|
| a) $0.\overline{3}$ | b) $1.\overline{2}$ | c) $0.\overline{12}$ | d) $1.2\overline{1}$ | e) $4.02\overline{5}$ |
| f) $0.\overline{23}$ | g) $1.\overline{25}$ | h) $2.0\overline{12}$ | i) $0.\overline{123}$ | j) $1.23\overline{456}$ |

F93. Write each non-terminating decimal as a fraction in lowest terms (see the example 4):

- | | | | | |
|----------------------|----------------------|-----------------------|-------------------------|------------------------|
| a) $0.\overline{7}$ | b) $1.\overline{3}$ | c) $2.5\overline{3}$ | d) $1.3\overline{2}$ | e) $1.12\overline{9}$ |
| f) $0.\overline{12}$ | g) $3.0\overline{1}$ | h) $1.3\overline{12}$ | i) $6.123\overline{45}$ | j) $1.0\overline{123}$ |

Order of operations (VI)

1. If an expression contains only decimal numbers, you can do all the operations using decimal numbers.

Example 1:

$$1.2 \ 0.5 \ 1.6 \ 2.4 \ 2 \ 1.2 \ 0.8 \ 1.2 \ 0.8$$

2. If an expression contains operations with *non-terminating repeating decimal numbers*, convert decimal numbers to fractions and do all the operations with fractions.

Example 2:

$$1.\bar{3} \ 1.\overline{25} \ 0.3\bar{1} \ 1\frac{1}{3} \ 1\frac{25}{99} \ 3\frac{1}{9} \ 1\frac{533}{1485} \ 1.359$$

3. If an expression contains both fractions and decimal numbers, it is *recommended* that you first convert the decimal numbers to fractions, and do all the operations with fractions.

Example 3:

$$1.2 \ \frac{2}{5} \ 0.5 \ \frac{4}{5} \ \frac{3}{10} \ 1.5 \ \frac{6}{5} \ \frac{2}{5} \ \frac{1}{2} \ \frac{4}{5} \ \frac{3}{10} \ \frac{3}{2} \ \frac{7}{5} \ 1.4$$

F94. Find the value of each expression (only use operations with decimals):

a) $0.25 \ 0.15$ b) $0.50 \ 0.15$ c) $2.4 \ 0.5$ d) $0.10 \ 0.05$ e) $0.5^2 \ 0.25$

F95. Find the value of each expression by converting decimals to fractions:

a) $0.75 \ 1.25$ b) $0.60 \ 0.25$ c) $0.5 \ 1.4$ d) $1.2 \ 0.75$ e) $1 \ 0.4^2$

F96. Find the value of each expression by converting repeating decimals to fractions:

a) $1.\bar{3} \ 0.\bar{6}$ b) $2.\bar{4} \ 0.\overline{45}$ c) $0.\bar{5} \ 2.\bar{2}$ d) $0.2\bar{8} \ 0.0\bar{1}$ e) $0.\bar{6}^2 \ 0.\bar{3}^2$

F97. Find the value of each expression by converting fractions to decimals:

a) $\frac{2}{5} \ 0.5 \ \frac{1}{2} \ 0.2 \ 0.2$ b) $1.5 \ \frac{2}{5} \ \frac{1}{2} \ 0.5^2$ c) $\frac{4}{5} \ 0.3 \ 1.5 \ 0.8^2$ d) $0.1 \ \frac{1}{5} \ \frac{3}{10} \ 0.2^2$

F98. Find the value of each expression by converting decimals to fractions:

a) $\frac{1}{4} \ 0.20 \ \frac{3}{2}$ b) $0.25 \ \frac{5}{6} \ 0.15 \ \frac{2}{3}$ c) $\frac{2}{3} \ 0.25 \ \frac{1}{4} \ 0.\bar{3}$ d) $0.5^2 \ \frac{1}{2}^3 \ 0.8$

F99. Find the value of each expression by converting decimals to fractions:

a) $\frac{1}{2} \ 0.5 \ \frac{3}{5} \ \frac{3}{4} \ 1.5 \ \frac{2}{5}$ b) $1.25 \ \frac{5}{4} \ 0.5 \ 0.5 \ 0.75^2 \ \frac{2}{3}$ c) $\frac{4}{5} \ 0.5 \ \frac{5}{3} \ 0.\bar{3} \ \frac{1}{2} \ 0.25 \ \frac{1}{3} \ 4$

d) $\frac{1.5^2 \ \frac{5}{6} \ \frac{3}{8} \ 24}{0.4 \ \frac{3}{4} \ 0.25}$ e) $\frac{0.25 \ \frac{1}{2}^2 \ \frac{3}{16}}{\frac{4}{5} \ 0.3 \ 3} \ 0.5$ f) $\frac{\frac{3}{5} \ 0.1 \ \frac{3}{4} \ 0.5}{1.25 \ \frac{1}{2} \ \frac{1}{4} \ 1.25} \ \frac{3}{4}$

g) $\frac{2}{3} \ 1.\bar{3}^2 \ 5 \ 0.4 \ 1.6 \ \frac{3}{5}^5 \ 0.2 \ \frac{2}{3}$ h) $2.5 \ \frac{4}{5} \ 0.7 \ \frac{1}{3}^3 \ 1.8 \ \frac{3}{5} \ \frac{5}{2}^2 \ \frac{1}{9}$

i) $\frac{2.\bar{4} \ \frac{4}{9}^3 \ 0.5^2}{2.4 \ 1.2 \ \frac{2}{5}}$ $\frac{0.\bar{6} \ 0.\bar{3} \ \frac{1}{9}}{1.\bar{5} \ \frac{1}{9}^2} \ 5^2$ j) $\frac{0.1 \ 0.25 \ 0.4^2}{\frac{1}{2} \ \frac{1}{5} \ \frac{1}{2}^1} \ \frac{0.2 \ \frac{1}{5}^2 \ 0.6 \ \frac{3}{5}}{1.6 \ 0.4 \ \frac{2}{5} \ 0.5} \ 7 \ \frac{1}{3}^1 \ 2^5$

Time and Fractions

1. One hour has 60 minutes. This relation can be written in two ways:

$$1 h = 60 \text{ min} \quad 1 \text{ min} = \frac{1 h}{60} = \frac{1}{60} h$$

2. To make conversions between hours and minutes, use the formulas above.

Example 1:

$$1\frac{1}{3} h = \frac{4}{3} h = \frac{4}{3} \cdot 60 \text{ min} = 80 \text{ min}$$

Example 2:

$$25 \text{ min} = 25 \cdot \frac{1 h}{60} = \frac{25}{60} h = \frac{5}{12} h$$

3. One minutes has 60 seconds. This relation can be written in two ways:

$$1 \text{ min} = 60 s \quad 1 s = \frac{1 \text{ min}}{60} = \frac{1}{60} \text{ min}$$

4. To make conversions between minutes and seconds, use the formulas above.

Example 3:

$$0.25 \text{ min} = \frac{3}{4} \text{ min} = \frac{3}{4} \cdot 60 s = 45 s$$

Example 4:

$$100 s = 100 \cdot \frac{1 \text{ min}}{60} = \frac{100}{60} \text{ min} = \frac{5}{3} \text{ min}$$

5. The conversion between seconds and hours is a two-step task.

$$4500 s = 4500 \cdot \frac{1 \text{ min}}{60} = 75 \text{ min} = 75 \cdot \frac{1 h}{60} = 1\frac{1}{4} h$$

F100. Convert hours to minutes. Write the results as mixed numbers in lowest terms:

- | | | | | |
|--------------------|--------------------|--------------------|---------------------|-------------------|
| a) 0.5 h | b) $\frac{1}{3} h$ | c) 1.75 h | d) $2\frac{3}{4} h$ | e) 0.1 h |
| f) $\frac{5}{6} h$ | g) 2.25 h | h) $\frac{3}{8} h$ | i) $\frac{7}{15} h$ | j) $1.2\bar{3} h$ |

F101. Convert minutes to hours. Write the results as mixed numbers in lowest terms:

- | | | | | |
|-----------|-----------|------------|------------------------------|------------|
| a) 5 min | b) 10 min | c) 15 min | d) 25 min | e) 50 min |
| f) 70 min | g) 36 min | h) 250 min | i) $\frac{3}{4} \text{ min}$ | j) 2.5 min |

F102. Convert minutes to seconds. Write the results as mixed numbers in lowest terms:

- | | | | | |
|-------------------------------|-------------------------------|-------------------------------|--------------------------------|---------------------------|
| a) 1.5 min | b) $1\frac{2}{3} \text{ min}$ | c) 0.25 min | d) $1\frac{2}{5} \text{ min}$ | e) 1.15 min |
| f) $\frac{7}{12} \text{ min}$ | g) 1.75 min | h) $\frac{5}{12} \text{ min}$ | i) $\frac{11}{45} \text{ min}$ | j) $7\bar{3} \text{ min}$ |

F103. Convert seconds to minutes. Write the results as mixed numbers in lowest terms:

- | | | | | |
|----------------------|-----------|----------------------|----------------------|------------------|
| a) 12 s | b) 10 s | c) 45 s | d) 90 s | e) 200 s |
| f) $10\frac{1}{2} s$ | g) 0.75 s | h) $15\frac{3}{4} s$ | i) $\frac{100}{3} s$ | j) $20\bar{6} s$ |

F104. Do the required conversions:

- | | | | | | |
|-------------------|-------------------|---------------------|--|---------------|---------------|
| a) 500 s ? h | b) 1500 s ? h | c) 9000 s ? h | d) 0.2 h ? s | e) 1.75 h ? s | f) 0.15 h ? s |
| g) 2 h 10 min ? s | h) 15 min 5 s ? s | i) 0.5 h 15 s ? min | j) $\frac{2}{3} h$ 10.5 min 10 s ? min | | |

Canadian coins and fractions

1. One dollar (loonie or \$) has 100 cents (pennies):

Example 1: $125 \text{ cents} = 125 \frac{1}{100} \$ = 1\frac{1}{4} \$$

$$1 \$ = 100 \text{ cents} \quad 1 \text{ cent} = \frac{1 \$}{100} = \frac{1}{100} \$$$

2. One twonie (or toonie) has 200 cents:

Example 2: $1.75 \text{ twonies} = \frac{175}{100} \cdot 200 \text{ cents} = 350 \text{ cents}$

$$1 \text{ twonie} = 200 \text{ cents} \quad 1 \text{ cent} = \frac{1 \text{ twonie}}{200} = \frac{1}{200} \text{ twonie}$$

3. One nickel has 5 cents:

Example 3: $1\frac{1}{5} \text{ nickels} = \frac{6}{5} \cdot 5 \text{ cents} = 6 \text{ cents}$

$$1 \text{ nickel} = 5 \text{ cents} \quad 1 \text{ cent} = \frac{1 \text{ nickel}}{5} = \frac{1}{5} \text{ nickel}$$

4. One dime has 10 cents:

Example 4: $25 \text{ cents} = 25 \frac{1}{10} \text{ dimes} = 2\frac{1}{2} \text{ dimes}$

$$1 \text{ dime} = 10 \text{ cents} \quad 1 \text{ cent} = \frac{1 \text{ dime}}{10} = \frac{1}{10} \text{ dime}$$

5. One quarter has 25 cents:

Example 5: $\frac{2}{5} \text{ quarters} = \frac{2}{5} \cdot 25 \text{ cents} = 10 \text{ cents}$

$$1 \text{ quarter} = 25 \text{ cents} \quad 1 \text{ cent} = \frac{1 \text{ quarter}}{25} = \frac{1}{25} \text{ quarter}$$

F105. Convert dollars to cents. Write the results as mixed numbers in lowest terms:

- a) 0.12 \$ b) $\frac{1}{125} \$$ c) 1.02 \$ d) $1\frac{2}{75} \$$ e) 0.07 \$ f) $\frac{7}{150} \$$

F106. Convert cents to dollars. Write the results as mixed numbers in lowest terms:

- a) 25 cents b) 20 cents c) 45 cents d) 160 cents e) 450 cents f) 5.5 cents

F107. Convert twonies to cents. Write the results as mixed numbers in lowest terms:

- a) 0.02 twonies b) 0.95 twonies c) $\frac{3}{100} \text{ twonies}$ d) $\frac{11}{150} \text{ twonies}$ e) $1\frac{3}{10} \text{ twonies}$ f) $\frac{1.2}{25} \text{ twonies}$

F108. Convert cents to twonies. Write the results as mixed numbers in lowest terms:

- a) 125 cents b) 250 cents c) 40 cents d) 120 cents e) 500 cents f) 10.2 cents

F109. Convert nickels to cents. Write the results as mixed numbers in lowest terms:

- a) 0.2 nickels b) 1.2 nickels c) $\frac{7}{20} \text{ nickels}$ d) $\frac{7}{5} \text{ nickels}$ e) $2\frac{3}{20} \text{ nickels}$ f) $\frac{10.5}{2} \text{ nickels}$

F110. Convert cents to nickels. Write the results as mixed numbers in lowest terms:

- a) 15 cents b) 25 cents c) 75 cents d) 4 cents e) 1.5 cents f) 0.5 cents

F111. Convert quarters to cents. Write the results as mixed numbers in lowest terms:

- a) 0.2 quarters b) 1.6 quarters c) $\frac{3}{25} \text{ quarters}$ d) $\frac{9}{125} \text{ quarters}$ e) $1\frac{3}{5} \text{ quarters}$ f) $\frac{1.2}{5} \text{ quarters}$

F112. Convert cents to quarters. Write the results as mixed numbers in lowest terms:

- a) 50 cents b) 125 cents c) 100 cents d) 20 cents e) 0.5 cents f) 55 cents

F113. Do the required conversions:

- a) 3 quarters ? nickels b) 1.5 dimes ? nickels c) 2.5 dimes ? quarters d) 0.2 quarters ? dimes
 e) 8 nickels ? dimes f) 15 nickels ? dimes g) 1 quarters 2 dimes ? nickels h) 5 quarters ? dimes

Fractions, ratio, percent, decimals, and proportions

1. A *fraction* is a comparison between a part and the whole. For example 40/100
2. A *ratio* is a comparison between two numbers. For example 40:100
3. A *percent* is a comparison between a number and 100. For example 40%
4. A *decimal* is a comparison between a number and 1. For example 0.40
5. The same number can be written as a fraction, ratio, percent, or decimal. To convert the number from one expression to another, use this two-step algorithm:

1) Express the number as a fraction

Ex. 1 $1\frac{2}{5}$ $\frac{7}{5}$ Ex. 2 7 to 5 $7:5$ $\frac{7}{5}$ Ex. 3 140% $\frac{140}{100}$ $\frac{7}{5}$ Ex. 4 1.4 $\frac{14}{10}$ $\frac{7}{5}$

