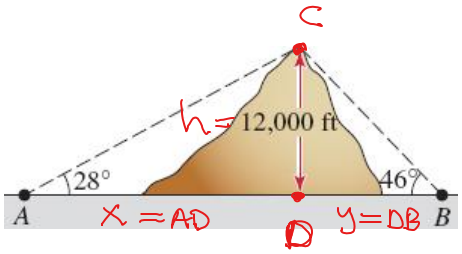
7.5 Solve Problems Involving Right Triangles

Example 1. Observers in two towns *A* and *B* on either side of a 12,000-ft mountain measure the angles of elevation between the ground and the top of the mountain. Assuming that the towns and the mountaintop lie in the same vertical plane, find the horizontal distance between them.



$AB = ?$

$\tan 28° = \frac{12{,}000}{x} \Rightarrow x = \frac{12{,}000}{\tan 28°}$

$y = \frac{12{,}000}{\tan 46°}$

$AB = x + y$

$= \frac{12{,}000}{\tan 28°} + \frac{12{,}000}{\tan 46°}$

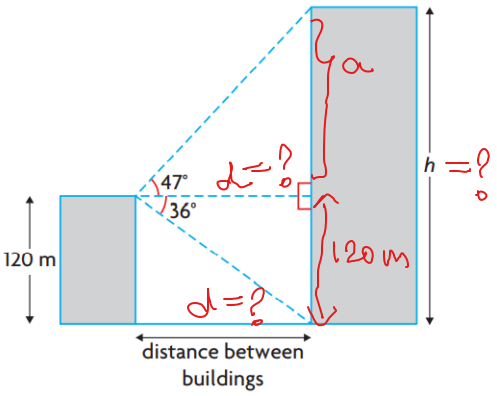
$\simeq 34{,}156.98$

∴ The horizontal distance between the towns A and B is about 34,157 ft

Example 2. A video camera is mounted on top of a building that is 120 m tall. The angle of depression from the camera to the base of another building is 36°. The angle of elevation from the camera to the top of the same building is 47°. Find the distance between the buildings and height of the second building.

$\tan 36° = \frac{120}{d} \Rightarrow d = \frac{120}{\tan 36°}$

$\simeq 165.17 \text{ m}$



$\tan 47° = \frac{a}{d}$

$a = d \cdot \tan 47°$

$= \text{Ans} \cdot \tan 47°$

$\simeq 177.12 \text{ m}$

$h = a + 120$

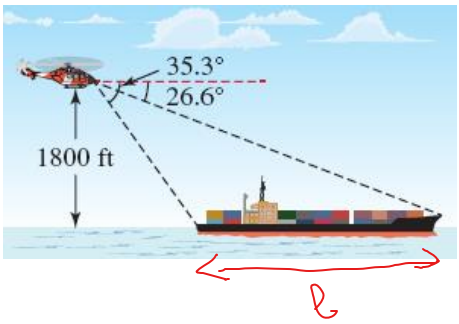
$= \text{Ans} + 120$

$\simeq 297.12 \text{ m}$

∴ The distance between these two buildings is about 165.17 m and the height of the second building is about 297.12 m

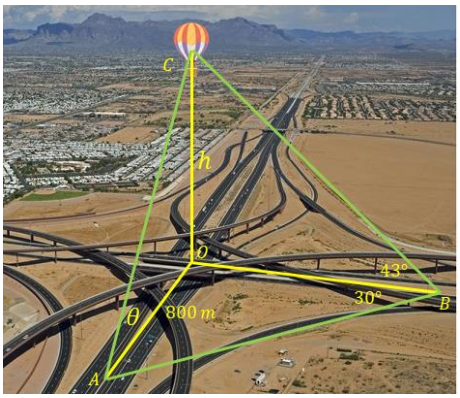
Example 3. On a rescue flight, a U.S. Coast Guard helicopter approaches a container ship at an altitude of 1800 ft. Measured from the front the helicopter, the angle of depression of the ship's stern is 35.3° and the angle of depression of its bow is 26.6°. Approximately how long is the container ship?

Let l be



Example 4. Two highways intersect each other at O. At noon the car A is at $800\,m$ from the intersection and is going North while another car B is going West towards the intersection. A balloon is above the intersection at the unknown height h. The angle of elevation of the balloon from the car B is $43°$ and the angle $\angle OBA = 30°$.

a) Use the information provided to find the height h of the balloon.

