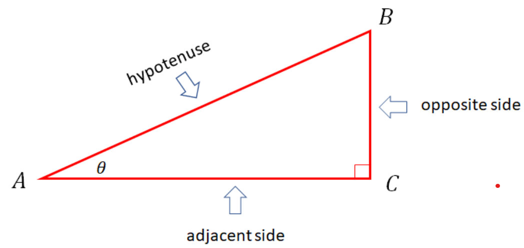
7.4 The Sine and Cosine Ratios

A Sine and Cosine Ratios

The sine and cosine ratios are defined by:

$$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{BC}{AB}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{AC}{AB}$$



Example 1. Use a scientific calculator to find the sine or the cosine of the indicated angles:

a) $\sin 0°$ $= 0$ b) $\cos 0°$ $= 1$ c) $\sin 30°$ $= \frac{1}{2}$ d) $\cos 45°$ $= \frac{\sqrt{2}}{2}$ $= 0.707$ e) $\sin 90°$ $= 1$ f) $\cos 90°$ $= 0$

Note. Be sure your scientific calculator **Mode** is in **Degrees**.

B Inverse sine and cosine functions

MCR3U $\Rightarrow$ obtuse angles $> 180°$

Given the sine or cosine ratio k of an acute angle θ ($\theta < 90°$), use the inverse sine or inverse cosine functions to find the value of that angle:

$$\sin\theta = k \quad \Leftrightarrow \quad \theta = \sin^{-1}(k)$$

$$\cos\theta = k \quad \Leftrightarrow \quad \theta = \cos^{-1}(k)$$

$\boxed{\text{ALT}}$ $\boxed{\text{2nd}}$

Note. Press **SHIFT** and then **sin** or **cos** keys to get the inverse functions on your scientific calculator.

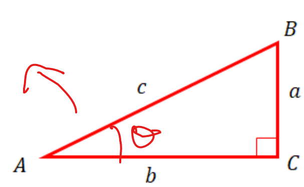
$\boxed{\begin{array}{l} -1 \le \sin\theta \le 1 \\ -1 \le \cos\theta \le 1 \end{array}}$

Example 2. For each case, find the angle θ.   θ is a complex number

a) $\sin\theta = 0.5$	b) $\sin\theta = 1$	c) $\cos\theta = 2$	d) $\cos\theta = 0.5$	e) $\sin\theta = 0.75$
$\theta = \sin^{-1}(0.5) = 30°$	$\theta = \sin^{-1}(1) = 90°$	θ is not a real number	$\theta = \cos^{-1} 0.5 = 60°$	$\theta = \sin^{-1} 0.75 = 48.59°$

Example 3. Use the definition of the trigonometric ratios (sine, cosine and tangent) to prove the following relationships:

$a^2 + b^2 = c^2$ (Pythagorean Theorem)



LS = left side
RS = Right side

a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

$RS = \frac{\sin\theta}{\cos\theta} = \frac{a/c}{b/c} = \frac{a}{\cancel{c}} \cdot \frac{\cancel{c}}{b}$

$= \frac{a}{b} = \tan\theta = LS$

b) $\underbrace{\sin^2\theta + \cos^2\theta}_{LS} = \underbrace{1}_{RS}$

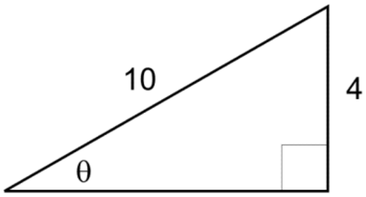
$LS = \sin^2\theta + \cos^2\theta$

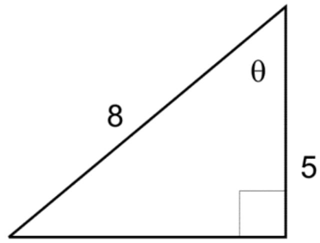
$= \left(\frac{a}{c}\right)^2 + \left(\frac{b}{c}\right)^2$

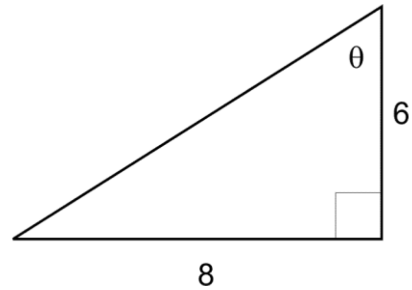
$= \frac{a^2 + b^2}{c^2}$

$= \frac{c^2}{c^2} = 1 = RS$

Example 4. For each case, find the value of the angle θ.