2) Convert the fraction using a proportion and the cross-multiplication rule

Ex. 5 $\frac{7}{5}$ $1\frac{2}{5}$ Ex. 6 $\frac{7}{5}$ x to 35 $\frac{x}{35}$; $x = \frac{7 \cdot 35}{5} = 49$; so $\frac{7}{5}$ 49 to 35

Ex. 7 $\frac{7}{5}$ 1.4 Ex. 8 $\frac{7}{5}$ $\frac{x}{100}$; $x = \frac{7 \cdot 100}{5} = 140$; so $\frac{7}{5}$ 140%

F114. Fill out the table:

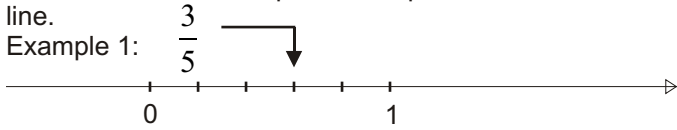
	Fraction	Ratio	Percent	Decimal
a)	1/4	... to 20		
b)	1/3	... out of 21		
c)	3/2	12 to ...		
d)	5/6	15 out of ...		
e)	3/5	42 out of ...		
f)		2 out of 5		
g)		10 to 45		
h)		3 out of 12		
i)		18 to 15		
j)		7 to 10		
k)		... to 70	30 %	
l)		36 out of ...	80 %	
m)		... to 30	130 %	
n)		... out of 5	10 %	
o)		... out of 64	75 %	
p)		... to 10		0.15
r)		... out of 35		1.4
s)		6 to ...		0.08
t)		80 out of ...		0.64
u)		... out of 256		0.125

Fractions and Number Line

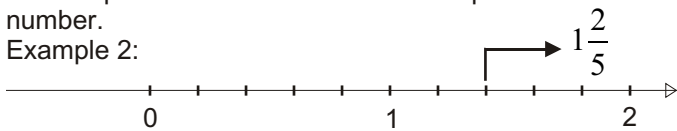
1. To create a number line assign points to 0 and 1:



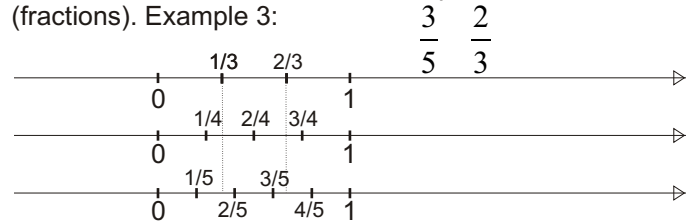
2. Each number corresponds to a point on the number line.



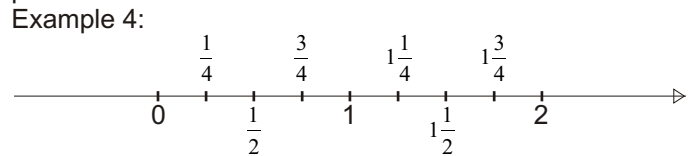
3. Each point on the number line corresponds to a number.



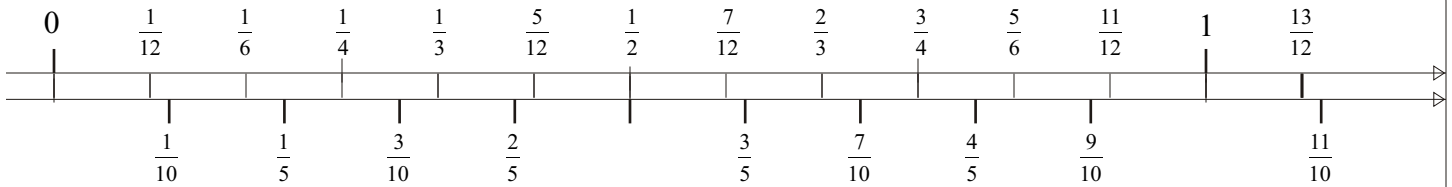
4. Number lines can be used to compare numbers (fractions). Example 3:



5. To calibrate a ruler means to assign numbers to the points on the ruler.



F115. Use the number line to compare fractions (use $>$ or $<$ symbols):



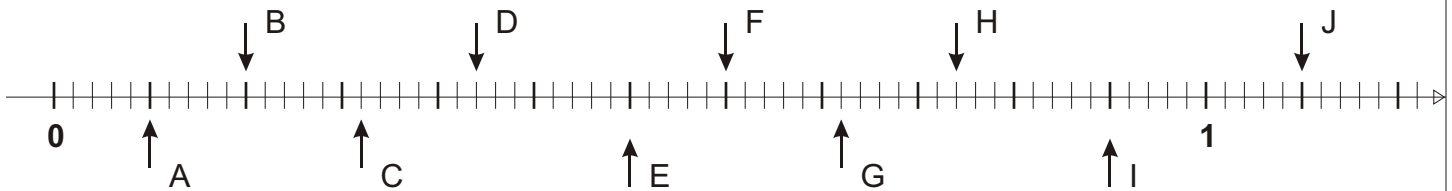
- a) $\frac{1}{2}$ $\frac{6}{12}$
- b) $\frac{1}{3}$ $\frac{3}{10}$
- c) $\frac{3}{5}$ $\frac{7}{12}$
- d) $\frac{4}{5}$ $\frac{5}{6}$
- e) $\frac{11}{12}$ $\frac{9}{10}$
- f) $\frac{2}{5}$ $\frac{5}{12}$
- g) $\frac{4}{5}$ $\frac{3}{4}$
- h) $\frac{1}{6}$ $\frac{1}{5}$
- i) $\frac{4}{5}$ $\frac{3}{4}$
- j) $\frac{1}{12}$ $\frac{1}{10}$

F116. Find a point on the number line corresponding to each fraction:



- a) $\frac{1}{2}$
- b) $\frac{3}{4}$
- c) $\frac{1}{12}$
- d) $1\frac{1}{4}$
- e) $\frac{3}{8}$
- f) $\frac{3}{24}$
- g) $\frac{7}{12}$
- h) $1\frac{5}{24}$
- i) $1\frac{1}{12}$
- j) $1\frac{1}{8}$

F117. Find the fraction corresponding to each point on the number line:



F118. Calibrate the ruler:



Comparing fractions

1. Fractions are *ordered numbers*. That means you can *compare* them and decide if they are *equal* (=) or which one is *greater* (>) or *less* (<) than the other. The main idea is that a small number is less (<) than a big number:

$$\frac{\text{small}}{\text{big}} \quad (1)$$

Example 1: $\frac{1}{3} < \frac{5}{2} < \frac{4}{4}$

2. If you divide relation (1) by any number *a*, you'll get:

$$\frac{\frac{\text{small}}{a}}{\frac{\text{big}}{a}} \quad (2)$$

So, if you compare two fractions having the same denominator, the smallest one has the smallest numerator.

Example 2: $\frac{2}{5} < \frac{4}{5} < \frac{7}{11} < \frac{5}{11} < \frac{3}{4} < \frac{3}{4}$

3. If you cross exchange the factors in relation (2), you'll get:

$$\frac{\frac{a}{\text{big}}}{\frac{a}{\text{small}}} \quad (3)$$

So, if you compare two fractions having like numerators, the smallest fraction has the biggest denominator.

Example 3: $\frac{1}{3} < \frac{1}{2} < \frac{2}{5} < \frac{2}{7} < \frac{3}{8} < \frac{3}{8}$

4. To compare two fractions in the general case:

$$\frac{n_1}{d_1} ? \frac{n_2}{d_2}$$

use cross multiplication to convert the initial comparison to another equivalent one:

$$n_1 d_2 ? n_2 d_1$$

Example 4: $\frac{3}{4} < \frac{2}{5}$ because $3 \cdot 5 < 4 \cdot 2$

5. To compare two fractions in the general case, you can also find the LCD (Least or Lowest Common Denominator), convert the original fractions to equivalent fractions having like denominators and then use the relation (2).

Example 5: $\frac{5}{12} < \frac{7}{16}$ because $\frac{5}{12} = \frac{20}{48} < \frac{21}{48} = \frac{7}{16}$

6. You can use the LCD method when you have to order a set of fractions:

Example 6: $\frac{2}{3} < \frac{3}{4} < \frac{5}{6}$ because $\frac{2}{3} = \frac{8}{12} < \frac{3}{4} = \frac{9}{12} < \frac{5}{6} = \frac{10}{12}$

F119. Compare the whole numbers. Use <, =, and > operators :

- a) 2 and 9 b) 10 and 10 c) 1 and 0 d) 7 and 5 e) 123 and 132 f) 0 and 3 g) 12 and 21

F120. Compare the fractions (see example 2):

- a) $\frac{3}{7}$ and $\frac{5}{7}$ b) $\frac{5}{6}$ and $\frac{1}{6}$ c) $\frac{2}{6}$ and $\frac{1}{3}$ d) $\frac{3}{11}$ and $\frac{5}{11}$ e) $\frac{13}{25}$ and $\frac{23}{25}$ f) $1\frac{2}{3}$ and $\frac{4}{3}$ g) $2\frac{4}{5}$ and $\frac{14}{5}$
 h) 2 and $\frac{7}{3}$ i) 1 and $\frac{5}{5}$ j) $1\frac{5}{4}$ and $2\frac{1}{4}$ k) $3\frac{2}{5}$ and $2\frac{4}{5}$ l) $2\frac{1}{5}$ and 3 m) $3\frac{1}{3}$ and $2\frac{4}{3}$ n) $\frac{11}{4}$ and $2\frac{1}{4}$

F121. Compare the fractions (see example 3):

- a) $\frac{1}{2}$ and $\frac{1}{3}$ b) $\frac{2}{7}$ and $\frac{2}{5}$ c) $1\frac{1}{2}$ and $\frac{3}{2}$ d) $3\frac{1}{4}$ and $2\frac{3}{5}$ e) $\frac{4}{7}$ and $\frac{4}{3}$ f) $\frac{10}{7}$ and $\frac{10}{11}$ g) $2\frac{4}{7}$ and $2\frac{4}{5}$
 h) $\frac{5}{4}$ and $\frac{5}{6}$ i) 1 and $\frac{7}{9}$ j) $3\frac{2}{7}$ and $3\frac{2}{3}$ k) 1 and $\frac{1}{3}$ l) $\frac{4}{5}$ and 4 m) $1\frac{2}{5}$ and $\frac{7}{6}$ n) $\frac{13}{7}$ and $2\frac{3}{5}$

F122. Compare the fractions (see example 4):

- a) $\frac{2}{3}$ and $\frac{3}{4}$ b) $\frac{4}{5}$ and $\frac{6}{7}$ c) $1\frac{1}{2}$ and $1\frac{3}{5}$ d) $\frac{3}{4}$ and $\frac{5}{7}$ e) $\frac{1}{2}$ and $\frac{3}{4}$ f) $\frac{7}{10}$ and $\frac{8}{11}$ g) $\frac{3}{8}$ and $\frac{4}{9}$
 h) $\frac{5}{4}$ and $\frac{15}{12}$ i) $\frac{5}{6}$ and $\frac{7}{9}$ j) $2\frac{2}{5}$ and $2\frac{5}{2}$ k) $\frac{3}{2}$ and $\frac{2}{3}$ l) $1\frac{4}{5}$ and $\frac{27}{15}$ m) $\frac{3}{10}$ and $\frac{4}{13}$ n) $\frac{3}{5}$ and $\frac{4}{7}$

F123. Compare the fractions (see example 5):

- a) $\frac{3}{5}$ and $\frac{8}{15}$ b) $\frac{3}{8}$ and $\frac{11}{24}$ c) $\frac{3}{4}$ and $\frac{5}{6}$ d) $\frac{4}{15}$ and $\frac{7}{20}$ e) $\frac{4}{12}$ and $\frac{5}{15}$ f) $\frac{5}{6}$ and $\frac{7}{10}$ g) $\frac{2}{9}$ and $\frac{4}{15}$

F124. Write each set of fractions in order from least to greatest:

- a) $\frac{2}{7}, \frac{1}{7}, \frac{3}{7}$ b) $\frac{3}{5}, \frac{3}{11}, \frac{3}{7}$ c) $\frac{3}{4}, \frac{2}{3}, \frac{1}{2}, \frac{4}{5}$ d) $\frac{5}{12}, \frac{3}{10}, \frac{2}{5}$ e) $\frac{1}{2}, \frac{7}{12}, \frac{3}{4}, \frac{4}{15}$ f) $1\frac{1}{4}, 1\frac{5}{24}, 1, \frac{11}{8}, 1\frac{5}{6}$

F125. Write each set of fractions in order from greatest to least:

- a) $\frac{3}{5}, \frac{4}{5}, \frac{1}{5}$ b) $\frac{2}{7}, \frac{2}{5}, \frac{2}{6}$ c) $\frac{5}{6}, \frac{3}{4}, \frac{2}{3}, \frac{6}{7}$ d) $\frac{1}{4}, \frac{3}{5}, \frac{1}{3}$ e) $\frac{7}{12}, \frac{3}{4}, \frac{2}{3}, \frac{9}{10}$ f) $1\frac{1}{4}, 1\frac{3}{8}, 1\frac{1}{2}, 1\frac{5}{16}, 1\frac{11}{32}$

Solving equations by working backward method

The *working backward* method requires to identify the operations applied to the unknown quantity x , and do the opposite operations in the opposite order.

1. If the operation is an additions with a number then apply a subtraction with the same number.

Example 1:

$$x \frac{1}{2} \frac{3}{4} \text{ so } x \frac{3}{4} \frac{1}{2} \frac{1}{4}$$

2. If the operation is a subtraction with a number then apply an addition with the same number.

Example 2:

$$x \frac{1}{2} \frac{1}{3} \text{ so } x \frac{1}{3} \frac{1}{2} \frac{5}{6}$$

3. If the operation is a multiplication by a number then apply a division by the same number.

Example 3:

$$x \frac{2}{3} \frac{3}{4} \text{ so } x \frac{3}{4} \frac{2}{3} \frac{9}{8}$$

4. If the operation is a division by a number then apply a multiplication by the same number.

Example 4:

$$x \frac{3}{5} \frac{4}{7} \text{ so } x \frac{4}{7} \frac{3}{5} \frac{12}{35}$$

5. If the operation is an inversion then apply another inversion.

Example 5:

$$\frac{1}{x} \frac{2}{3} \text{ so } \frac{x}{1} \frac{3}{2} \text{ so } x \frac{3}{2}$$

6. If more than one operation are implied then identify the operations and the order in which they appear and then do the opposite operations in the opposite order.

