# 6.4 Quadratic Formula

## A Quadratic Equation

The equation

$$ax^2 + bx + c = 0 \qquad a \neq 0$$

is called the *quadratic equation*.

## B Quadratic Formula

The *roots (solutions)* of the quadratic equation may be found by using the *quadratic formula*:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1. Prove the quadratic formula by following these steps.

$$ax^2 + bx + c = 0$$

- ✓ Move $c$ on the right side

$$ax^2 + bx = -c$$

- ✓ Divide both sides by $a$

$$x^2 + \frac{b}{a}x = -\frac{c}{a}$$

- ✓ Add to both sides $\left(\frac{b}{2a}\right)^2$ to form a perfect square on the left side

$$x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 = -\frac{c}{a} + \left(\frac{b}{2a}\right)^2$$

- ✓ Write the left side as a square and get the common denominator on the right side

$$\sqrt{\left(x + \frac{b}{2a}\right)^2} = \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}}$$

$$\sqrt{-\frac{c}{a}\cdot\frac{4a}{4a} + \frac{b^2}{4a^2}} = \sqrt{\frac{b^2-4ac}{4a^2}} = \frac{\sqrt{b^2-4ac}}{2a}$$

- ✓ Square root both sides
- ✓ Isolate the variable $x$

$$x + \frac{b}{2a} = \pm\sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} \Rightarrow x = -\frac{b}{2a} \pm \sqrt{-\frac{c}{a} + \frac{b^2}{4a^2}} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

Example 2. Solve the following (non-factorable) quadratic equations given in standard form by using the algorithm presented at example 1. For each case, find the exact and the approximate value rounded to the nearest hundredths.

a) $2x^2 + 5x - 3 = 0$

1. $2x^2 + 5x = 3$
2. $x^2 + \frac{5}{2}x = \frac{3}{2}$
3. $x^2 + \frac{5}{2}x + \left(\frac{5}{4}\right)^2 = \frac{3}{2} + \left(\frac{5}{4}\right)^2$
4. $\left(x + \frac{5}{4}\right)^2 = \frac{3\cdot 8}{16} + \frac{25}{16}$
5. $x + \frac{5}{4} = \pm\sqrt{\frac{49}{16}}$
6. $x = -\frac{5}{4} \pm \frac{7}{4}$

$\therefore x = -3, \frac{1}{2}$

b) $x^2 - 4x + 2 = 0$

1. $x^2 - 4x = -2$
2. $x^2 - 4x = -2$
3. $x^2 - 4x + 4 = -2 + 4$
4. $(x-2)^2 = 2$
5. $x - 2 = \pm\sqrt{2}$
6. $x = 2 \pm \sqrt{2}$

$\therefore x = 2 \pm \sqrt{2}$ (exact values)

$x \simeq 0.59, 3.41$

Method #2
$(x-3)(x-3)=0$
$\therefore x = \pm 3$

(approximate values)

Example 3. Solve each quadratic equation by using the quadratic formula.

a) $x^2 - 9 = 0$

$1 \cdot x^2 + 0 \cdot x - 9 = 0$

$a = 1$; $b = 0$; $c = -9$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-0 \pm \sqrt{0^2 - 4(1)(-9)}}{2(1)}$

$= \frac{\pm 6}{2}$ $\therefore x = \pm 3$

b) $4x^2 - 4x + 1 = 0$

$a = 4$; $b = -4$; $c = 1$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-(-4) \pm \sqrt{(-4)^2 - 4(4)(1)}}{2(4)}$

$= 4$

c) $x^2 - x - 20 = 0$

d) $6x^2 - 13x + 6 = 0$

e) $x^2 - 2x - 4 = 0$

f) $x^2 + x + 1 = 0$

**C Vieta's Formulas**

The roots $x_1$ and $x_2$ of the quadratic equation $ax^2 + bx + c = 0$ satisfy Vieta's formulas:

$$x_1 + x_2 = -\frac{b}{a} \quad \text{and} \quad x_1 x_2 = \frac{c}{a}$$

Example 4. Prove Vieta's formulas.

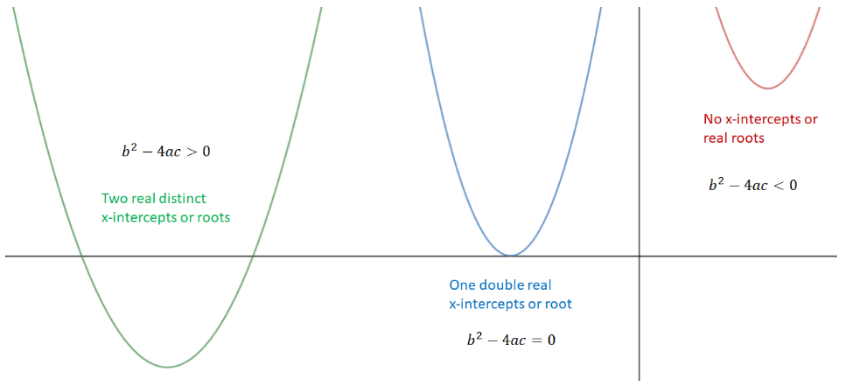

 

**D Quadratic Relations and Equations**

The *quadratic relation* $y = ax^2 + bx + c$ and the *quadratic equation* $ax^2 + bx + c = 0$ are close related to each other.
The *zeros* or the *x-intercepts* of the quadratic relation are identical to the *roots* (*solutions*) of the quadratic equation.

The quantity $D = b^2 - 4ac$ is called *discriminant*.

✓ If $D = b^2 - 4ac > 0$, then the quadratic equation has *two real distinct roots* (solutions). The corresponding quadratic function has *two distinct x-intercepts* (zeros).
✓ If $D = b^2 - 4ac = 0$, then the quadratic equation has *one double real root* (solution) (or *two coincident roots*). The corresponding quadratic function has *one double x-intercept* (zero) (or *two coincident x-intercepts or zeros*).
✓ If $D = b^2 - 4ac < 0$, then the quadratic equation has *no real roots* (solutions). In this case the quadratic equation has *two distinct complex* roots. The corresponding quadratic function has *no x-intercept*.




Example 5. For each case, find how many real distinct roots each quadratic equation has.

a) $x^2 + x + 1 = 0$ b) $x^2 + x - 2 = 0$ c) $x^2 + 2x + 1 = 0$

Example 6. Find the parameter $k$, such that the following quadratic equation has one double real root.

$2x^2 + kx + k + 6 = 0$

**E Technology**

Example 7. Use technology to solve the following quadratic equations. Find the exact values of the roots and the approximate values to the nearest hundredths.

a) $0.1x^2 - x + 0.25 = 0$
(use a scientific calculator)

b) $\pi x^2 - 5x + \frac{1}{3} = 0$
(use Wolfram Alpha)

Notes: Textbook Pages 292-300
Homework: Textbook Pages 300-303 # 1ab, 2ab, 3ab, 6, 9ab, 11, 16