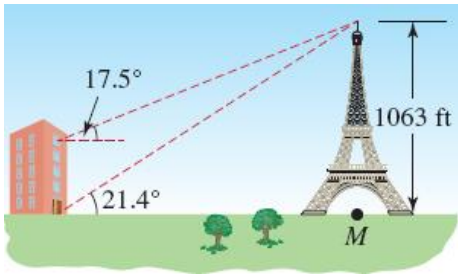


b) Find the angle of elevation θ of the balloon from the car A.

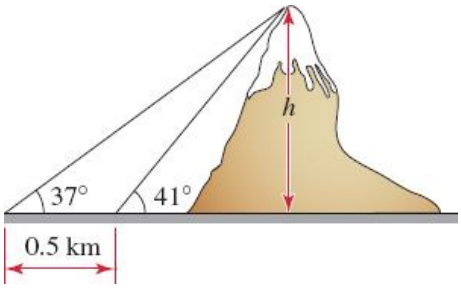
Notes: Textbook Pages 378-380
Homework: Textbook Pages 380-385 # 5, 11, 18, 23, 28, 30

More examples to practice

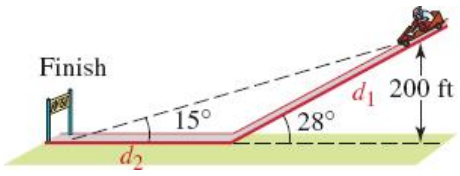
Example 4. From a room on the top floor of a nearby hotel, the angle of elevation to the top of the Eiffel Tower is 17.5°; from street level in front of the hotel the angle of elevation to the top of the tower is 21.4°. What is the distance from the hotel to the midpoint M, shown in the figure, of the base of the tower? How far is the hotel from the Eiffel Tower and how high is the hotel building?



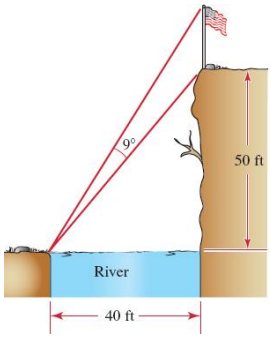
Example 5. A surveyor uses an instrument called a theodolite to measure the angle of elevation between ground level and the top of a mountain. At one point the angle of elevation is measured to be 41°. A half kilometre farther from the base of the mountain, the angle of elevation is measured to be 37°. How high is the mountain?



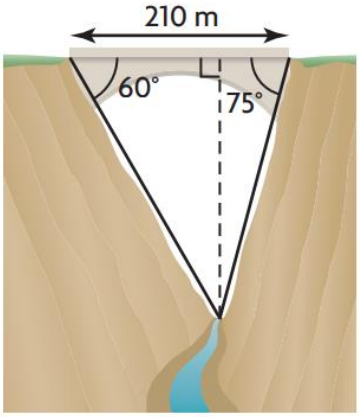
Example 6. An entry in a soapbox derby rolls down a hill. Using the information given in the figure, find the total distance $d_1 + d_2$ that the soapbox travels.



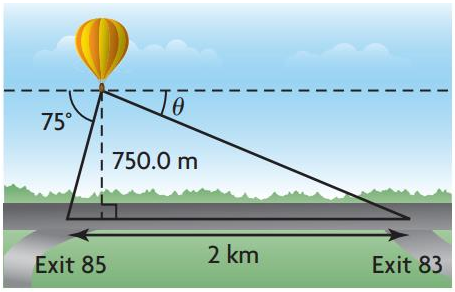
Example 7. A flagpole is located at the edge of a sheer 50-ft cliff at the bank of a river of width 40 ft. An observer on the opposite side of the river measures an angle of 9° between her line of sight to the top of the flagpole and her line of sight to the top of the cliff. Find the height of the flagpole.