Example 6:

$$\frac{x \frac{2}{3} \frac{1}{2}}{4} \text{ so } x \frac{2}{3} \frac{1}{2} 4 2 \text{ so } x 2 \frac{2}{3} \frac{8}{3}$$

Example 7:

$$\frac{\frac{1}{3} x \frac{1}{2} \frac{4}{3}}{2} 2 \text{ so } \frac{1}{3} x \frac{1}{2} \frac{4}{3} 4 \text{ so } \frac{1}{3} x \frac{1}{2} \frac{8}{3}$$

$$\text{so } 3 x \frac{1}{2} \frac{3}{8} \text{ so } 3 x \frac{7}{8} \text{ so } x \frac{7}{24}$$

F126. Solve for x working backward:

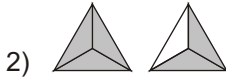
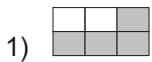
- | | | | | |
|--|--|--|--|--|
| a) $x \frac{2}{5} \frac{1}{2}$ | b) $x \frac{1}{3} \frac{4}{5}$ | c) $x \frac{1}{2} \frac{1}{3}$ | d) $x \frac{4}{7} 2 \frac{1}{3}$ | e) $\frac{1}{x} \frac{5}{7}$ |
| f) $2 x \frac{2}{3} \frac{4}{5}$ | g) $3 x \frac{1}{2} \frac{1}{3}$ | h) $\frac{x}{3} \frac{2}{3} \frac{3}{4}$ | i) $\frac{x}{3} \frac{2}{3} \frac{4}{5}$ | j) $\frac{1}{x} 2 \frac{2}{3}$ |
| k) $\frac{x}{3} \frac{1}{2} \frac{2}{3}$ | l) $\frac{x}{5} \frac{2}{3} \frac{1}{2}$ | m) $(x-1) \frac{1}{2} \frac{4}{5}$ | n) $x \frac{1}{5} \frac{2}{3} \frac{1}{2}$ | o) $\frac{1}{x} \frac{1}{3} \frac{3}{4}$ |
| p) $\frac{x-3}{2} \frac{1}{3} \frac{4}{5}$ | q) $\frac{x-2}{3} \frac{1}{2} \frac{1}{3}$ | r) $\frac{x-2}{2} \frac{2}{3} \frac{4}{5}$ | s) $\frac{x}{2} 1 \frac{2}{3} \frac{3}{5} \frac{2}{5}$ | |
| t) $\frac{x-2}{3} \frac{1}{2} \frac{2}{3} \frac{4}{3}$ | u) $\frac{x-3}{4} \frac{1}{2} \frac{1}{3} \frac{2}{3} \frac{4}{5}$ | v) $\frac{x-2}{3} \frac{1}{2} \frac{2}{3} \frac{1}{6}$ | w) $\frac{2}{x-1} \frac{2}{3} \frac{3}{4}$ | |

F127. Solve for x working backward:

- | | |
|--|--|
| a) $\frac{\frac{x-3}{8} \frac{5}{8} \frac{1}{2}}{\frac{3}{4} \frac{2}{3}} \frac{1}{2} \frac{1}{6} \frac{1}{2} \frac{3}{4} \frac{1}{4}$ | b) $\frac{\frac{1}{2} \frac{3}{2} \frac{1}{6}}{\frac{1}{x} \frac{1}{3} \frac{1}{2} \frac{4}{3}} \frac{2}{3} \frac{1}{4} \frac{4}{5} \frac{2}{3} \frac{1}{5} \frac{2}{3} \frac{5}{4}$ |
| c) $\frac{1}{10} \frac{\frac{1}{4} \frac{1}{5}}{\frac{2}{3} x \frac{2}{5}} \frac{4}{5} \frac{1}{5} \frac{3}{5} \frac{1}{2}$ | d) $\frac{1}{3} \frac{4}{5} \frac{3}{5} \frac{1}{9} \frac{1}{x-2} \frac{2}{3} \frac{2}{5} \frac{3}{10} \frac{2}{3} 0$ |

Final Test

Find the fractions:



- 3) three eighths 4) five out of seven 5) four and three quarters

Express the numbers as improper fractions:



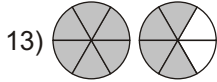
7) $2\frac{1}{3}$

8) $3\frac{2}{5}$

9) $3\frac{10}{7}$

- 10) five and two thirds 11) 2.4 12) one and five thirds

Express the numbers as mixed numbers:



14) $\frac{10}{4}$

15) $\frac{15}{4}$

16) $1\frac{5}{3}$

- 17) eight fifths 18) 1.25 19) $\frac{15}{7}$ 20) eleven quarters

Write the fractions in lowest terms:

21) $\frac{24}{36}$

22) $\frac{20}{8}$

23) $\frac{12}{18}$

24) $\frac{18}{32}$

25) $\frac{10}{12}$ and $\frac{15}{20}$

26) $\frac{4}{12}$ and $\frac{5}{15}$

27) $\frac{100}{64}$ and $\frac{75}{48}$

Check each pair of fractions for equivalence:

Add the fractions:

28) $1\frac{2}{3}$

29) $\frac{2}{5} + \frac{1}{5}$

30) $1\frac{1}{6} + 2\frac{5}{6}$

31) $\frac{1}{3} + \frac{3}{4}$

32) $\frac{10}{3} + 1\frac{5}{6}$

33) $1\frac{2}{15} + 2\frac{3}{20}$

34) $\frac{5}{4} + 1\frac{2}{3}$

Subtract the fractions:

35) $1\frac{2}{3}$

36) $\frac{10}{3} - 2$

37) $\frac{5}{7} - \frac{3}{7}$

38) $\frac{3}{4} - \frac{2}{3}$

39) $3\frac{4}{5} - 2\frac{7}{10}$

40) $5\frac{1}{4} - 2\frac{2}{3}$

41) $\frac{9}{5} - 1\frac{1}{4}$

Multiply the fractions:

42) $\frac{2}{3} \times \frac{3}{5}$

43) $2 \times \frac{3}{5}$

44) $1\frac{1}{2} \times \frac{2}{3}$

45) $1\frac{2}{3} \times 1\frac{2}{5}$

46) $\frac{3}{4} \times \frac{4}{9}$

47) $\frac{5}{3} \times 1\frac{2}{3}$

48) $1\frac{3}{4} \times 1\frac{1}{3}$

Divide the fractions:

49) $\frac{2}{3} \div \frac{3}{4}$

50) $2 \div \frac{2}{5}$

51) $\frac{8}{9} \div 4$

52) $1\frac{4}{5} \div \frac{6}{10}$

53) $1\frac{1}{6} \div 3\frac{1}{2}$

54) $\frac{22}{3} \div 1\frac{5}{6}$

55) $2\frac{1}{4} \div 2\frac{2}{5}$

Convert each fraction to a decimal:

56) $\frac{2}{5}$

57) $2\frac{2}{5}$

58) $\frac{4}{10}$

59) $\frac{15}{12}$

60) $\frac{2}{3}$

61) $1\frac{2}{7}$

62) $\frac{5}{16}$

63) $\frac{7}{8}$

64) $\frac{100}{32}$

Convert each decimal to a fraction:

65) 0.12

66) 1.5

67) 2.25

68) 0.0125

69) $0.\bar{4}$

70) $1.\bar{63}$

71) 0.725

72) 1.65

73) 0.875

Do the required conversions (use a fraction to express the result if possible):

74) 0.45 min ? s

75) 0.1 h ? min

76) 100 s ? min

77) 2000 s ? h

78) 20 min 20 s ? h

79) 1.5 dimes ? nickels

80) 7.5 quarters ? \$

81) 0.35 \$? twonies

82) 1 \$ 1 quarter ? dimes

83) $\frac{2}{3}$? twelfths

84) $\frac{1}{5}$? tenths

85) $1\frac{1}{2}$? sixths

86) $\frac{3}{16}$? eighths

87) $\frac{2}{3}$? fifteenths

Do the required operations:

88) $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$

89) $\frac{1}{2} - \frac{4}{3} + \frac{1}{6}$

90) $1\frac{2}{3} \times \frac{5}{6} \div \frac{2}{3}$

91) $2\frac{1}{2} - 3\frac{3}{4} + \frac{1}{2}$

92) $\frac{2}{3} \times \frac{1}{4} \div \frac{2}{3}$

93) $\frac{2}{3} \div 1\frac{1}{3} \times \frac{5}{6}$

94) $1 - \frac{2}{3} + 2 - \frac{4}{3}$

95) $\frac{1}{2} \times 2\frac{1}{2} \div \frac{2}{5} + \frac{3}{10}$

Solve for x:

96) $x \times \frac{1}{3} = \frac{1}{4}$

97) $x \div \frac{1}{3} = \frac{2}{5}$

98) $\frac{x}{3} = \frac{5}{6}$

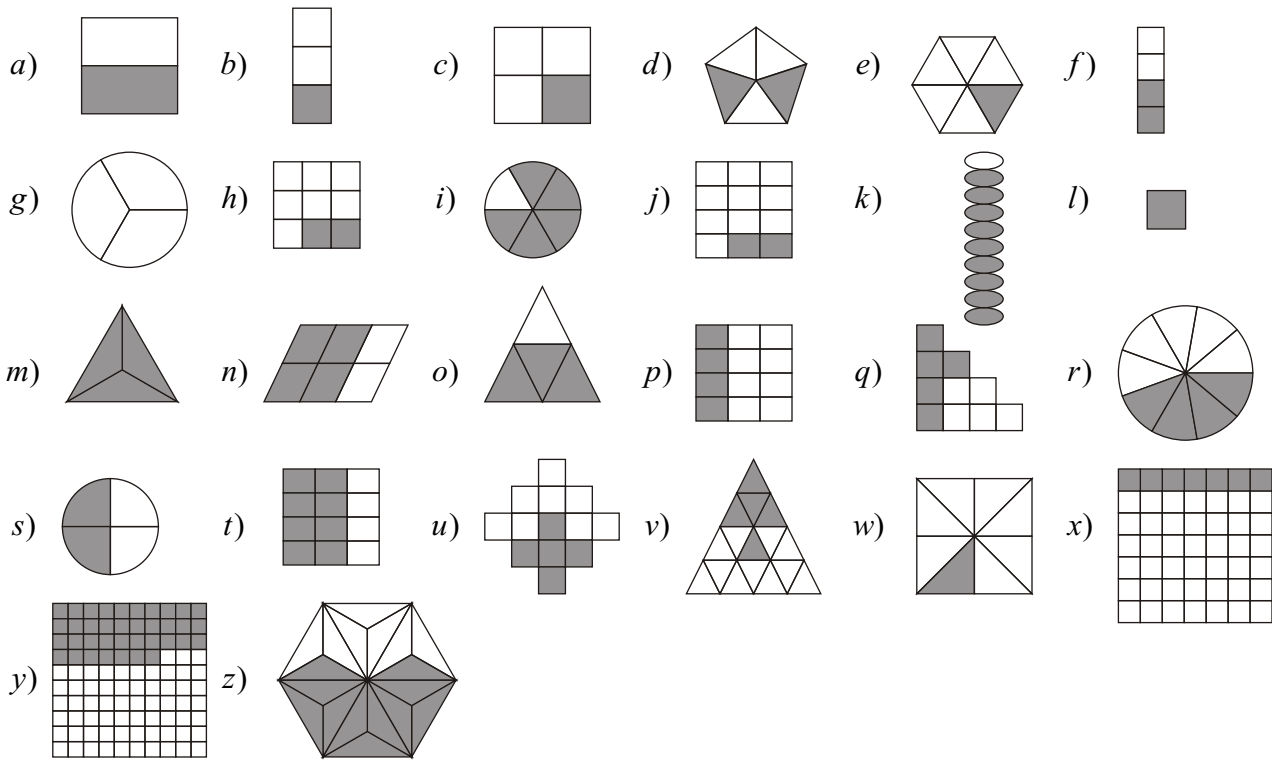
99) $\frac{2}{x} = \frac{3}{2} \div \frac{3}{4}$

100) $\frac{1}{2} \div \frac{3}{x} = \frac{2}{5}$

Answers

F01. a) $\frac{1}{2}$ b) $\frac{2}{3}$ c) $\frac{1}{4}$ d) $\frac{5}{9}$ e) $\frac{58}{100}$ f) $\frac{6}{10}$ g) $\frac{5}{6}$ h) $\frac{4}{6}$ i) $\frac{2}{6}$ j) $\frac{1}{3}$ k) $\frac{6}{18}$ l) $\frac{1}{1}$ m) $\frac{2}{5}$
 n) $\frac{6}{10}$ o) $\frac{2}{4}$ p) $\frac{9}{13}$ q) $\frac{1}{4}$ r) $\frac{3}{8}$ s) $\frac{7}{9}$ t) $\frac{21}{49}$ u) $\frac{6}{12}$ v) $\frac{0}{12}$ w) $\frac{12}{12}$ x) $\frac{7}{10}$ y) $\frac{10}{16}$

F02. The answers may vary.



F03.

