

### 7.3 The Tangent Ration

#### A The tangent of the slope angle

Let  $m$  be the slope of the hypotenuse  $AB$ .

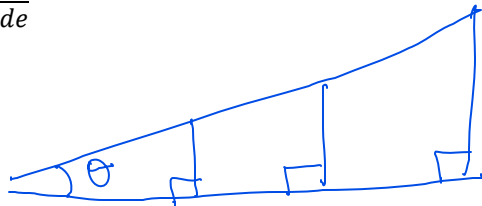
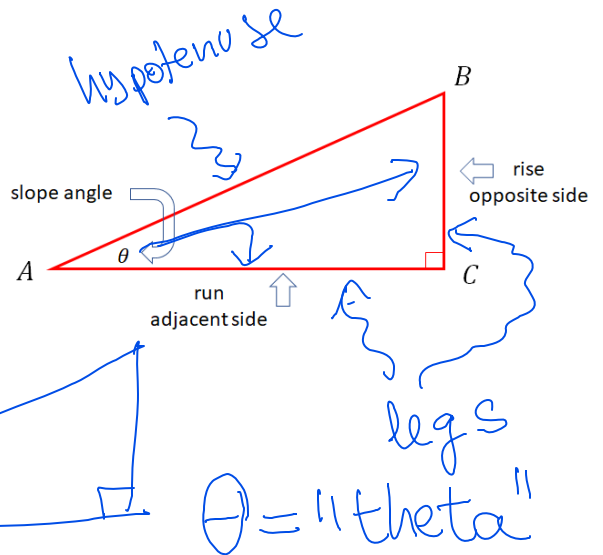
$$m = \frac{\text{rise}}{\text{run}}$$

The tangent ratio of the slope angle  $\theta$  is defined by:

$$\tan \theta = \frac{\text{opposite side}}{\text{adjacent side}}$$

Therefore:

$$m = \tan \theta$$



#### B Technology

Example 1. Use a scientific calculator to find the tangent of the following angles. Write the answer as an exact value and as an approximate value rounded to the nearest hundredths.

Note. Be sure your scientific calculator **Mode** is in **Degrees**.



a)  $0^\circ$

b)  $30^\circ$

c)  $45^\circ$

d)  $60^\circ$

e)  $90^\circ$

$$\tan 0^\circ = 0$$

$$\tan 30^\circ = \frac{\sqrt{3}}{3} \approx 0.57$$

$$\tan 45^\circ = 1$$

$$\tan 60^\circ = \sqrt{3} \approx 1.732$$

$\tan 90^\circ$  is undefined

#### C The inverse tangent function

If  $\theta$  is an acute angle ( $\theta < 90^\circ$ ) and  $\tan \theta = k$  then  $\theta = \tan^{-1}(k)$ .

Press **SHIFT** and then **tan** keys to get  $\tan^{-1}()$



$\tan^{-1} \Rightarrow$  "inverse tangent"



Example 2. For each case, find the angle  $\theta$ . Round the answers to two decimal places.

a)  $\tan \theta = 0.2$

$$\theta = \tan^{-1}(0.2) = 11.31^\circ$$

b)  $\tan \theta = 1$

$$\theta = \tan^{-1}(1) = 45^\circ$$

c)  $\tan \theta = 100$

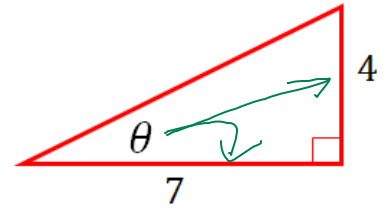
$$\theta = \tan^{-1}(100) = 89.43^\circ$$

Example 3. For each case, find the value of the angle  $\theta$ . Round the answers to two decimal places.