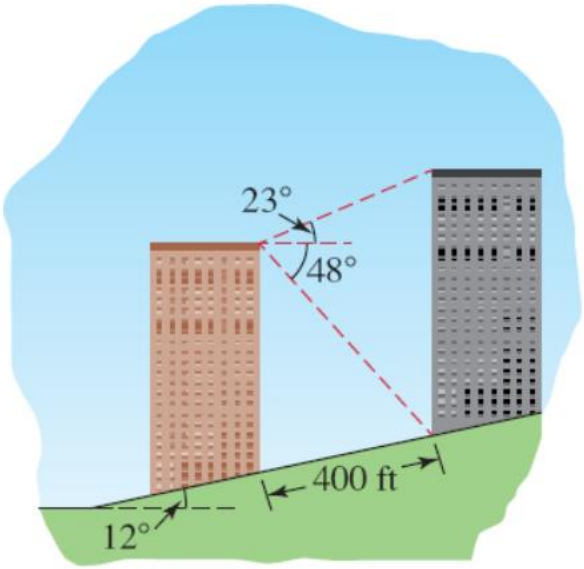
Example 8. A bridge across a gorge is 210 m long, as shown in the diagram at the left. The walls of the gorge make angles of and with the bridge. Determine the depth of the gorge to the nearest metre.



Example 9. Reid's hot-air balloon is 750.0 m directly above a highway. When Reid is looking west, the angle of depression to Exit 85 is 75 . Exit 83 is located 2 km to the east of Exit 85. What is the angle of depression to Exit 83 when Reid is looking east?

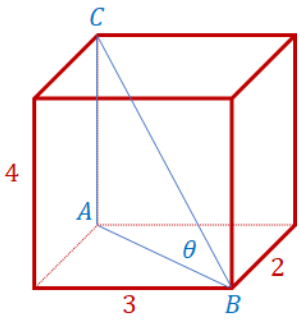


Example 10. Two buildings were constructed on a inclined lot. The angle of elevation from the right side of the roof of the brown building to the left side of the roof of the gray building is 23°. From the same spot on the roof of the brown building, the angle of depression to the base of the gray building is 48°. Use the additional information in the figure to determine the heights of the facing sides of the buildings relative to the inclined lot.

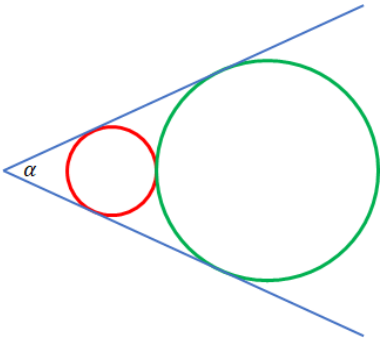


More Math Fun!

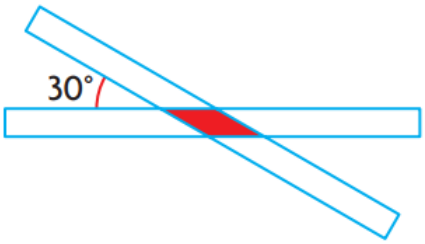
Example 11. In the diagram below is shown a rectangular box. Find the angle $\theta = \angle ABC$.



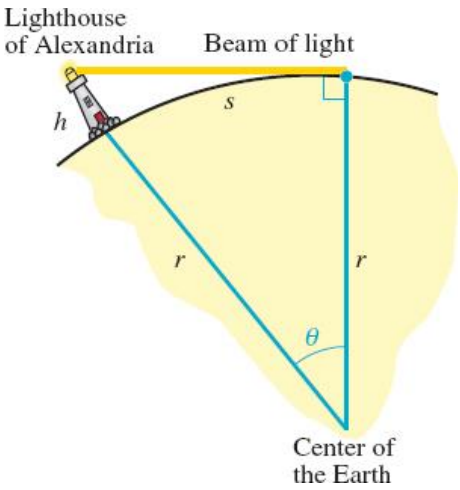
Example 12. The circles in the diagram below have a radius of 2 and 5 units. Find the exact value of the angle α, and the approximate value rounded to the nearest tenth of a degree.



Example 13. Two paper strips, each 5 cm wide, are laid across each other at an angle of 30, as shown at the right. Determine the area of the overlapping region. Round your answer to the nearest tenth of a square centimetre.