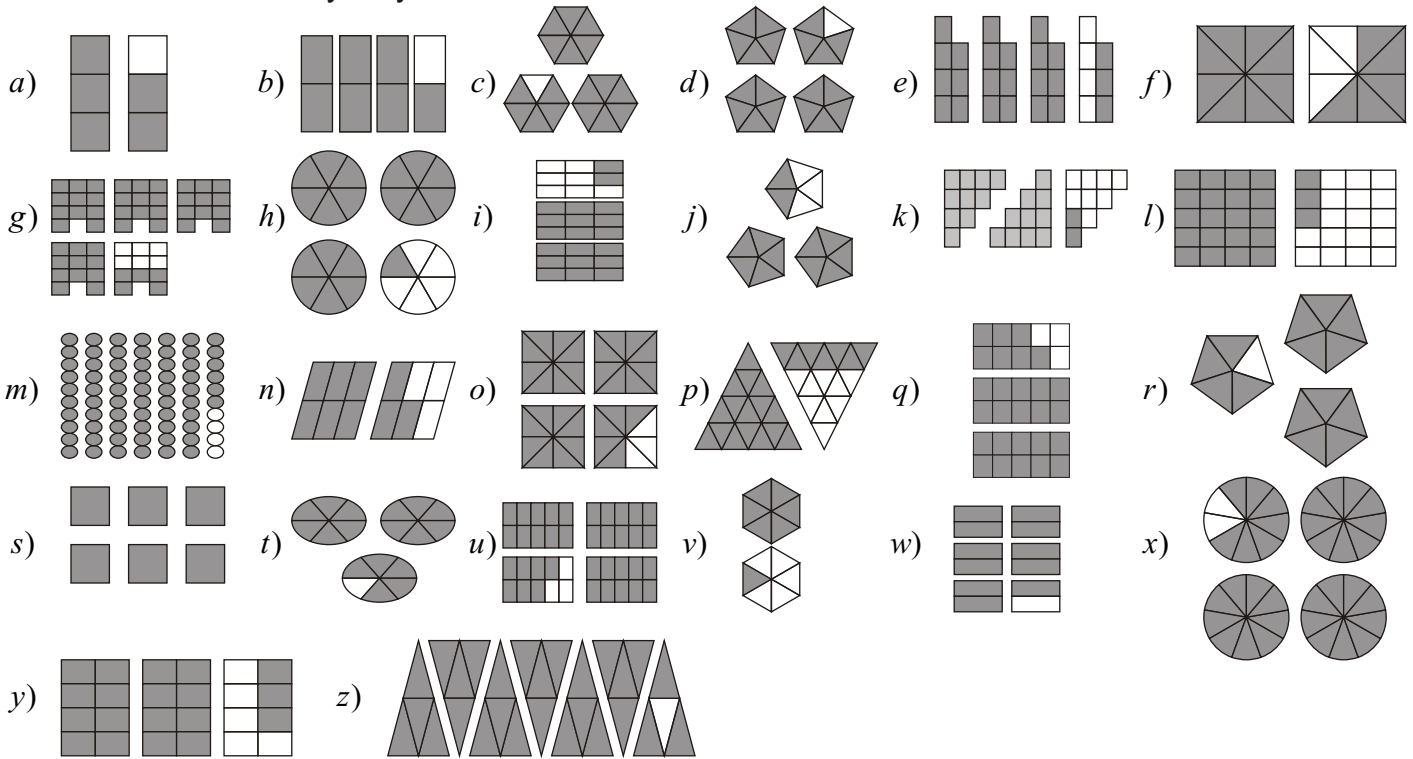
a) two thirds b) three hundredths c) one tenth d) one half e) three sevenths f) three twentieths g) one thousandth
 h) four fifths i) eight thirtieths j) eight thirteenths k) eight ninths l) five sixths m) five eighths n) seven thousandths
 o) three fiftieths p) two fifths q) twenty one hundredths r) six twelfths s) seven elevenths t) eleven fiftieths
 u) eleven millionths v) two ninths w) seven tenths x) eleven twelfths y) two fiftieths z) nine billionths

F04. a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{6}$ d) $\frac{2}{5}$ e) $\frac{4}{7}$ f) $\frac{7}{8}$ g) $\frac{11}{50}$ h) $\frac{7}{20}$ i) $\frac{5}{12}$ j) $\frac{8}{9}$ k) $\frac{6}{10}$ l) $\frac{9}{1000}$ m) $\frac{15}{1000000}$
 n) $\frac{8}{6}$ o) $\frac{3}{50}$ p) $\frac{11}{1000000000}$ q) $\frac{23}{100}$ r) $\frac{7}{13}$ s) $\frac{11}{12}$ t) $\frac{3}{1000000000}$ u) $\frac{13}{30}$ v) $\frac{1}{5}$ w) $\frac{1}{11}$ x) $\frac{8}{9}$ y) $\frac{6}{10}$ z) $\frac{6}{12}$

F05. a) $\frac{2}{3}$ b) $\frac{1}{4}$ c) $\frac{3}{5}$ d) $\frac{3}{6}$ e) $\frac{3}{12}$ f) $\frac{2}{5}$ g) $\frac{2}{7}$ h) $\frac{3}{4}$ i) $\frac{5}{12}$ j) $\frac{3}{10}$ k) $\frac{4}{6}$ l) $\frac{3}{5}$

F06. a) $1\frac{1}{2}$ b) $2\frac{1}{4}$ c) $1\frac{2}{3}$ d) $2\frac{1}{3}$ e) $1\frac{3}{6}$ f) $2\frac{1}{4}$ g) $3\frac{5}{6}$ h) 6 i) $1\frac{7}{9}$ j) $1\frac{33}{100}$ k) $4\frac{7}{10}$ l) $2\frac{1}{6}$ m) $3\frac{2}{5}$
 n) $1\frac{8}{18}$ o) $5\frac{6}{10}$ p) $6\frac{3}{4}$ q) $2\frac{9}{13}$ r) $2\frac{13}{49}$ s) $3\frac{5}{9}$ t) $2\frac{6}{8}$ u) $1\frac{6}{12}$ v) $5\frac{10}{16}$ w) $1\frac{3}{10}$ x) $2\frac{7}{10}$

F07. The answers may vary.



F08.

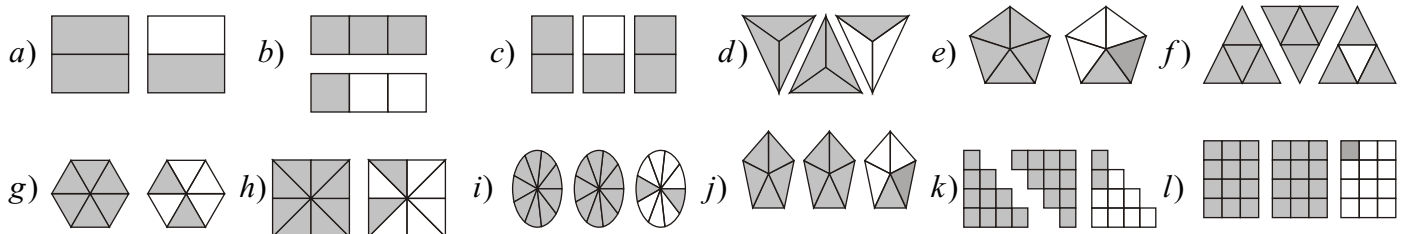
- a) one and one half b) two and one third c) one and one quarter d) two and three fifths e) one and five sixths
 f) two and three sevenths g) three and five eighths h) one and five ninths i) two and three tenths j) one and two elevenths
 k) three and five twelfths l) one and two fifteenths m) three and seven twentieths n) two and nine thirtieths
 o) two and seven fiftieths p) two and three hundredths q) three and nine thousandths r) two and seven millionths
 s) one and three fortieths t) two and seven nineteenths u) three and five sixteenths v) two and three seventeenths
 w) four and three fourteenths x) two and five fifteenths y) two and one sixtieth
 z) two and three ninetieths

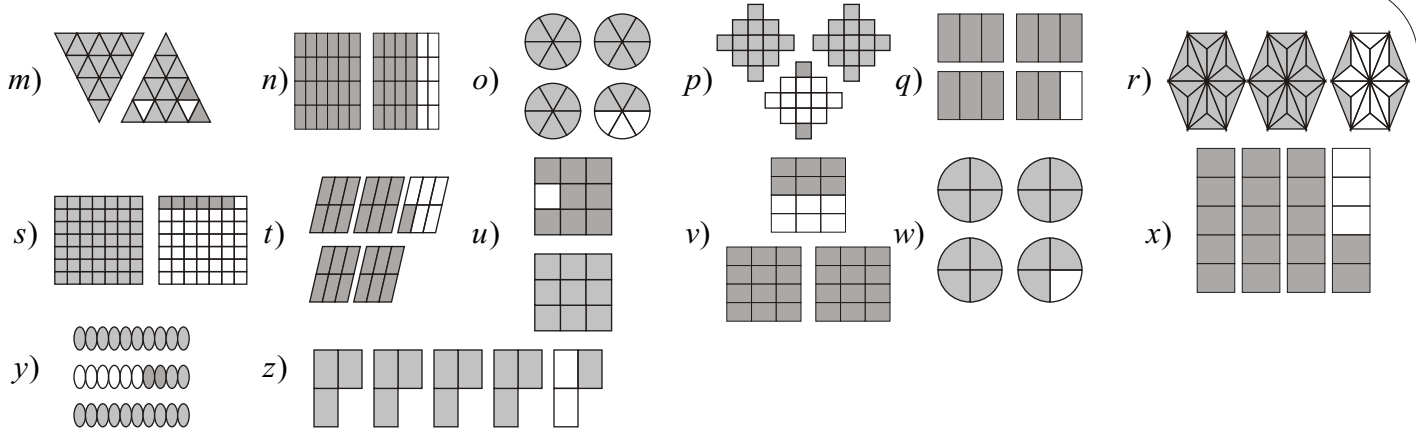
- F09. a) $2\frac{2}{3}$ b) $3\frac{1}{2}$ c) $5\frac{5}{6}$ d) $2\frac{1}{3}$ e) $4\frac{5}{7}$ f) $7\frac{5}{50}$ g) $2\frac{3}{4}$ h) $3\frac{2}{9}$ i) $6\frac{7}{100}$ j) $9\frac{1}{2}$ k) $8\frac{11}{50}$ l) $1\frac{5}{100000000}$ m) $1\frac{2}{11}$
 n) $8\frac{5}{6}$ o) $3\frac{2}{12}$ p) $5\frac{3}{1000000}$ q) $20\frac{3}{100}$ r) $6\frac{4}{15}$ s) $11\frac{4}{30}$ t) $8\frac{7}{10}$ u) $4\frac{1}{3}$ v) $1\frac{2}{5}$ w) $3\frac{2}{11}$ x) $8\frac{6}{9}$ y) $5\frac{9}{10}$ z) $1\frac{11}{12}$

- F10. a) $2\frac{3}{5}$ b) $2\frac{1}{3}$ c) $1\frac{3}{4}$ d) $1\frac{4}{6}$ e) $2\frac{3}{5}$ f) $3\frac{2}{5}$ g) $2\frac{2}{3}$ h) $3\frac{1}{2}$ i) $2\frac{4}{6}$ j) $3\frac{3}{5}$ k) $4\frac{2}{3}$ l) $2\frac{3}{4}$

- F11. a) $\frac{5}{3}$ b) $\frac{3}{1}$ c) $\frac{11}{4}$ d) $\frac{15}{6}$ e) $\frac{10}{6}$ f) $\frac{25}{9}$ g) $\frac{5}{2}$ h) $\frac{14}{4}$ i) $\frac{10}{3}$ j) $\frac{6}{5}$ k) $\frac{33}{10}$ l) $\frac{11}{8}$ m) $\frac{19}{4}$ n) $\frac{21}{10}$ o) $\frac{42}{18}$ p) $\frac{13}{10}$
 q) $\frac{17}{10}$ r) $\frac{15}{6}$ s) $\frac{31}{9}$ t) $\frac{48}{13}$ u) $\frac{32}{12}$ v) $\frac{133}{100}$ w) $\frac{74}{16}$ x) $\frac{62}{49}$

F12. The answers may vary.





F13. a) $\frac{5}{4}$ b) $\frac{7}{4}$ c) $\frac{7}{5}$ d) $\frac{4}{3}$ e) $\frac{16}{12}$ f) $\frac{12}{5}$ g) $\frac{7}{5}$ h) $\frac{11}{4}$ i) $\frac{7}{3}$ j) $\frac{13}{10}$ k) $\frac{13}{6}$ l) $\frac{13}{5}$

F14. a) $\frac{7}{3}$ $2\frac{1}{3}$ b) $\frac{3}{2}$ $1\frac{1}{2}$ c) $\frac{9}{4}$ $2\frac{1}{4}$ d) $\frac{20}{6}$ $3\frac{2}{6}$ e) $\frac{13}{6}$ $2\frac{1}{6}$ f) $\frac{23}{9}$ $2\frac{5}{9}$ g) $\frac{7}{3}$ $2\frac{1}{3}$ h) $\frac{11}{4}$ $2\frac{3}{4}$ i) $\frac{8}{3}$ $2\frac{2}{3}$
 j) $\frac{12}{5}$ $2\frac{2}{5}$ k) $\frac{25}{10}$ $2\frac{5}{10}$ l) $\frac{20}{8}$ $2\frac{4}{8}$ m) $\frac{15}{4}$ $3\frac{3}{4}$ n) $\frac{21}{10}$ $2\frac{1}{10}$ o) $\frac{24}{18}$ $1\frac{6}{18}$ p) $\frac{13}{10}$ $1\frac{3}{10}$ q) $\frac{34}{10}$ $3\frac{4}{10}$
 r) $\frac{16}{6}$ $2\frac{4}{6}$ s) $\frac{24}{9}$ $2\frac{6}{9}$ t) $\frac{32}{13}$ $2\frac{6}{13}$ u) $\frac{24}{12}$ 2 v) $\frac{136}{100}$ $1\frac{36}{100}$ w) $\frac{54}{16}$ $3\frac{6}{16}$ x) $\frac{111}{49}$ $2\frac{13}{49}$

F15. a) $\frac{3}{2}$ b) $\frac{8}{3}$ c) $\frac{15}{4}$ d) $\frac{7}{2}$ e) $\frac{17}{7}$ f) $\frac{103}{20}$ g) $\frac{53}{10}$ h) $\frac{23}{4}$ i) $\frac{67}{30}$ j) $\frac{18}{13}$ k) $\frac{26}{9}$ l) $\frac{23}{6}$ m) $\frac{21}{8}$
 n) $\frac{1207}{100}$ o) $\frac{103}{50}$ p) $\frac{24}{5}$ q) $\frac{221}{100}$ r) $\frac{25}{12}$ s) $\frac{35}{11}$ t) $\frac{149}{50}$ u) $\frac{211}{100}$ v) $\frac{2}{9}$ w) $\frac{20}{10}$ x) *not defined* y) $\frac{12}{3}$ z) $\frac{35}{10}$

F16. a) $1\frac{1}{2}$ b) $1\frac{1}{3}$ c) $1\frac{1}{4}$ d) $4\frac{1}{2}$ e) $1\frac{6}{7}$ f) $1\frac{13}{20}$ g) $12\frac{5}{10}$ h) $2\frac{4}{5}$ i) $2\frac{2}{3}$ j) $6\frac{2}{3}$ k) $8\frac{8}{9}$ l) $8\frac{2}{6}$ m) $8\frac{6}{8}$
 n) $1\frac{7}{10}$ o) $2\frac{3}{5}$ p) $4\frac{2}{5}$ q) $2\frac{1}{10}$ r) $1\frac{4}{12}$ s) $6\frac{4}{11}$ t) $2\frac{11}{50}$ u) $11\frac{1}{10}$ v) $0\frac{0}{9}$ w) *not defined* x) $1\frac{4}{7}$ y) $1\frac{5}{15}$ z) $7\frac{7}{9}$

F17. a) 1 b) 1 c) 1 d) 2 e) 3 f) 8 g) 10 h) 2 i) 3 j) 4 k) 6 l) 15 m) 4 n) 5 o) 0 p) *not defined*
 q) *not defined* r) 1 s) 2 t) 0 u) *not defined*