Note. Use  $\tan \theta = \frac{\text{opposite}}{\text{adjacent}}$ , then  $\theta = \tan^{-1} \dots$

a)  $\tan \theta = \frac{4}{7}$

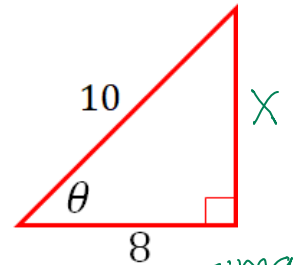
$\theta = \tan^{-1} \left( \frac{4}{7} \right) \approx 29.74^\circ$



b)  $10^2 = 8^2 + x^2 \Rightarrow x = 6$

$\tan \theta = \frac{6}{8}$

$\theta = \tan^{-1} (6/8) \approx 36.87^\circ$



Example 4. Find the unknown side lengths  $x$  and  $y$ . Round the answers to two decimal places.

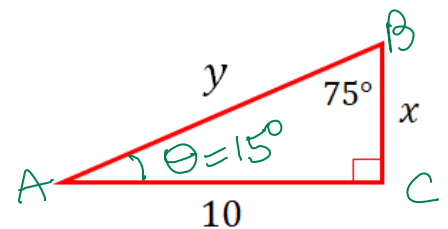
Hint. Use  $\tan \alpha = \frac{\text{opposite}}{\text{adjacent}}$  to find the first side, and Pythagorean Theorem to find the second side.

$\angle A = 180^\circ - 90^\circ - 75^\circ = 15^\circ = \theta$

$\frac{\tan 15^\circ}{1} = \frac{x}{10} \Rightarrow$

ANS =  $x = 10 \cdot \tan 15^\circ \approx 2.68$

$y = \sqrt{10^2 + \text{ANS}^2} \approx 10.35$



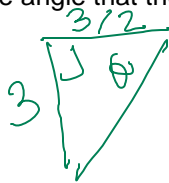
$\therefore x \approx 2.68$

$y \approx 10.35$

$\approx$  (approximate equal to)

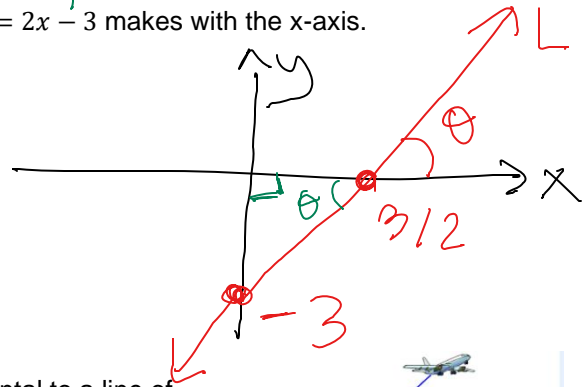
Example 5. Find the measure of the acute angle that the line  $y = 2x - 3$  makes with the x-axis.

$\tan \theta = \frac{3}{3/2} = 2$



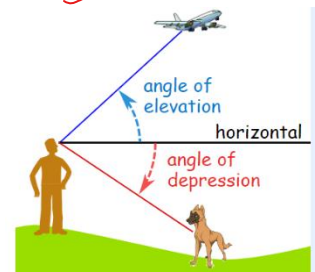
$\theta = \tan^{-1} (2)$

$\approx 63.43^\circ$



**D Angles of Elevation or Depression**

- ✓ Angle of Depression: The *downwards angle* from the horizontal to a line of sight from the observer to some point of interest.
- ✓ Angle of Elevation: The *upwards angle* from the horizontal to a line of sight from the observer to some point of interest.

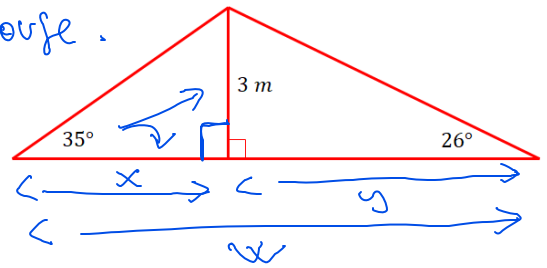


Example 6. The angles of inclination of the rafters of the roof of a house are  $35^\circ$  and  $26^\circ$ . The roof support is  $3\text{ m}$  high. How wide is the house, to the nearest metre?

