6.1 Maxima and Minima

A Terminology

$\{-2, 7, 10, -15\}$

maximum

minimum

- ✓ The *maximum* element of a set is the *largest* element in that set
- ✓ The *minimum* element of a set is the *smallest* element in that set
- ✓ An *extremum* is either maximum or a minimum
- ✓ *Maxima* is the plural of maximum
- ✓ *Minima* is the plural of minimum
- ✓ *Extrema* is the plural of extremum

"Latin"!

B Maximum for a quadratic relation

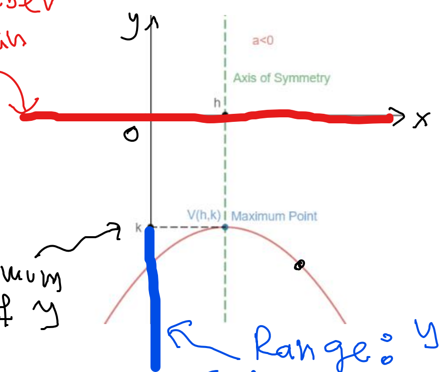
set of all real number

Domain

$D = \mathbb{R}$

If $a < 0$ for a quadratic relation, then

- ✓ The vertex point $V(h, k)$ is a *maximum point*
- ✓ The y-coordinate of the vertex point k is the *maximum value*
- ✓ The *range* of the quadratic relation is $y \le k$



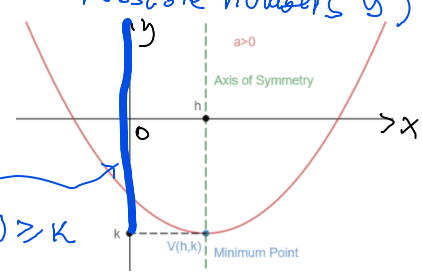
maximum value of y

Range: $y \le K$
(set of all possible numbers y)

C Minimum for a quadratic relation

If $a > 0$ for a quadratic relation, then

- ✓ The vertex point $V(h, k)$ is a *minimum point*
- ✓ The y-coordinate of the vertex point k is the *minimum value*
- ✓ The *range* of the quadratic relation is $y \ge k$



R: $y \ge K$

D Vertex Form

$y = a(x-h)^2 + K$

If the quadratic relation is given in the vertex form

- ✓ The vertex point $V(h, k)$ is an extremum point (either a maximum or a minim point)

Example 1. Complete the following table.

$a = -2 < 0$ $\qquad$ $a = 0.1 > 0$

Quadratic Relation	$y = 7 - 2(x+3)^2$	$y = 0.1(x-5)^2$
Vertex Point	$V(-3, 7)$	$V(5, 0)$
Minimum/Maximum Point	maximum	minimum
Minimum/Maximum Value for the variable y	$y_{max} = 7$	$y_{min} = 0$
Range (the set of all y-values)	$y \le 7$	$y \ge 0$

$ax^2+bx+c \longrightarrow a(x-h)^2+k$

standard form vertex form

E Completing the Square (Method #1)

To convert the *standard form* into the *vertex form* by *completing the square*: $y = ax^2+bx+c$

- ✓ Factor out the coefficient a from the first two terms $\quad y = a\left(x^2 + \frac{b}{a}x\right) + c$ $\quad b$
- ✓ Add and subtract the square of the half of the coefficient of x $\quad y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$
- ✓ Write the first three terms as a square $\quad y = a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$
- ✓ Remove the bracket $\quad y = a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$

Example 2. Find the vertex form by completing the square (non-factorable trinomials).

	$a=1$	$a=-2$	$a=2$
Quadratic Relation	$y = x^2 + 2x + 3$	$y = -2x^2 + 4x + 1$	$y = 2x^2 + 3x - 1$ (~~challenge~~)
Factor out the coefficient a from the first two terms	$y=1(x^2+2x)+3$	$y=-2(x^2-2x)+1$	$y=2\left(x^2+\frac{3}{2}x\right)-1$
Add and subtract the square of the half of the coefficient of x	$\frac{2}{2}=1 \rightarrow 1^2=1$ $y=(x^2+2x+1-1)+3$	$\frac{-2}{2}=-1 \rightarrow (-1)^2=1$ $y=-2(x^2-2x+1-1)+1$	$\frac{\frac{3}{2}}{2}=\frac{3}{4} \rightarrow \left(\frac{3}{4}\right)^2=\frac{9}{16}$ $y=2\left(x^2+\frac{3}{2}x+\frac{9}{16}-\frac{9}{16}\right)-1$
Write the first three terms as a square	$y=[(x+1)^2-1]+3$	$y=-2[(x-1)^2-1]+1$	$y=2\left[\left(x+\frac{3}{4}\right)^2-\frac{9}{16}\right]-1$
Remove the bracket	$y=(x+1)^2-1+3$	$y=-2(x-1)^2+2+1$	$y=2\left(x+\frac{3}{4}\right)^2+2\left(-\frac{9}{16}\right)-1$
Vertex Form	$\therefore y=(x+1)^2+2$	$\therefore y=-2(x-1)^2+3$	$y=2\left(x+\frac{3}{4}\right)^2-\frac{17}{8}$
Minimum/Maximum Value for the variable y	$y_{min}=2$	$y_{max}=3$	$y_{min}=-\frac{17}{8}$

F Convert Standard Form into Vertex Form (Method #2)

To convert the *standard form* into the *vertex form*:

- ✓ Find h by using $h = \frac{-b}{2a}$
- ✓ Find k by substitution $x = h$ into the standard form
- ✓ Write the vertex form $y = a(x-h)^2 + k$

Example 3. Prove that the x-coordinate of the vertex point is given by $h = \frac{-b}{2a}$

Example 3. Complete the following table. Note that all these trinomials are non-factorable.

Quadratic Relation	$y = x^2 + 4x + 1$	$y = -2x^2 + 3x - 4$ (challenge)
Find h		
Find k		
Vertex Form		
Minimum/Maximum Value for the variable y		

Example 4. The drag on a small aircraft is made up of induced drag from the wings as they produce lift and parasitic drag from the airframe. Over a limited speed range, the drag, d, in newtons, produced by a speed, v, in kilometres per hour, can be modelled by the quadratic relation $d = 0.15v^2 - 9v + 195$. Determine the speed that results in minimum drag.

G Technology

Example 5. Use Wolfram Alpha to find the vertex form for $y = -15x^2 + 27x - 11$.

Hint. Go to Wolfram Alpha and enter:
`vertex form -15x^2+27x-11` or `complete the square -15x^2+27x-11`

Notes: Textbook Pages 264-269
Homework: Textbook Pages 270-273 # 3ag, 4a, 5, 7cd, 8ad, 9e, 10ae, 11a, 17a, 22 (use Desmos or Wolfram Alpha)