F18. The answers may vary.

a) $\frac{2}{2}; \frac{5}{5}$ b) $\frac{6}{2}; \frac{9}{3}$ c) $\frac{14}{2}; \frac{21}{3}$ d) $\frac{8}{2}; \frac{12}{3}$ e) $\frac{2}{1}; \frac{12}{6}$ f) $\frac{10}{2}; \frac{45}{9}$ g) $\frac{100}{10}; \frac{30}{3}$ h) $\frac{0}{2}; \frac{0}{3}$ i) $\frac{50}{2}; \frac{125}{5}$ j) $\frac{100}{1}; \frac{700}{7}$
 k) $\frac{22}{2}; \frac{55}{5}$ l) $\frac{16}{2}; \frac{24}{3}$ m) $\frac{26}{2}; \frac{39}{3}$ n) $\frac{34}{2}; \frac{51}{3}$

F19. a) *improper* b) *whole* c) *improper* d) *proper* e) *mixed* f) *improper* g) *mixed* h) *not defined* i) *proper*
 j) *whole* k) *improper* l) *proper* m) *mixed* n) *mixed* o) *mixed* p) *improper* q) *whole* r) *improper* s) *improper*
 t) *mixed* u) *proper* v) *mixed* w) *improper* x) *proper* y) *improper* z) *improper*

F20. a) 1 b) $\frac{2}{3}$ c) $\frac{3}{4}$ d) $\frac{5}{9}$ e) $\frac{52}{100}$ f) $\frac{7}{10}$ g) $\frac{4}{6}$ h) $\frac{5}{6}$ i) $\frac{4}{6}$ j) $\frac{2}{3}$ k) $\frac{10}{18}$ l) $\frac{4}{6}$ m) $\frac{3}{5}$ n) $\frac{7}{10}$ o) $\frac{3}{4}$ p) $\frac{10}{13}$ q) $\frac{3}{4}$
 r) $\frac{6}{8}$ s) $\frac{7}{9}$ t) $\frac{25}{49}$ u) $\frac{10}{12}$ v) $\frac{5}{9}$ w) $\frac{6}{12}$ x) $\frac{7}{10}$ y) $\frac{12}{16}$

F21. a) $\frac{2}{2}$ 1 b) $\frac{4}{3}$ $1\frac{1}{3}$ c) $\frac{9}{4}$ $2\frac{1}{4}$ d) $\frac{12}{9}$ $1\frac{3}{9}$ e) $\frac{120}{100}$ $1\frac{20}{100}$ f) $\frac{24}{10}$ $2\frac{4}{10}$ g) $\frac{8}{6}$ $1\frac{2}{6}$ h) $\frac{20}{6}$ $3\frac{2}{6}$ i) $\frac{7}{6}$ $1\frac{1}{6}$
 j) $\frac{4}{3}$ $1\frac{1}{3}$ k) $\frac{19}{18}$ $1\frac{1}{18}$ l) $\frac{19}{6}$ $3\frac{1}{6}$ m) $\frac{6}{5}$ $1\frac{1}{5}$ n) $\frac{13}{10}$ $1\frac{3}{10}$ o) $\frac{5}{4}$ $1\frac{1}{4}$ p) $\frac{15}{13}$ $1\frac{2}{13}$ q) $\frac{10}{4}$ $2\frac{2}{4}$

- r) $\frac{10}{8}$ $1\frac{2}{8}$ s) $\frac{13}{9}$ $1\frac{4}{9}$ t) $\frac{63}{49}$ $1\frac{14}{49}$ u) $\frac{19}{12}$ $1\frac{7}{12}$ v) $\frac{13}{9}$ $1\frac{4}{9}$ w) $\frac{18}{12}$ $1\frac{6}{12}$ x) $\frac{28}{16}$ $1\frac{12}{16}$ y) $\frac{8}{6}$ $1\frac{2}{6}$
- F22. a) $\frac{3}{4}$ b) $\frac{2}{3}$ c) $\frac{3}{5}$ d) $\frac{8}{11}$ e) $\frac{3}{6}$ f) $\frac{5}{7}$ g) $\frac{12}{19}$ h) $\frac{30}{100}$ i) $\frac{29}{35}$ j) $\frac{12}{41}$ k) $\frac{11}{13}$ l) $\frac{32}{54}$ m) $\frac{5}{10}$ n) $\frac{9}{12}$ o) $\frac{13}{17}$ p) $\frac{0}{4}$ q) $\frac{5}{13}$
- r) $\frac{10}{20}$ s) $\frac{18}{25}$ t) $\frac{7}{9}$ u) $\frac{7}{19}$ v) $\frac{5}{10}$ w) $\frac{20}{30}$ x) $\frac{11}{13}$ y) $\frac{40}{400}$
- F23. a) $1\frac{1}{3}$ b) $1\frac{1}{4}$ c) $1\frac{2}{5}$ d) $1\frac{2}{6}$ e) $1\frac{1}{7}$ f) $1\frac{4}{8}$ g) $1\frac{6}{9}$ h) $1\frac{4}{10}$ i) $1\frac{7}{11}$ j) $1\frac{2}{12}$ k) $1\frac{6}{15}$ l) $1\frac{3}{20}$ m) $1\frac{5}{25}$ n) $1\frac{38}{50}$ o) $1\frac{10}{100}$
- p) $1\frac{4}{9}$ q) $1\frac{7}{10}$ r) $1\frac{12}{40}$ s) $4\frac{1}{3}$ t) $2\frac{4}{10}$ u) $1\frac{8}{9}$ v) $3\frac{2}{10}$ w) $1\frac{49}{50}$ x) $8\frac{1}{3}$ y) $4\frac{9}{10}$
- F24. a) 4 b) $3\frac{2}{3}$ c) $3\frac{3}{4}$ d) $4\frac{4}{5}$ e) $3\frac{5}{6}$ f) $5\frac{5}{7}$ g) $5\frac{5}{8}$ h) $3\frac{3}{9}$ i) $2\frac{8}{10}$ j) $5\frac{11}{15}$ k) 4 l) $7\frac{3}{7}$ m) $6\frac{3}{10}$ n) $8\frac{3}{20}$ o) $7\frac{5}{11}$
- p) $8\frac{1}{9}$ q) $10\frac{3}{10}$ r) $6\frac{3}{30}$ s) $10\frac{1}{3}$ t) 9 u) $4\frac{14}{19}$ v) $8\frac{21}{30}$ w) $5\frac{1}{23}$ x) $7\frac{20}{35}$ y) $4\frac{10}{100}$
- F25. a) $3\frac{1}{2}$ b) $3\frac{2}{3}$ c) $4\frac{1}{4}$ d) $7\frac{2}{5}$ e) $7\frac{5}{6}$ f) $9\frac{2}{7}$ g) $13\frac{5}{8}$ h) $4\frac{1}{9}$ i) $12\frac{1}{10}$ j) 2 k) $6\frac{4}{5}$ l) $7\frac{1}{7}$ m) $5\frac{1}{10}$ n) $5\frac{2}{20}$
- o) $27\frac{2}{11}$ p) $20\frac{1}{9}$ q) 40 r) $14\frac{11}{30}$ s) $15\frac{1}{3}$ t) $8\frac{4}{10}$ u) 11 v) $9\frac{14}{30}$ w) $5\frac{17}{23}$ x) $3\frac{2}{35}$ y) $113\frac{11}{100}$
- F26. a) 6 b) $1\frac{2}{4}$ c) $7\frac{1}{4}$ d) $7\frac{1}{2}$ e) 8 f) 3 g) $6\frac{7}{10}$ h) 9 i) $4\frac{13}{50}$ j) $12\frac{4}{6}$ k) $3\frac{2}{5}$ l) $2\frac{8}{9}$ m) 8 n) $4\frac{3}{12}$
- o) $2\frac{3}{11}$ p) $3\frac{8}{9}$ q) $3\frac{9}{10}$ r) $12\frac{1}{2}$ s) $6\frac{52}{100}$ t) 11 u) $2\frac{4}{9}$ v) $8\frac{1}{4}$ w) 8 x) $3\frac{9}{10}$ y) 3
- F27. a) 3 b) $2\frac{3}{4}$ c) 4 d) 3 e) $4\frac{1}{5}$ f) 5 g) $3\frac{7}{10}$ h) 8 i) $4\frac{18}{50}$ j) $5\frac{4}{6}$ k) $4\frac{2}{5}$ l) $6\frac{5}{9}$ m) 7 n) 5
- o) $10\frac{3}{11}$ p) $5\frac{6}{9}$ q) $7\frac{2}{10}$ r) $16\frac{1}{2}$ s) $6\frac{33}{100}$ t) $7\frac{1}{10}$ u) 4 v) $8\frac{2}{4}$ w) $6\frac{2}{3}$ x) $4\frac{6}{10}$ y) $9\frac{1}{10}$
- F28. a) $\frac{1}{2}$ $\frac{2}{4}$ b) $\frac{1}{3}$ $\frac{2}{6}$ c) $\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$ d) $\frac{2}{3}$ $\frac{6}{9}$ e) $\frac{1}{5}$ $\frac{2}{10}$ f) $\frac{1}{2}$ $\frac{3}{6}$ g) $\frac{1}{2}$ $\frac{2}{4}$ h) $\frac{1}{3}$ $\frac{2}{6}$ i) $\frac{2}{3}$ $\frac{6}{9}$ j) $\frac{1}{4}$ $\frac{2}{8}$ k) $\frac{1}{6}$ $\frac{3}{18}$
- l) $\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$ m) $\frac{1}{2}$ $\frac{2}{4}$ n) $\frac{1}{6}$ $\frac{2}{12}$ $\frac{4}{24}$ o) $\frac{1}{2}$ $\frac{2}{4}$ $\frac{4}{8}$ $\frac{8}{16}$ $\frac{16}{32}$ p) $\frac{1}{4}$ $\frac{5}{20}$ q) $\frac{1}{6}$ $\frac{2}{12}$ r) $\frac{1}{4}$ $\frac{2}{8}$ s) $\frac{1}{5}$ $\frac{2}{10}$ $\frac{4}{20}$ $\frac{8}{40}$
- t) $\frac{1}{4}$ $\frac{3}{12}$ u) $\frac{1}{9}$ $\frac{2}{18}$ $\frac{4}{36}$ v) $\frac{1}{4}$ $\frac{3}{12}$ w) $\frac{1}{4}$ $\frac{2}{8}$ $\frac{4}{16}$ x) $\frac{2}{3}$ $\frac{4}{6}$ $\frac{8}{12}$ $\frac{16}{24}$ y) $\frac{3}{4}$ $\frac{6}{8}$ $\frac{12}{16}$ $\frac{24}{32}$ $\frac{48}{64}$
- F29. The answers may vary.
- a) $\frac{2}{4}$ $\frac{3}{6}$ $\frac{4}{8}$ b) $\frac{2}{6}$ $\frac{3}{9}$ $\frac{4}{12}$ c) $\frac{4}{6}$ $\frac{6}{9}$ $\frac{8}{12}$ d) $\frac{2}{8}$ $\frac{3}{12}$ $\frac{4}{16}$ e) $\frac{6}{8}$ $\frac{9}{12}$ $\frac{12}{16}$ f) $\frac{2}{10}$ $\frac{3}{15}$ $\frac{4}{20}$ g) $\frac{6}{10}$ $\frac{9}{15}$ $\frac{12}{20}$
- h) $\frac{2}{12}$ $\frac{3}{18}$ $\frac{4}{24}$ i) $\frac{10}{12}$ $\frac{15}{18}$ $\frac{20}{24}$ j) $\frac{2}{14}$ $\frac{3}{21}$ $\frac{4}{28}$ k) $\frac{4}{14}$ $\frac{6}{21}$ $\frac{8}{28}$ l) $\frac{2}{20}$ $\frac{3}{30}$ $\frac{4}{40}$ m) $\frac{2}{200}$ $\frac{3}{300}$ $\frac{4}{400}$
- n) $\frac{10}{24}$ $\frac{15}{36}$ $\frac{20}{48}$ o) $\frac{4}{22}$ $\frac{6}{33}$ $\frac{8}{44}$ p) $\frac{4}{18}$ $\frac{6}{27}$ $\frac{8}{36}$ q) $\frac{6}{14}$ $\frac{9}{21}$ $\frac{12}{28}$ r) $\frac{10}{22}$ $\frac{15}{33}$ $\frac{20}{44}$ s) $\frac{4}{30}$ $\frac{6}{45}$ $\frac{8}{60}$
- t) $\frac{8}{10}$ $\frac{12}{15}$ $\frac{16}{20}$ u) $\frac{10}{14}$ $\frac{15}{21}$ $\frac{20}{28}$ v) $\frac{2}{60}$ $\frac{3}{90}$ $\frac{4}{120}$ w) $\frac{6}{100}$ $\frac{9}{150}$ $\frac{12}{200}$ x) $\frac{6}{400}$ $\frac{9}{600}$ $\frac{12}{800}$ y) $\frac{6}{2000}$ $\frac{9}{3000}$ $\frac{12}{4000}$
- F30. a) $\frac{1}{2}$ b) $\frac{1}{3}$ c) $\frac{2}{8}$ $\frac{1}{4}$ d) $\frac{1}{5}$ e) $\frac{3}{18}$ $\frac{2}{12}$ $\frac{1}{6}$ f) $\frac{5}{50}$ $\frac{2}{20}$ $\frac{1}{10}$ g) $\frac{2}{6}$ $\frac{1}{3}$ h) $\frac{4}{5}$ i) $\frac{2}{3}$ j) $\frac{3}{4}$ k) $\frac{10}{15}$ $\frac{6}{9}$ $\frac{2}{3}$ l) $\frac{5}{8}$
- m) $\frac{9}{30}$ $\frac{6}{20}$ $\frac{3}{10}$ n) $\frac{30}{45}$ $\frac{20}{30}$ $\frac{12}{18}$ $\frac{10}{15}$ $\frac{6}{9}$ $\frac{4}{6}$ $\frac{2}{3}$ o) $\frac{30}{75}$ $\frac{20}{50}$ $\frac{12}{30}$ $\frac{10}{25}$ $\frac{6}{15}$ $\frac{4}{10}$ $\frac{2}{5}$ p) $\frac{4}{7}$ q) $\frac{25}{40}$ $\frac{15}{24}$ $\frac{5}{8}$
- r) $\frac{33}{77}$ $\frac{6}{14}$ $\frac{3}{7}$ s) $\frac{28}{210}$ $\frac{14}{105}$ $\frac{8}{60}$ $\frac{4}{30}$ $\frac{2}{15}$ t) $\frac{16}{64}$ $\frac{8}{32}$ $\frac{4}{16}$ $\frac{2}{8}$ $\frac{1}{4}$ u) $\frac{9}{27}$ $\frac{3}{9}$ $\frac{1}{3}$ v) $\frac{9}{11}$ w) $\frac{2}{11}$ x) $\frac{45}{125}$ $\frac{9}{25}$
- y) $\frac{32}{512}$ $\frac{16}{256}$ $\frac{8}{128}$ $\frac{4}{64}$ $\frac{2}{32}$ $\frac{1}{16}$

