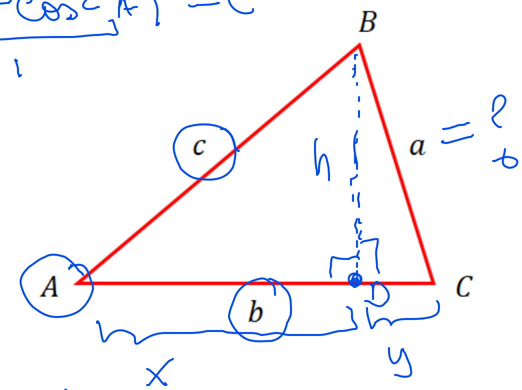
8.2 8.3 The Cosine Law

A The Cosine Law

The Cosine Law states that in any triangle

$c^2(\underbrace{\sin^2 A + \cos^2 A}_{1}) = c^2$

$$c^2 = a^2 + b^2 - 2ab\cos C$$
$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$



Example 1. Prove the cosine law.

$\sin A = \frac{h}{c} \Rightarrow h = c\cdot\sin A$

$\cos A = \frac{x}{c} \Rightarrow x = c\cdot\cos A$

$x + y = b \Rightarrow y = b - x = b - c\cdot\cos A$

$a^2 = h^2 + y^2 = (c\cdot\sin A)^2 + (b - c\cdot\cos A)^2$

$= c^2\sin^2 A + b^2 + c^2\cdot\cos^2 A - 2bc\cos A = c^2 + b^2 - 2bc\cos A$

Example 2. Rewrite the cosine laws for a right triangle with $\angle C = 90°$.

$\cos 90° = 0$

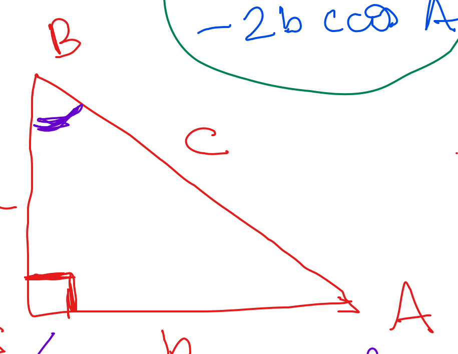
$c^2 = a^2 + b^2 - 2ab\cancel{\cos C}$

$c^2 = a^2 + b^2$ (Pythagorean Th.)

$b^2 = a^2 + c^2 - 2ac\cdot\cos B \Rightarrow$

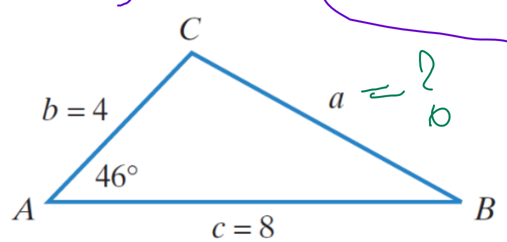
$\cancel{b^2} = a^2 + a^2 + \cancel{b^2} - 2ac\cos B$

$\cancel{2}\cancel{a}c\cos B = \cancel{2}\cancel{a}^2 \Rightarrow \cos B = \frac{a}{c}$



B Using the Cosine Law to find a Side

Example 3. Find the unknown side $a = BC$.



