

6.5 Solve Problems using Quadratic Equations

A An Algorithm to Solving Problems involving Quadratic Equations

1. Read the question carefully
2. Make diagram if necessary
3. Choose a quantity to be labelled as x
4. Read the question again and label all other quantities in terms of x
5. Based on the text of the problem, set up an quadratic equation in x
6. Solve the equation for x
7. Given the value(s) of x , find the values of all quantities labeled in terms of x . Interpret your results within the context of the problem
8. Check your results against the conditions stated in the problem
9. Read the question again and write down a conclusion statement

Problem 1. We throw an object upward from the top of a building. The height of the object, h (measured in feet) t seconds after we threw it is $h = -16t^2 + 160t + 1200$. How long does it take for the object to hit the ground?

Problem 2. Find the area of a right triangle whose sides are equal to three consecutive even integers.

Let x be the smallest side

$$(x+4)^2 = x^2 + (x+2)^2 \quad (\text{Pythagorean Relation})$$

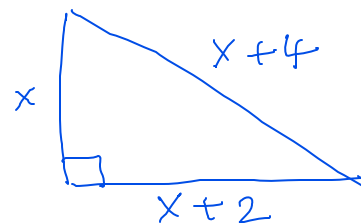
$$\cancel{x^2} + 8x + 16 = \cancel{x^2} + x^2 + 4x + 4$$

$$\begin{matrix} \hookrightarrow & \hookrightarrow \\ 0 = x^2 + 4x - 8x + 4 - 16 \end{matrix}$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6 \text{ or } x = -2 \quad (\text{reject, } x > 0)$$



$$x = 6$$

$$x+2 = 6+2 = 8$$

$$b = \text{base} = 8$$

$$h = \text{height} = 6$$

$$A = \frac{bh}{2} = \frac{8 \cdot 6}{2} = 24$$

∴ The area of a triangle is 24.

Problem 3. Tickets to a school dance cost \$5, and the projected attendance is 300 people. For every \$0.50 increase in the ticket price, the dance committee projects that attendance will decrease by 20. What ticket price will generate \$1562.50 in revenue?

Old Revenue = $(300)(\$5) = \1500 ← new price

New Revenue = $(300 - 20x)(5 + 0.5x) = R$

Let x be the number of times we increase the price by \$0.50.

$$(300 - 20x)(5 + 0.5x) = 1562.5$$

$$1500 + 150x - 100x - 10x^2 = 1562.5$$

$$10x^2 - 50x + 62.5 = 0$$

$$x = \frac{50 \pm \sqrt{2500 - 4(10)(62.5)}}{20}$$

$$x = 2.5$$

New Price
 $= 5 + 0.5(2.5)$
 $= 6.25$
 \therefore in order to collect \$1562.50 in revenue, the price should be \$6.25

Problem 4. The sum of a number and its reciprocal is 3. Find that number.

Note. The reciprocal of a number a is $1/a$.

Let a the number.

$$a + \frac{1}{a} = 3 \quad | \cdot a$$

$$a^2 + 1 = 3a$$

$$a^2 - 3a + 1 = 0$$

$$a = \frac{3 \pm \sqrt{9 - 4}}{2}$$

$$a = \frac{3 \pm \sqrt{5}}{2}$$

\therefore There are two numbers
 $a = \frac{3 \pm \sqrt{5}}{2}$
 such that the sum of the number and its reciprocal

Problem 5. A photograph measuring 12 cm by 8 cm is to be surrounded by a mat before framing. The width of the mat is to be the same on all sides of the photograph. The area of the mat is equal to the area of the photograph. Find the width of the mat.

Problem 6. Find the points of intersection between the parabolas $y = (x + 1)^2 - 2$ and $y = -x^2 + 3$.

Notes: Textbook Pages 304-311

Homework: Textbook Pages 311-314 # 2, 3, 4, 6, 12, 18, 20, 23, 27a