F31. a) $\frac{1}{12}$ b) $\frac{3}{7}$ c) $\frac{2}{3}$ d) $\frac{1}{4}$ e) $\frac{3}{4}$ f) $\frac{1}{5}$ g) $\frac{3}{5}$ h) $\frac{1}{6}$ i) $\frac{5}{6}$ j) $\frac{1}{7}$ k) $\frac{2}{7}$ l) $\frac{3}{14}$ m) $\frac{7}{9}$ n) $\frac{1}{8}$ o) $\frac{2}{11}$ p) $\frac{2}{9}$ q) $\frac{3}{7}$
 r) $\frac{5}{22}$ s) $\frac{2}{15}$ t) $\frac{1}{2}$ u) $\frac{5}{7}$ v) $\frac{7}{15}$ w) $\frac{11}{25}$ x) $\frac{3}{20}$ y) $\frac{7}{10}$

F32. a) $\frac{3}{4}$ b) $\frac{4}{5}$ c) $\frac{2}{3}$ d) $\frac{3}{4}$ e) $\frac{7}{9}$ f) $\frac{2}{7}$ g) $\frac{8}{15}$ h) $\frac{2}{5}$ i) $\frac{7}{13}$ j) $\frac{4}{5}$ k) $\frac{2}{3}$ l) $\frac{5}{8}$ m) $\frac{3}{10}$ n) $\frac{4}{5}$ o) $\frac{2}{5}$ p) $\frac{4}{9}$ q) $\frac{5}{8}$
 r) $\frac{3}{7}$ s) $\frac{2}{15}$ t) $\frac{1}{4}$ u) $\frac{1}{3}$ v) $\frac{9}{11}$ w) $\frac{2}{11}$ x) $\frac{9}{25}$ y) $\frac{1}{16}$

F33. a) yes b) yes c) no d) yes e) no f) yes g) no h) yes i) no j) no k) no l) yes m) yes
 n) no o) no p) yes q) no r) yes s) no t) yes u) yes v) no w) yes x) yes y) no

F34. a) no b) yes c) no d) yes e) no f) no g) yes h) yes i) no j) yes k) no l) yes m) no
 n) yes o) no p) no q) yes r) no s) no t) yes u) yes v) no w) no x) yes y) yes

F35. a) 4 b) 2 c) 16 d) 4 e) 20 f) 8 g) 3 h) 4 i) 15 j) 60 k) 15 l) 54 m) 30 n) 72 o) $5\frac{5}{11}$

F36. a) 3 b) 21 c) 15 d) 9 e) 6 f) 15 g) 30 h) 56 i) 27 j) 84

F37. a) 2 b) 15 c) 5 d) 25 e) 7 f) 3 g) 8 h) 12 i) 24 j) 30 k) 64 l) 36 m) 18 n) 48 o) 35

F38. a) $1\frac{1}{2}$ b) $1\frac{1}{5}$ c) $3\frac{3}{4}$ d) $3\frac{1}{3}$ e) $1\frac{11}{16}$ f) $6\frac{6}{7}$ g) $3\frac{3}{4}$ h) $3\frac{3}{5}$ i) $\frac{1}{2}$ j) $38\frac{4}{7}$

F39. a) $\frac{5}{6}$ b) $1\frac{5}{12}$ c) $\frac{13}{20}$ d) $\frac{11}{30}$ e) $\frac{29}{35}$ f) $1\frac{3}{56}$ g) $\frac{67}{72}$ h) $\frac{77}{90}$ i) $\frac{43}{110}$ j) $\frac{31}{110}$ k) $\frac{17}{30}$ l) $3\frac{19}{30}$ m) $1\frac{2}{9}$ n) $\frac{7}{30}$ o) $\frac{2}{5}$
 p) $2\frac{1}{6}$ q) $3\frac{7}{15}$ r) $5\frac{1}{6}$ s) $3\frac{1}{110}$ t) $3\frac{13}{20}$ u) $\frac{3}{10}$ v) $1\frac{79}{200}$ w) $2\frac{1}{6}$ x) $3\frac{13}{24}$ y) $1\frac{5}{32}$

F40. a) $1\frac{1}{6}$ b) $\frac{7}{12}$ c) $\frac{19}{20}$ d) $1\frac{7}{30}$ e) $\frac{26}{35}$ f) $\frac{53}{56}$ g) $\frac{5}{6}$ h) $1\frac{1}{18}$ i) $\frac{8}{15}$ j) $\frac{2}{3}$ k) $\frac{8}{15}$ l) $3\frac{7}{12}$ m) $1\frac{2}{9}$ n) $\frac{7}{30}$ o) $\frac{1}{2}$
 p) $2\frac{1}{6}$ q) $4\frac{4}{15}$ r) $4\frac{5}{6}$ s) $2\frac{7}{15}$ t) $4\frac{7}{20}$ u) $\frac{9}{20}$ v) $1\frac{13}{40}$ w) $1\frac{7}{12}$ x) $4\frac{1}{12}$ y) $1\frac{5}{12}$

F41. a) $1\frac{1}{12}$ b) $\frac{25}{48}$ c) $\frac{17}{80}$ d) $\frac{7}{20}$ e) $\frac{18}{35}$ f) $1\frac{13}{28}$ g) $\frac{67}{144}$ h) $\frac{23}{36}$ i) $\frac{19}{55}$ j) $\frac{5}{22}$ k) $\frac{17}{30}$ l) $\frac{11}{15}$ m) $\frac{13}{18}$ n) $\frac{9}{50}$ o) $\frac{22}{75}$
 p) $1\frac{41}{150}$ q) $2\frac{7}{75}$ r) $3\frac{47}{144}$ s) $1\frac{13}{150}$ t) $3\frac{37}{450}$ u) $\frac{27}{80}$ v) $1\frac{59}{200}$ w) $1\frac{79}{120}$ x) $3\frac{3}{28}$ y) $1\frac{13}{192}$

F42. a) $1\frac{11}{12}$ b) $1\frac{11}{24}$ c) $1\frac{29}{120}$ d) $1\frac{41}{48}$ e) $1\frac{2}{15}$ f) $\frac{2}{5}$ g) $\frac{197}{252}$ h) $1\frac{9}{20}$ i) $\frac{181}{240}$ j) $\frac{11}{30}$ k) $\frac{31}{180}$ l) $1\frac{7}{60}$ m) $1\frac{7}{15}$ n) $\frac{38}{75}$
 o) $\frac{107}{525}$ p) $1\frac{199}{2400}$ q) $1\frac{17}{60}$ r) $\frac{19}{20}$ s) $1\frac{7}{60}$ t) $\frac{227}{300}$

F43. a) $\frac{2}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ b) $\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ c) $\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{4}$ d) $\frac{5}{9}$ $\frac{3}{9}$ $\frac{2}{9}$ e) $\frac{50}{100}$ $\frac{20}{100}$ $\frac{3}{10}$ f) $\frac{7}{10}$ $\frac{5}{10}$ $\frac{1}{5}$ g) $\frac{5}{6}$ $\frac{3}{6}$ $\frac{1}{3}$
 h) $\frac{3}{6}$ $\frac{1}{6}$ $\frac{1}{3}$ i) $\frac{4}{6}$ $\frac{2}{6}$ $\frac{1}{3}$ j) $\frac{2}{3}$ $\frac{1}{3}$ $\frac{1}{3}$ k) $\frac{16}{18}$ $\frac{8}{18}$ $\frac{4}{9}$ l) $\frac{3}{6}$ $\frac{2}{6}$ $\frac{1}{6}$ m) $\frac{3}{5}$ $\frac{2}{5}$ $\frac{1}{5}$ n) $\frac{6}{10}$ $\frac{2}{10}$ $\frac{2}{5}$
 o) $\frac{3}{4}$ $\frac{1}{4}$ $\frac{1}{2}$ p) $\frac{9}{13}$ $\frac{4}{13}$ $\frac{5}{13}$ q) $\frac{3}{4}$ $\frac{2}{4}$ $\frac{1}{4}$ r) $\frac{6}{8}$ $\frac{2}{8}$ $\frac{1}{2}$ s) $\frac{5}{9}$ $\frac{2}{9}$ $\frac{3}{9}$ $\frac{1}{3}$ t) $\frac{23}{49}$ $\frac{8}{49}$ $\frac{15}{49}$
 u) $\frac{8}{12}$ $\frac{4}{12}$ $\frac{1}{3}$ v) $\frac{5}{9}$ $\frac{2}{9}$ $\frac{1}{3}$ w) $\frac{8}{12}$ $\frac{2}{12}$ $\frac{1}{2}$ x) $\frac{7}{10}$ $\frac{3}{10}$ $\frac{2}{5}$ y) $\frac{11}{16}$ $\frac{4}{16}$ $\frac{7}{16}$

F44. a) $\frac{1}{4}$ b) $\frac{2}{3}$ c) $\frac{3}{5}$ d) $\frac{7}{11}$ e) $\frac{1}{2}$ f) $\frac{3}{7}$ g) $\frac{2}{19}$ h) $\frac{1}{5}$ i) $\frac{2}{7}$ j) $\frac{1}{10}$ k) $\frac{1}{3}$ l) $\frac{1}{9}$ m) $\frac{6}{9}$ $\frac{2}{3}$ n) $\frac{1}{8}$ o) 2
 p) $\frac{1}{2}$ q) $\frac{3}{4}$ r) $\frac{1}{4}$ s) $\frac{3}{10}$ t) $\frac{1}{250}$ u) $\frac{1}{10}$ v) $\frac{1}{5}$ w) $\frac{1}{5}$ x) $\frac{1}{3}$ y) $\frac{6}{60}$ $\frac{1}{10}$