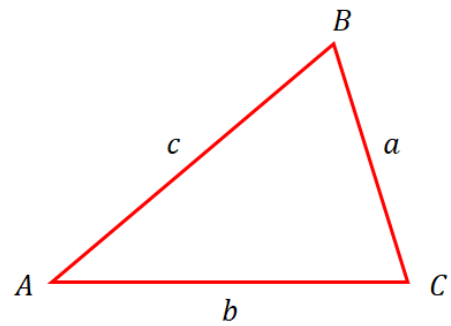
$a^2 = b^2 + c^2 - 2bc\cos A$

$= 4^2 + 8^2 - 2(4)(8)\cos 46°$

$a = \sqrt{4^2 + 8^2 - 2(4)(8)\cos 46°}$

$\therefore a \simeq 5.96$

C Using the Cosine Law to find an Angle



From the Cosine Law

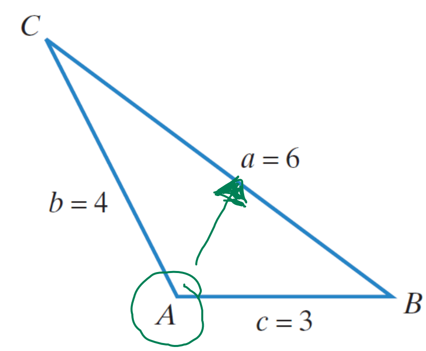
$c^2 = a^2 + b^2 - 2ab\cos C$

$\frac{\cancel{2ab}\cos C}{\cancel{2ab}} = \frac{a^2+b^2-c^2}{2ab}$

By solving for the angle C we get $\boxed{\cos C = \frac{a^2+b^2-c^2}{2ab}}$ or

$$\boxed{\angle C = \cos^{-1}\frac{a^2+b^2-c^2}{2ab}}$$

Example 4. Use the cosine law to find the angles $\angle A$ and $\angle B$.



$a^2 = b^2 + c^2 - 2bc\cos A$

$6^2 = 4^2 + 3^2 - 2(4)(3)\cos A$

$36 = 16 + 9 - 24\cos A$

$24\cos A = 25 - 36$

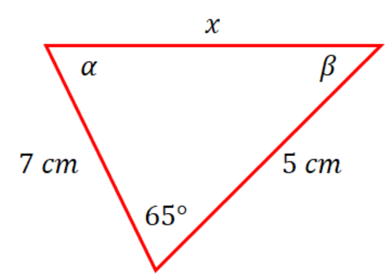
$\cos A = \frac{25-36}{24}$

D Using the Cosine Law to solve a Triangle

Solving a triangle means finding all the side lengths and all the angle measures.

Example 5. Solve each triangle.

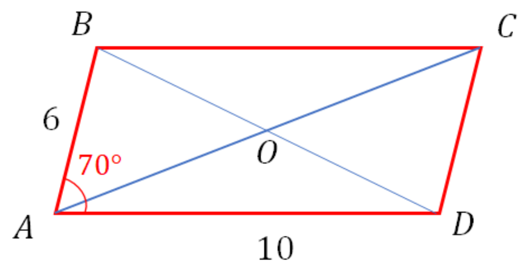
a)



b) $a = 6\ cm$, $b = 12\ cm$, $c = 8\ cm$

Example 6. For the parallelogram $ABCD$ on the figure below, find

a) the length of the diagonal BD



b) the length of the diagonal AC

c) the obtuse angle $\angle BOC$ between the diagonals

Notes: Textbook Pages 405-408 and 412-417
Homework: Textbook Pages 409-411 # 1a, 4a, 9, 11 and 418-419 #1a, 3a, 5a, 9, 10, 11

Applications

Example 1. Find the area of a triangle with the side lengths equal to $5m$, $7m$, and $10m$.

Example 2. The diagonals of a parallelogram are $16cm$ and $24cm$ respectively and the acute angle between the diagonals is $36°$. Find the sides of the parallelogram.

Example 3. Two ships leave a port at the same time. One sails in a direction $[N32°W]$ at 15 knots, and the other sails $[N78°E]$ at 18 knots. How far apart are the ships two hours after they have left the port? (A knot is one nautical mile per hour.)

Example 4. The hands of a clock are 4.25 in and 24 in respectively. What is the distance between the tips of the two hands at 8 o'clock?

Example 5. Two sides of a parallelogram are 65 cm and 85 cm and one of the diagonals is 45 cm. Find the angles of the parallelogram.

Example 6. A triangle has the side lengths $2m$, $3m$, and $4m$. Find the radius of the inscribed circle of the triangle.

Example 7. A triangle has the side lengths $2m$, $3m$, and $4m$. Find the radius of the circumscribed circle of the triangle.

Example 8. A triangle has two sides of 7 cm and 12 cm, respectively. If the area of the triangle is 27.6 cm^2, find the angle between the two sides.

Example 9. Solve ΔABC with vertices $A(2,1)$, $B(7,4)$, and $C(1,5)$.