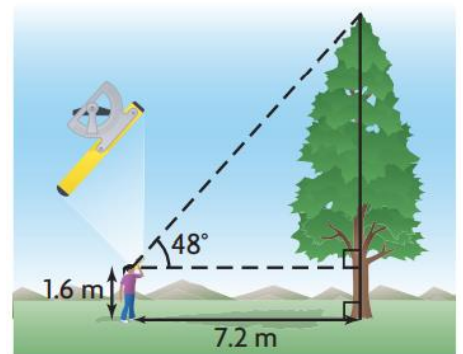
Let  $w$  be the width of the house.

$$w = x + y$$

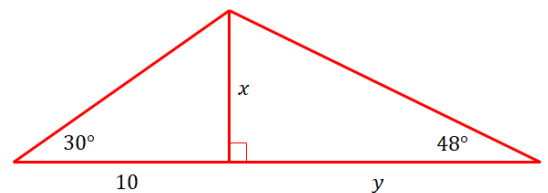
$$\frac{\tan 35^\circ}{1} = \frac{3}{x} \Rightarrow /$$



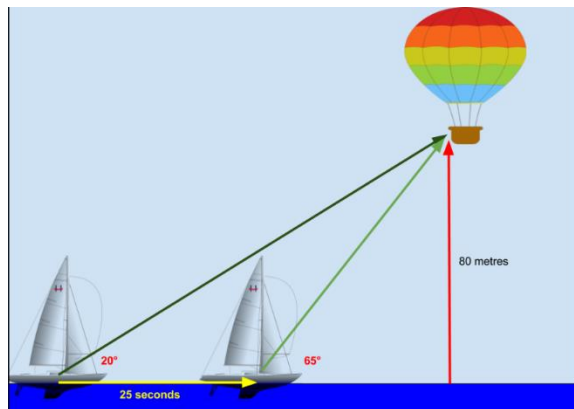
Example 7. Ayesha is a forester. She uses a clinometer (a device used to measure angles of elevation) to sight the top of a tree. She measures an angle of  $48^\circ$ . She is standing  $7.2\text{ m}$  from the tree, and her eyes are  $1.6\text{ m}$  above ground. How tall is the tree?



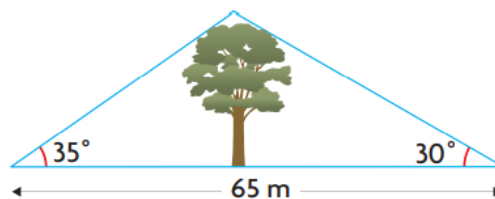
Example 8. Find the length of  $x$ , then the length of  $y$ .



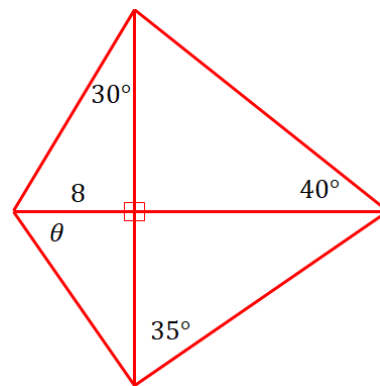
Example 9. A balloon is 80 metres above the water and a sailboat is approaching the vertical of the balloon. In 25 seconds, the angle of elevation changes from  $20^\circ$  to  $65^\circ$ . How fast (in metres per seconds) is the sailboat moving?



Example 10. Angles were measured from two points on opposite sides of a tree, as shown. How tall is the tree?



Example 11. Find the angle  $\alpha$  on the diagram to the right.



Notes: Textbook Pages 352-361

Homework: Textbook Pages 362-365 # 1a, 3a, 4a, 5a, 6a, 7e, 11, 17