- F45. a) $\frac{2}{3}$ b) 1 c) $1\frac{1}{5}$ d) $\frac{9}{11}$ e) $2\frac{1}{2}$ f) $2\frac{5}{7}$ g) $1\frac{8}{19}$ h) 9 i) $1\frac{5}{7}$ j) $1\frac{19}{20}$ k) $1\frac{5}{6}$ l) $3\frac{2}{7}$ m) $2\frac{5}{9}$ n) $\frac{13}{16}$ o) 0
 p) $1\frac{1}{2}$ q) $5\frac{5}{8}$ r) $\frac{5}{6}$ s) $2\frac{1}{4}$ t) $1\frac{1}{500}$ u) $\frac{24}{25}$ v) $3\frac{1}{5}$ w) $1\frac{3}{25}$ x) $\frac{10}{11}$ y) $3\frac{1}{10}$
- F46. a) $1\frac{1}{3}$ b) 3 c) $8\frac{2}{5}$ d) $6\frac{5}{11}$ e) $2\frac{1}{2}$ f) $3\frac{3}{7}$ g) $1\frac{10}{19}$ h) $3\frac{4}{5}$ i) $1\frac{1}{7}$ j) $2\frac{1}{4}$ k) $3\frac{1}{6}$ l) $2\frac{2}{7}$ m) $1\frac{2}{3}$ n) $3\frac{1}{4}$ o) $3\frac{3}{7}$
 p) $3\frac{1}{2}$ q) $4\frac{1}{2}$ r) $2\frac{1}{2}$ s) $2\frac{1}{4}$ t) $3\frac{1}{50}$ u) $2\frac{1}{5}$ v) $2\frac{1}{5}$ w) $5\frac{3}{25}$ x) $2\frac{2}{11}$ y) $2\frac{1}{3}$
- F47. a) $\frac{2}{3}$ b) $2\frac{5}{7}$ c) $2\frac{4}{5}$ d) $2\frac{8}{11}$ e) $4\frac{1}{2}$ f) $1\frac{5}{9}$ g) $1\frac{8}{19}$ h) 9 i) $1\frac{5}{7}$ j) $1\frac{19}{20}$ k) $1\frac{5}{6}$ l) $5\frac{1}{2}$ m) $1\frac{2}{3}$ n) $1\frac{1}{2}$ o) $2\frac{4}{7}$
 p) $3\frac{1}{4}$ q) $4\frac{3}{8}$ r) $\frac{5}{6}$ s) $3\frac{1}{4}$ t) $2\frac{1}{100}$ u) $3\frac{24}{25}$ v) $5\frac{4}{5}$ w) $4\frac{18}{25}$ x) $\frac{7}{11}$ y) $2\frac{13}{15}$
- F48. a) $\frac{1}{6}$ b) $\frac{1}{12}$ c) $\frac{3}{20}$ d) $\frac{1}{30}$ e) $\frac{1}{35}$ f) $\frac{11}{56}$ g) $\frac{13}{72}$ h) $\frac{23}{90}$ i) $\frac{23}{110}$ j) $\frac{9}{110}$ k) $\frac{1}{30}$ l) $\frac{11}{30}$ m) $\frac{1}{9}$ n) $\frac{1}{30}$ o) 0
 p) $\frac{5}{6}$ q) $2\frac{2}{15}$ r) $\frac{1}{6}$ s) $\frac{89}{110}$ t) $\frac{17}{20}$ u) $\frac{1}{10}$ v) $\frac{151}{200}$ w) $1\frac{1}{3}$ x) $\frac{19}{24}$ y) $\frac{31}{32}$
- F49. a) $\frac{1}{6}$ b) $\frac{1}{12}$ c) $\frac{11}{20}$ d) $\frac{13}{30}$ e) $\frac{6}{35}$ f) $\frac{11}{56}$ g) $\frac{1}{6}$ h) $\frac{1}{18}$ i) $\frac{2}{15}$ j) $\frac{1}{12}$ k) $\frac{2}{15}$ l) $\frac{1}{4}$ m) $\frac{1}{9}$ n) $\frac{1}{6}$ o) $\frac{1}{10}$
 p) $\frac{5}{6}$ q) $1\frac{1}{15}$ r) $1\frac{5}{6}$ s) $\frac{13}{15}$ t) $\frac{17}{20}$ u) $\frac{1}{10}$ v) $\frac{37}{40}$ w) $1\frac{1}{12}$ x) $\frac{7}{12}$ y) $1\frac{5}{24}$
- F50. a) $\frac{7}{12}$ b) $\frac{7}{48}$ c) $\frac{7}{80}$ d) $\frac{11}{60}$ e) $\frac{3}{35}$ f) $1\frac{1}{28}$ g) $\frac{13}{144}$ h) $\frac{7}{36}$ i) $\frac{14}{55}$ j) $\frac{7}{66}$ k) $\frac{1}{30}$ l) $\frac{4}{15}$ m) $\frac{5}{18}$ n) $\frac{1}{50}$ o) $\frac{2}{75}$
 p) $\frac{121}{150}$ q) $\frac{73}{75}$ r) $1\frac{11}{144}$ s) $\frac{7}{150}$ t) $\frac{7}{450}$ u) $\frac{1}{16}$ v) $\frac{191}{200}$ w) $\frac{1}{120}$ x) $1\frac{1}{28}$ y) $\frac{187}{192}$
- F51. a) $\frac{11}{12}$ b) $\frac{23}{24}$ c) $\frac{19}{120}$ d) $\frac{47}{48}$ e) $\frac{8}{15}$ f) $\frac{1}{5}$ g) $\frac{31}{252}$ h) $\frac{23}{60}$ i) $\frac{17}{48}$ j) $\frac{2}{15}$ k) $\frac{7}{60}$ l) $\frac{31}{60}$ m) $\frac{1}{5}$ n) $\frac{4}{75}$ o) $\frac{22}{525}$
 p) $\frac{1}{48}$ q) $\frac{7}{60}$ r) $\frac{13}{60}$ s) $\frac{5}{12}$ t) $\frac{31}{300}$
- F52. a) $1\frac{17}{60}$ b) $\frac{13}{60}$ c) $8\frac{13}{60}$ d) $\frac{23}{60}$ e) 1 f) $\frac{1}{2}$ g) 1 h) $\frac{23}{60}$ i) $\frac{53}{60}$ j) 0 k) $\frac{13}{60}$ l) $1\frac{5}{12}$ m) $2\frac{25}{28}$ n) $4\frac{7}{60}$ o) $\frac{3}{20}$
 p) $\frac{7}{12}$ q) $2\frac{3}{40}$ r) $\frac{18}{25}$ s) $1\frac{7}{12}$ t) 1 u) $\frac{7}{30}$
- F53. a) $\frac{3}{8}$ b) $\frac{1}{3}$ c) $\frac{3}{20}$ d) $\frac{3}{2}$ e) $\frac{3}{8}$ f) 2 g) $\frac{1}{4}$ h) $\frac{1}{8}$ i) $\frac{5}{8}$ j) $\frac{1}{40}$ k) $\frac{4}{9}$ l) $\frac{2}{5}$ m) 2 n) $2\frac{2}{5}$ o) $\frac{1}{4}$
- F54. a) $\frac{3}{4}$ b) $1\frac{1}{6}$ c) 2 d) $\frac{4}{5}$ e) $1\frac{1}{6}$ f) $1\frac{1}{2}$ g) $3\frac{3}{4}$ h) 4 i) $4\frac{3}{8}$ j) $2\frac{4}{11}$ k) 4 l) $4\frac{2}{5}$ m) $\frac{1}{2}$ n) 4 o) 3
 p) $3\frac{1}{3}$ q) $\frac{1}{2}$ r) 6 s) 4 t) 6
- F55. a) $\frac{3}{4}$ b) 1 c) $\frac{4}{5}$ d) 2 e) $3\frac{3}{4}$ f) 1 g) 3 h) $\frac{2}{3}$ i) $4\frac{1}{2}$ j) 3 k) $4\frac{1}{2}$ l) $10\frac{2}{3}$ m) $8\frac{1}{2}$ n) 22 o) $2\frac{1}{3}$
- F56. a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) $\frac{1}{25}$ d) $\frac{8}{15}$ e) $\frac{1}{15}$ f) $1\frac{7}{8}$ g) $2\frac{1}{4}$ h) 1 i) $\frac{1}{4}$ j) 1 k) 60 l) 5 m) 6 n) $\frac{1}{3}$ o) $3\frac{3}{10}$
- F57. a) $\frac{1}{3}$ b) $\frac{1}{2}$ c) 1 d) $\frac{3}{5}$ e) $3\frac{1}{2}$ f) 1 g) $\frac{1}{4}$ h) $\frac{3}{5}$ i) $\frac{4}{5}$ j) $\frac{1}{20}$ k) 1 l) $\frac{1}{4}$ m) $\frac{1}{2}$ n) $\frac{11}{18}$ o) $8\frac{4}{7}$
- F58. a) $\frac{3}{4}$ b) 1 c) 1 d) $\frac{2}{3}$ e) 1 f) 1 g) $2\frac{2}{3}$ h) 1 i) $7\frac{1}{2}$ j) $1\frac{1}{10}$ k) 1 l) $2\frac{1}{3}$ m) 6 n) 24 o) 4
 p) $3\frac{1}{3}$ q) $\frac{1}{2}$ r) $\frac{7}{8}$ s) $1\frac{1}{4}$ t) $4\frac{1}{2}$

- F59. a) $\frac{1}{4}$ b) $\frac{1}{5}$ c) $\frac{1}{8}$ d) 3 e) $\frac{1}{15}$ f) $2\frac{3}{4}$ g) $1\frac{3}{4}$ h) $2\frac{1}{2}$ i) 1 j) $1\frac{1}{2}$ k) $\frac{15}{32}$ l) $\frac{1}{6}$ m) $\frac{7}{36}$ n) $\frac{63}{256}$
- F60. a) $\frac{9}{10}$ b) 1 c) $\frac{1}{2}$ d) $\frac{1}{6}$ e) $\frac{1}{30}$ f) $\frac{1}{6}$ g) $\frac{1}{2}$ h) $\frac{5}{6}$ i) $1\frac{1}{6}$ j) 1 k) $\frac{1}{12}$ l) $\frac{11}{12}$ m) $\frac{13}{24}$ n) 3 o) $3\frac{1}{2}$
- F61. a) $\frac{2}{3}$ b) $\frac{1}{6}$ c) $\frac{1}{10}$ d) $\frac{1}{4}$ e) $\frac{17}{144}$ f) 0 g) $\frac{1}{3}$ h) $1\frac{1}{3}$ i) $\frac{2}{15}$ j) $\frac{2}{7}$ k) $\frac{1}{6}$ l) $\frac{1}{2}$ m) $\frac{1}{4}$ n) $\frac{2}{3}$ o) 1
p) 0 q) $\frac{65}{77}$
- F62. a) $\frac{2}{1}$ b) $1\frac{1}{2}$ c) $\frac{4}{5}$ d) $\frac{3}{7}$ e) *not defined* f) $\frac{6}{7}$ g) $\frac{3}{13}$ h) 2 i) $1\frac{2}{3}$ j) *not defined*
- F63. a) $\frac{2}{3}$ b) $\frac{3}{8}$ c) $\frac{5}{12}$ d) $\frac{7}{8}$ e) $\frac{3}{11}$ f) $\frac{1}{4}$ g) $\frac{11}{24}$ h) $\frac{7}{19}$ i) $\frac{4}{5}$ j) $\frac{4}{15}$
- F64. a) 1 b) $\frac{1}{2}$ c) $\frac{1}{5}$ d) $\frac{1}{100}$ e) *not defined*
- F65. a) *yes* b) *no* c) *no* d) *yes* e) *no* f) *no* g) *yes* h) *no* i) *yes* j) *yes*
- F66. a) 4 b) 2 c) $2\frac{1}{2}$ d) $\frac{1}{5}$ e) $\frac{4}{9}$ f) $3\frac{1}{2}$ g) $\frac{1}{7}$ h) $\frac{9}{11}$ i) $\frac{4}{7}$ j) $\frac{5}{14}$
- F67. a) $\frac{2}{3}$ b) $1\frac{1}{2}$ c) $1\frac{1}{2}$ d) $\frac{3}{4}$ e) $1\frac{1}{4}$ f) $\frac{5}{9}$ g) $3\frac{1}{3}$ h) 2 i) 3 j) $\frac{2}{3}$ k) $\frac{3}{8}$ l) $\frac{5}{8}$ m) $\frac{2}{5}$ n) $\frac{3}{4}$ o) 2
- F68. a) 5 b) $\frac{2}{3}$ c) $1\frac{1}{8}$ d) 4 e) $\frac{4}{7}$ f) 1 g) $\frac{2}{3}$ h) $1\frac{1}{2}$ i) $\frac{1}{6}$ j) 2 k) $\frac{2}{3}$ l) $\frac{1}{2}$ m) 10 n) 3 o) $1\frac{5}{7}$
p) $\frac{3}{5}$ q) $\frac{2}{3}$ r) 6 s) $6\frac{1}{4}$ t) $\frac{27}{28}$
- F69. a) 2 b) $\frac{1}{6}$ c) 5 d) $\frac{1}{6}$ e) 6 f) $2\frac{1}{3}$ g) $\frac{1}{6}$ h) $1\frac{1}{4}$ i) $\frac{1}{8}$ j) $2\frac{1}{4}$ k) $\frac{1}{2}$ l) $2\frac{1}{5}$ m) $3\frac{1}{2}$ n) $\frac{1}{28}$ o) $1\frac{2}{3}$
- F70. a) $\frac{5}{6}$ b) $2\frac{1}{4}$ c) $\frac{2}{3}$ d) 7 e) $\frac{4}{5}$
- F71. a) $\frac{3}{2}$ b) 6 c) $\frac{4}{7}$ d) $\frac{1}{10}$ e) $\frac{3}{7}$
- F72. a) 1 b) $1\frac{1}{2}$ c) $1\frac{5}{6}$ d) $\frac{2}{7}$ e) $\frac{6}{11}$ f) 3 g) $1\frac{1}{2}$ h) $1\frac{1}{3}$ i) 9 j) 3 k) $\frac{1}{6}$ l) $\frac{5}{8}$ m) $\frac{9}{50}$ n) $\frac{1}{9}$ o) $\frac{3}{5}$
- F73. a) $\frac{3}{4}$ b) $1\frac{1}{9}$ c) $2\frac{7}{9}$ d) $\frac{2}{3}$ e) $1\frac{1}{2}$ f) $1\frac{1}{3}$ g) $\frac{1}{6}$ h) $\frac{3}{4}$ i) $\frac{2}{5}$ j) $\frac{4}{9}$ k) $\frac{2}{3}$ l) 4 m) $2\frac{1}{4}$ n) $\frac{4}{7}$ o) $\frac{7}{2}$
- F74. a) $\frac{15}{16}$ b) $\frac{2}{15}$ c) $\frac{1}{120}$ d) 9 e) $\frac{32}{35}$ f) $\frac{1}{4}$ g) $1\frac{1}{2}$ h) $\frac{1}{32}$ i) $\frac{1}{3}$ j) $\frac{4}{81}$ k) $\frac{3}{2}$ l) $\frac{1}{4}$
- F75. a) $\frac{5}{9}$ b) $4\frac{1}{8}$ c) $\frac{7}{16}$ d) $\frac{8}{15}$ e) $\frac{1}{6}$ f) $\frac{1}{4}$ g) $\frac{3}{8}$ h) 8 i) $\frac{4}{9}$ j) $1\frac{1}{15}$ k) $1\frac{1}{2}$
- F76. a) $\frac{1}{4}$ b) $\frac{1}{2}$ c) $1\frac{1}{2}$ d) $5\frac{20}{21}$ e) $4\frac{1}{4}$ f) 2 g) $2\frac{1}{2}$ h) $\frac{11}{12}$
- F77. a) $4\frac{3}{10}$ b) 1 c) $3\frac{2}{5}$ d) $\frac{3}{5}$ e) $\frac{1}{24}$ f) $\frac{1}{2}$ g) $\frac{47}{60}$
- F78. a) $\frac{4}{5}$ b) $\frac{3}{8}$ c) $\frac{5}{27}$ d) 12 e) $\frac{9}{35}$ f) $\frac{5}{14}$ g) $1\frac{1}{3}$ h) $\frac{1}{2}$ i) 1 j) $1\frac{2}{3}$ k) $4\frac{1}{3}$
- F79. a) $\frac{1}{2}^4$ b) $\frac{3}{4}^2$ c) $\frac{3}{2}^5$ d) $\frac{5}{4}^4$ e) 2^4 f) $1\frac{2}{3}^3$ g) $\frac{4}{5}^2$ h) $\frac{1}{4}^5$
- F80. a) $\frac{3}{4} \frac{3}{4}$ b) $\frac{5}{3} \frac{5}{3} \frac{5}{3} \frac{5}{3}$ c) $\frac{1}{5} \frac{1}{5} \frac{1}{5}$ d) $\frac{3}{2} \frac{3}{2}$ e) $\frac{3}{7} \frac{3}{7}$ f) $\frac{3}{2} \frac{3}{2} \frac{3}{2}$ g) $\frac{4}{5}$ h) $\frac{4}{5} \frac{4}{5} \frac{4}{5}$