a) $\sin\theta = \frac{4}{10}$

$\theta = \sin^{-1}(4/10)$

$\simeq 23.58°$

b) $\cos\theta = \frac{5}{8}$

$\theta = \cos^{-1}(5/8)$

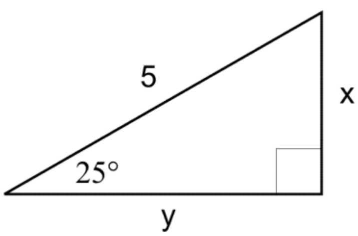
$\simeq 51.32°$

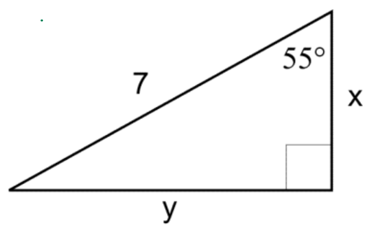
c) $\tan\theta = \frac{8}{6}$

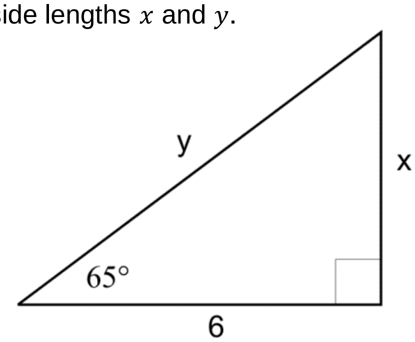
$\theta = \tan^{-1}(8/6)$

$\simeq 53.13°$

Example 5. For each case, use the sine or the cosine ratio to find the unknown side lengths x and y.







a) $\frac{\sin 25°}{1} = \frac{x}{5}$

$x = 5\sin 25° \simeq 2.11$

$\frac{\cos 25°}{1} = \frac{y}{5}$

$y = 5\cos 25° \simeq 4.53$

b) $\frac{\cos 55°}{1} = \frac{x}{7}$

$x = 7\cos 55°$

$\simeq 4.02$

$\sin 55° = \frac{y}{7}$

$y = 7\sin 55°$

$\simeq 5.73$

c) tan

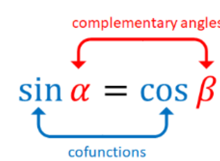
C Cofunction Identities

If $\alpha + \beta = 90°$ (α and β are called complementary angles), then the following cofunction identities are true:

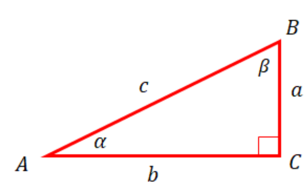
$$\sin\alpha = \cos\beta \qquad \cos\alpha = \sin\beta$$

or

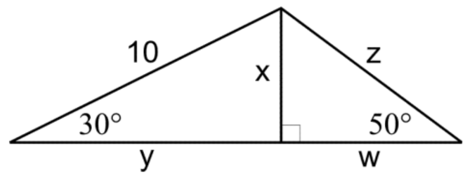
$$\sin(90° - \alpha) = \cos\alpha \qquad \cos(90° - \alpha) = \sin\alpha$$



Example 6. Prove the cofunction identities.



Example 7. Find the side lengths x, y, z, and w.

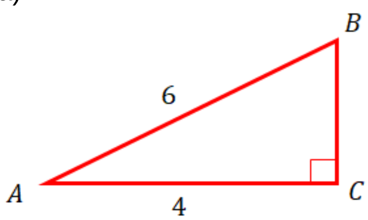


D Solving a Triangle

Solving a triangle means finding all sides and all angles

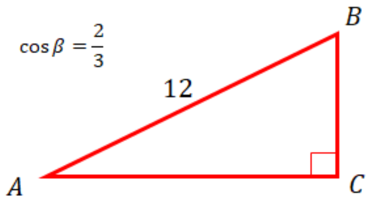
Example 8. Solve each triangle.

a)

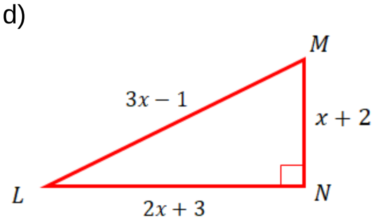


b) ΔPQR with $\angle P = 25°$, $\angle R = 90°$, $p = 10$

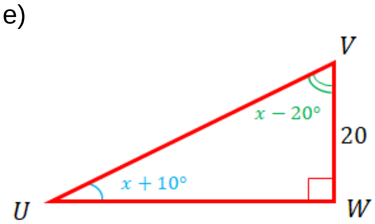
c)



d)



e)



Notes: Textbook Pages 366-371
Homework: Textbook Pages 372-377 # 1a, 2a, 6a, 7a, 8a, 9a, 10a, 11a, 12a, 13, 21, 24, 33