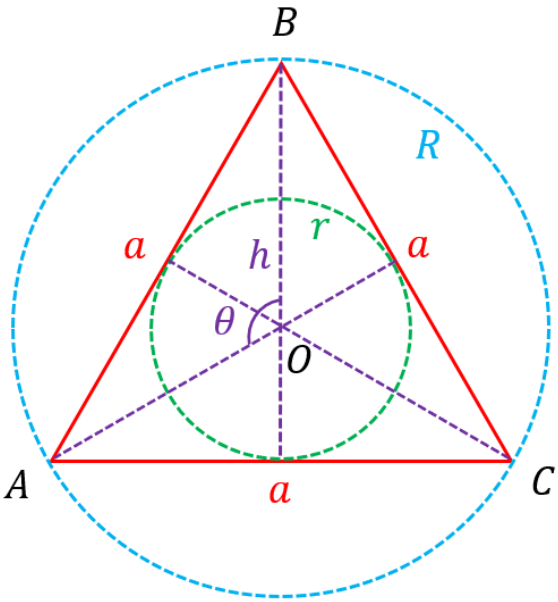


Example 14. There are ancient claims that the light of the Lighthouse of Alexandria could be seen on the ocean up to $s = 29$ miles away. Assume that the radius of the Earth is $r = 3963$ miles, find the hight h of the Lighthouse.



Investigations

1. Investigate the Equilateral Triangle



Let consider an equilateral triangle ΔABC of side a.

Find the exact values of the:

a) height h

b) area of the triangle

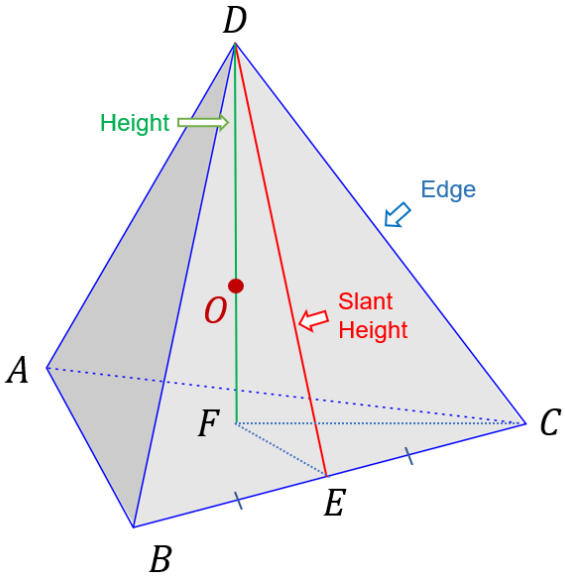
c) radius of the incircle r

d) radius of the circumcircle R

e) angle $\theta = \angle AOB$, where O is a point equidistant from all vertexes

Hint. Use the symmetry of the shape.

2. Investigate the Tetrahedron



Let consider a tetrahedron $ABCD$ of side a.

Find the exact values of the:

a) slant height h

b) total surface area T

c) height H

d) volume of the tetrahedron V

e) radius of the in-sphere r (tangent to faces)

f) radius of the mid-sphere ρ (tangent to edges)

g) radius of the circum-sphere R

h) angle $\theta = \angle AOB$, where O is a point equidistant from all vertexes

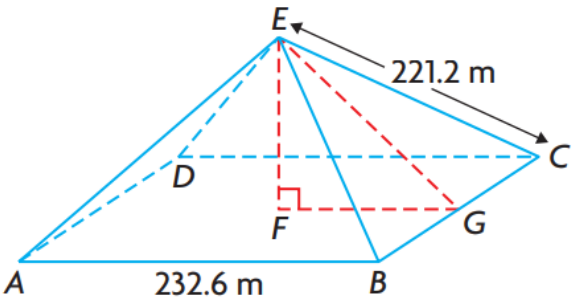
i) angle $\alpha = \angle ODC$ between a height and an edge from a common vertex

j) angle $\beta = \angle ODE$ between the height and a slant height from a common vertex

Hint. Use the symmetry of the shape.

3. The Great Pyramid of Giza

The Great Pyramid of Giza in Egypt has a square base with sides that are 232.6 m in length. The distance from the top of the pyramid to each corner of the base was originally 221.2 m.



a) Determine, to the nearest tenths, the height $h = EF$ of the pyramid.

b) Determine, to the nearest degree, the angle that each face makes with the base ($\angle EGF$).

c) Determine, to the nearest degree, the size of the apex angle of a face of the pyramid ($\angle AEB$).

d) Determine, to the nearest degree, the angle $\angle FEC$).