- F81. a) 1 b) $3\frac{3}{8}$ c) $\frac{4}{25}$ d) $\frac{27}{125}$ e) 8 f) $\frac{1}{81}$ g) 16 h) $5\frac{1}{16}$
- F82. a) $\frac{1}{9}$ b) $\frac{1}{25}$ c) 16 d) $3\frac{3}{8}$ e) $1\frac{7}{9}$ f) $1\frac{2}{5}$ g) $\frac{9}{16}$ h) $15\frac{5}{8}$
- F83. a) $\frac{49}{144}$ b) $\frac{36}{49}$ c) $\frac{1}{4}$ d) $\frac{13}{18}$ e) $\frac{1}{18}$ f) 5 g) 1 h) $\frac{49}{144}$
- F84. a) $\frac{2}{3}$ b) $\frac{9}{25}$ c) $\frac{1}{16}$ d) $\frac{4}{25}$ e) 1 f) $1\frac{9}{16}$ g) $\frac{1}{16}$ h) $\frac{1}{12}$
- F85. a) $3\frac{1}{2}$ b) 1 c) $1\frac{1}{2}$ d) $\frac{15}{16}$ e) $2\frac{1}{2}$ f) 2 g) $2\frac{1}{2}$ h) 3 i) 1 j) $5\frac{1}{3}$ k) $\frac{1}{2}$ l) $1\frac{1}{3}$
- F86. Legend : t terminating; nt non terminating a) t b) nt c) t d) t e) nt f) nt g) nt h) t i) nt j) nt k) t l) nt m) t n) t o) nt
- F87. a) 0.5 b) 0.75 c) 0.625 d) 0.2 e) 0.15 f) 0.1375 g) 0.13 h) 0.008 i) 0.006 j) 0.0016
- F88. a) $0.\bar{3}$ b) $0.1\bar{6}$ c) $0.30\bar{5}$ d) $0.\bar{7}$ e) $0.1\bar{8}$ f) $0.0\bar{6}$ g) $0.2\bar{3}$ h) $0.\overline{285714}$ i) $0.15384\bar{6}$ j) $0.0\bar{37}$
- F89. a) 1.5 b) 0.4 c) $1.\bar{3}$ d) $0.01\bar{3}$ e) $2.\bar{3}$ f) 2.25 g) 0.078125 h) 0.03125 i) 0.046875 j) 0.0078125 k) 0.012 l) $0.14285\bar{7}$ m) $1.42857\bar{1}$ n) 0.416 o) $2.23076\bar{9}$
- F90. a) $\frac{1}{10}$ b) $\frac{1}{2}$ c) $1\frac{2}{5}$ d) $1\frac{1}{4}$ e) $\frac{3}{4}$ f) $\frac{7}{200}$ g) $2\frac{1}{8}$ h) $10\frac{1}{8}$ i) $5\frac{3}{40}$ j) $100\frac{29}{40}$
- F91. a) $\frac{5}{8}$ b) $1\frac{1}{2}$ c) $\frac{1}{8}$ d) $\frac{2}{5}$ e) $2\frac{4}{25}$ f) $\frac{11}{40}$ g) $\frac{6}{25}$ h) $\frac{7}{20}$ i) $2\frac{9}{20}$ j) $\frac{41}{64}$
- F92. a) $\frac{1}{3}$ b) $1\frac{2}{9}$ c) $\frac{11}{90}$ d) $1\frac{19}{90}$ e) $4\frac{23}{900}$ f) $\frac{23}{99}$ g) $1\frac{25}{99}$ h) $2\frac{2}{165}$ i) $\frac{41}{333}$ j) $1\frac{7811}{33300}$
- F93. a) $\frac{7}{9}$ b) $1\frac{1}{3}$ c) $2\frac{8}{15}$ d) $1\frac{29}{90}$ e) $1\frac{13}{100}$ f) $\frac{4}{33}$ g) $3\frac{1}{99}$ h) $1\frac{103}{330}$ i) $6\frac{679}{5500}$ j) $1\frac{41}{3330}$
- F94. a) 0.4 b) 0.35 c) 1.2 d) 2 e) 0.5
- F95. a) 2 b) $\frac{7}{20}$ c) $\frac{7}{10}$ d) $1\frac{3}{5}$ e) $\frac{21}{25}$
- F96. a) 2 b) $1\frac{98}{99}$ c) $1\frac{19}{81}$ d) 26 e) $\frac{1}{3}$
- F97. a) 1.45 b) 0.95 c) 1.24 d) 0.05
- F98. a) $\frac{11}{20}$ b) $\frac{1}{5}$ c) $\frac{1}{3}$ d) $\frac{3}{20}$
- F99. a) $\frac{3}{5}$ b) $\frac{11}{16}$ c) $\frac{5}{6}$ d) $1\frac{1}{4}$ e) $\frac{1}{4}$ f) $1\frac{1}{3}$ g) $\frac{7}{10}$ h) 1 i) $\frac{5}{6}$ j) 1
- F100. a) 30 min b) 20 min c) 105 min d) 165 min e) 6 min f) 50 min g) 135 min h) $22\frac{1}{2}$ min i) 28 min j) 74 min
- F101. a) $\frac{1}{12}$ h b) $\frac{1}{6}$ h c) $\frac{1}{4}$ h d) $\frac{5}{12}$ h e) $\frac{5}{6}$ h f) $1\frac{1}{6}$ h g) $\frac{3}{5}$ h h) $4\frac{1}{6}$ h i) $\frac{1}{80}$ h j) $\frac{1}{24}$ h
- F102. a) 90 s b) 100 s c) 15 s d) 84 s e) 69 s f) 35 s g) 105 s h) 25 s i) $14\frac{2}{3}$ s j) 440 s
- F103. a) $\frac{1}{5}$ min b) $\frac{1}{6}$ min c) $\frac{3}{4}$ min d) $1\frac{1}{2}$ min e) $3\frac{1}{3}$ min f) $\frac{7}{40}$ min g) $\frac{1}{80}$ min h) $\frac{21}{80}$ min i) $\frac{5}{9}$ min j) $\frac{31}{90}$ min
- F104. a) $\frac{5}{36}$ h b) $\frac{5}{12}$ h c) $2\frac{1}{2}$ h d) 720 s e) 6300 s f) 540 s g) 7800 s h) 905 s i) $30\frac{1}{4}$ min j) $50\frac{2}{3}$ min

F105. a) 12 cents b) $\frac{4}{5}$ cents c) 102 cents d) $102\frac{2}{3}$ cents e) 7 cents f) $4\frac{2}{3}$ cents

F106. a) $\frac{1}{4}$ \$ b) $\frac{1}{5}$ \$ c) $\frac{9}{20}$ \$ d) $1\frac{3}{5}$ \$ e) $4\frac{1}{2}$ \$ f) $\frac{11}{200}$ \$

F107. a) 4 cents b) 190 cents c) 6 cents d) $14\frac{2}{3}$ cents e) 260 cents f) $9\frac{3}{5}$ cents

F108. a) $\frac{5}{8}$ twonies b) $1\frac{1}{4}$ twonies c) $\frac{1}{5}$ twonies d) $\frac{3}{5}$ twonies e) $2\frac{1}{2}$ twonies f) $\frac{51}{1000}$ twonies

F109. a) 1 cent b) 6 cents c) $1\frac{3}{4}$ cents d) 7 cents e) $10\frac{3}{4}$ cents f) $26\frac{1}{4}$ cents

F110. a) 3 nickels b) 5 nickels c) 15 nickels d) $\frac{4}{5}$ nickels e) $\frac{3}{10}$ nickels f) $\frac{1}{10}$ nickels

F111. a) 5 cents b) 40 cents c) 3 cents d) $1\frac{4}{5}$ cents e) 40 cents f) 6 cents

F112. a) 2 quarters b) 5 quarters c) 4 quarters d) $\frac{4}{5}$ quarters e) $\frac{1}{50}$ quarters f) $2\frac{1}{5}$ quarters

F113. a) 15 nickels b) 3 nickels c) 1 quarter d) $\frac{1}{2}$ dimes e) 4 dimes f) $7\frac{1}{2}$ dimes g) 9 nickels h) $12\frac{1}{2}$ dimes

F114.

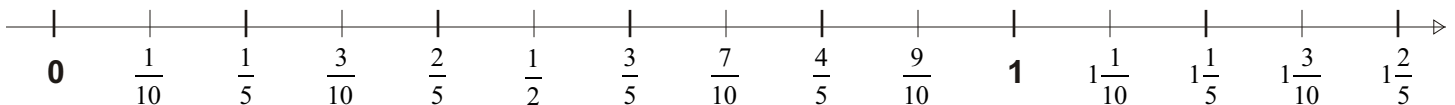
a) $\frac{1}{4}$ 5 to 20 25 % 0.25 b) $\frac{1}{3}$ 7 out of 21 $33.\bar{3}\%$ $0.\bar{3}$ c) $\frac{3}{2}$ 12 to 8 150 % 1.5 d) $\frac{5}{6}$ 15 out of 18 $83.\bar{3}\%$ $0.8\bar{3}$
 e) $\frac{3}{5}$ 42 out of 70 60 % 0.6 f) $\frac{2}{5}$ 2 out of 5 40 % 0.4 g) $\frac{2}{9}$ 10 to 45 $22.\bar{2}\%$ $0.\bar{2}$ h) $\frac{1}{4}$ 3 out of 12 25 % 0.25
 i) $\frac{6}{5}$ 18 to 15 120 % 1.2 j) $\frac{7}{10}$ 7 to 10 70 % 0.7 k) $\frac{3}{10}$ 21 to 70 30 % 0.3 l) $\frac{4}{5}$ 36 out of 45 80 % 0.8
 m) $\frac{13}{10}$ 39 to 30 130 % 1.3 n) $\frac{1}{10}$ 0.5 out of 5 10 % 0.1 o) $\frac{3}{4}$ 48 out of 64 75 % 0.75 p) $\frac{3}{20}$ 1.5 to 10 15 % 0.15
 r) $\frac{7}{5}$ 49 out of 35 140 % 1.4 s) $\frac{2}{25}$ 6 to 75 8 % 0.08 t) $\frac{16}{25}$ 80 out of 125 64 % 0.64
 u) $\frac{1}{8}$ 32 out of 256 12.5 % 0.125

F115. a) b) c) d) e) f) g) h) i) j)

F116. a) M b) S c) C d) e e) J f) D g) O h) d i) a j) b

F117. A $\frac{1}{12}$ B $\frac{1}{6}$ C $\frac{4}{15}$ D $\frac{11}{30}$ E $\frac{1}{2}$ F $\frac{7}{12}$ G $\frac{41}{60}$ H $\frac{47}{60}$ I $\frac{11}{12}$ J $1\frac{1}{12}$

F118.



F119. a) b) c) d) e) f) g)

F120. a) b) c) d) e) f) g) h) i) j) k) l) m) n)

F121. a) b) c) d) e) f) g) h) i) j) k) l) m) n)

F122. a) b) c) d) e) f) g) h) i) j) k) l) m) n)

F123. a) b) c) d) e) f) g)

F124. a) $\frac{1}{7}$ $\frac{2}{7}$ $\frac{3}{7}$ b) $\frac{3}{11}$ $\frac{3}{7}$ $\frac{3}{5}$ c) $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$ $\frac{4}{5}$ d) $\frac{3}{10}$ $\frac{2}{5}$ $\frac{5}{12}$ e) $\frac{4}{15}$ $\frac{1}{2}$ $\frac{7}{12}$ $\frac{3}{4}$ f) 1 $1\frac{5}{24}$ $1\frac{1}{4}$ $\frac{11}{8}$ $1\frac{5}{6}$

F125. a) $\frac{4}{5}$ $\frac{3}{5}$ $\frac{1}{5}$ b) $\frac{2}{5}$ $\frac{2}{6}$ $\frac{2}{7}$ c) $\frac{6}{7}$ $\frac{5}{6}$ $\frac{3}{4}$ $\frac{2}{3}$ d) $\frac{3}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ e) $\frac{9}{10}$ $\frac{3}{4}$ $\frac{2}{3}$ $\frac{7}{12}$ f) $1\frac{1}{2}$ $1\frac{3}{8}$ $1\frac{11}{32}$ $1\frac{5}{16}$ $1\frac{1}{4}$

F126. a) $\frac{1}{10}$ b) $1\frac{2}{15}$ c) $\frac{2}{3}$ d) $1\frac{1}{3}$ e) $1\frac{2}{5}$ f) $\frac{1}{15}$ g) $\frac{5}{18}$ h) $3\frac{3}{8}$ i) $1\frac{3}{5}$ j) $3\frac{1}{2}$ k) $\frac{1}{2}$ l) $5\frac{5}{6}$ m) $2\frac{3}{5}$ n) $\frac{2}{15}$ o) 1
 p) $\frac{8}{15}$ q) 1 r) $1\frac{9}{20}$ s) $7\frac{11}{25}$ t) $\frac{5}{36}$ u) $\frac{8}{9}$ v) $1\frac{2}{3}$ w) $7\frac{2}{3}$

F127. a) $3\frac{5}{12}$ b) $\frac{13}{44}$ c) $\frac{11}{15}$ d) 1

FT. 1) $\frac{2}{3}$ 2) $1\frac{2}{3}$ 3) $\frac{3}{8}$ 4) $\frac{5}{7}$ 5) $4\frac{3}{4}$ 6) $\frac{3}{2}$ 7) $\frac{7}{3}$ 8) $\frac{17}{5}$ 9) $\frac{31}{7}$ 10) $\frac{17}{3}$ 11) $\frac{12}{5}$ 12) $\frac{8}{3}$ 13) $1\frac{2}{3}$ 14) $2\frac{1}{2}$ 15) $3\frac{3}{4}$
 16) $2\frac{2}{3}$ 17) $1\frac{3}{5}$ 18) $1\frac{1}{4}$ 19) $2\frac{1}{7}$ 20) $2\frac{3}{4}$ 21) $\frac{2}{3}$ 22) $2\frac{1}{2}$ 23) $\frac{2}{3}$ 24) $\frac{9}{16}$ 25) no 26) yes 27) yes 28) $3\frac{2}{3}$ 29) $\frac{3}{5}$ 30) 4
 31) $1\frac{1}{12}$ 32) $5\frac{1}{6}$ 33) $3\frac{17}{60}$ 34) $2\frac{11}{12}$ 35) $\frac{1}{3}$ 36) $1\frac{1}{3}$ 37) $\frac{2}{7}$ 38) $\frac{1}{12}$ 39) $1\frac{1}{10}$ 40) $2\frac{7}{12}$ 41) $\frac{11}{20}$ 42) $\frac{2}{5}$ 43) $1\frac{1}{5}$ 44) 1
 45) $2\frac{1}{3}$ 46) $\frac{1}{3}$ 47) $2\frac{7}{9}$ 48) $2\frac{1}{3}$ 49) $\frac{8}{9}$ 50) 5 51) $\frac{2}{9}$ 52) 3 53) $\frac{1}{3}$ 54) 4 55) $\frac{15}{16}$ 56) 0.4 57) 2.4 58) 0.4 59) 1.25
 60) $0.\bar{6}$ 61) $1.\overline{1285714}$ 62) 0.3125 63) 0.875 64) 3.125 65) $\frac{3}{25}$ 66) $1\frac{1}{2}$ 67) $2\frac{1}{4}$ 68) $\frac{1}{80}$ 69) $\frac{4}{9}$ 70) $1\frac{7}{11}$ 71) $\frac{29}{40}$ 72) $1\frac{13}{20}$
 73) $\frac{7}{8}$ 74) 27 75) 6 76) $1\frac{2}{3}$ 77) $\frac{5}{9}$ 78) $\frac{61}{180}$ 79) 3 80) $1\frac{7}{8}$ 81) $\frac{7}{40}$ 82) $12\frac{1}{2}$ 83) 8 84) 2 85) 9 86) $\frac{1}{2}$ 87) 10 88) $\frac{5}{12}$
 89) $\frac{1}{2}$ 90) $2\frac{2}{3}$ 91) $\frac{1}{3}$ 92) $\frac{5}{18}$ 93) $1\frac{1}{3}$ 94) $\frac{1}{2}$ 95) $\frac{1}{10}$ 96) $\frac{7}{12}$ 97) $\frac{2}{15}$ 98) $2\frac{1}{2}$ 99) $4\frac{1}{6}$ 100) 6