

Sine Ratio



$$\sin \alpha = \frac{a}{c}$$

8.1 The Sine Law

A The Sine Law

The Sine Law states that in any triangle $\triangle ABC$:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

equivalent 3 proportions to

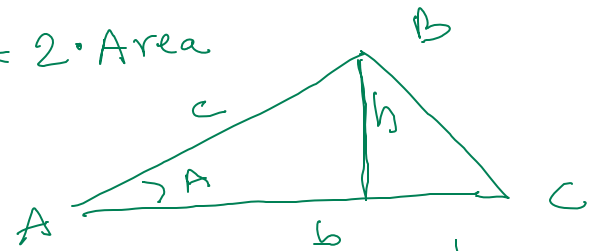
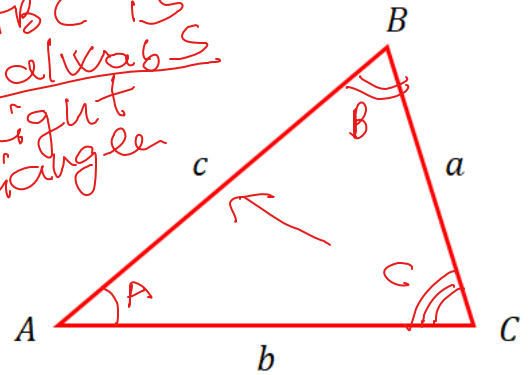
Example 1. Prove the Sine Law.

$$\text{Area} = \frac{\text{Base} \times \text{Height}}{2} = \frac{b \cdot c \sin A}{2}$$

$$\frac{b \cdot c \sin A}{a \cdot b \cdot c} = \frac{a \cdot c \sin B}{a \cdot b \cdot c} = \frac{b \cdot a \sin C}{a \cdot b \cdot c} = 2 \cdot \text{Area}$$

SOH = sine = $\frac{\text{opp}}{\text{hyp}}$
 CAH = cosine = $\frac{\text{adj}}{\text{hyp}}$
 TOA = tangent = $\frac{\text{opp}}{\text{adj}}$

$\triangle ABC$ is not always a right triangle



$$\sin A = \frac{h}{c} \Rightarrow h = c \sin A$$

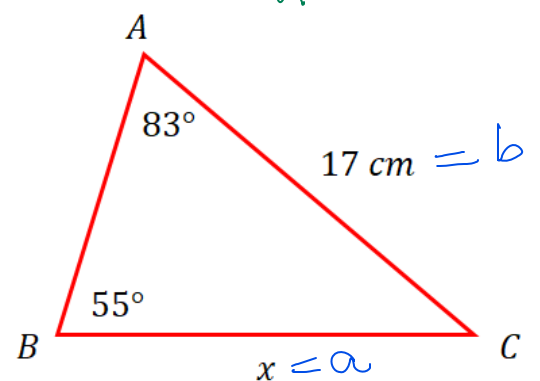
B Finding an unknown side.

Example 2. Use the sine law to find the unknown side x .

a) $\frac{a}{\sin A} = \frac{b}{\sin B}$ (sine law)

$$\frac{x}{\sin 83^\circ} = \frac{17}{\sin 55^\circ} \Rightarrow$$

$$x = \frac{17 \sin 83^\circ}{\sin 55^\circ} \Rightarrow \therefore x \approx 20.6$$

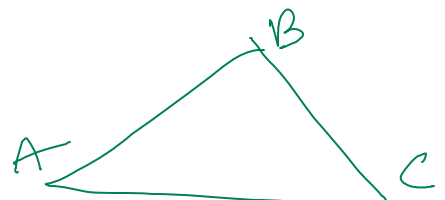
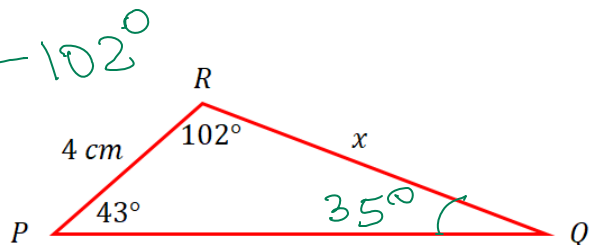


b) $\angle Q = \hat{Q} = 180^\circ - 43^\circ - 102^\circ$

$$\angle Q = 35^\circ$$

$$\frac{x}{\sin 43^\circ} = \frac{4}{\sin 35^\circ} = \frac{PQ}{\sin 102^\circ}$$

$$x = \frac{4 \sin 43^\circ}{\sin 35^\circ} \Rightarrow \therefore x \approx 4.76$$

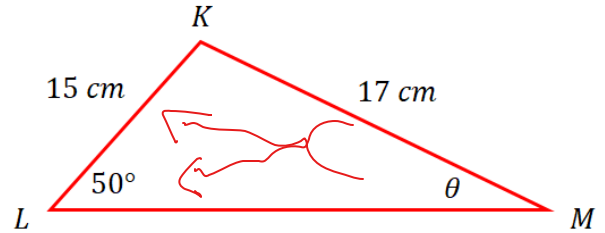


$$\angle A + \angle B + \angle C = 180^\circ$$

C Finding an unknown angle.

Example 3. Assuming that the triangle $\triangle KLM$ is acute, use the sine law to find the unknown angle θ .

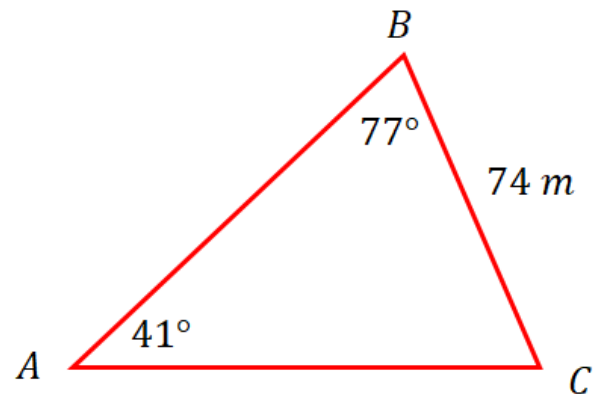
$$\frac{\sin \theta}{15} = \frac{\sin 50^\circ}{17}$$
$$\sin \theta = \frac{15 \sin 50^\circ}{17} = \dots$$
$$\theta = \sin^{-1} (15 \sin 50^\circ / 17)$$



D Solving a triangle

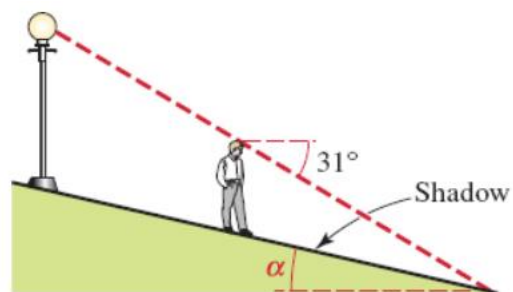
Solving a triangle means finding all the side lengths and all the angle measures.

Example 4. Solve the acute triangle.



Example 5. Solve the acute triangle with $\angle A = 60^\circ$, $a = 8$, $b = 5$.

Example 6. (Application) A man 5 ft 9 in. tall stands on a sidewalk that slopes down at a constant angle. A vertical street lamp directly behind him causes his shadow to be 25 ft long. The angle of depression from the top of the man to the tip of his shadow is 31° . Find the angle α , that the sidewalk makes with the horizontal.



Notes: Textbook Pages 396-401

Homework: Textbook Pages 402-404 Page # 1a, 2a, 3a, 5a, 6a, 9, 10, 12

Investigation on the Circumcircle of a Triangle (under construction)