5.6 Factor a Perfect Square Trinomial and a Difference of Squares

A Factor a Perfect Square Trinomial

If an expression is a *perfect square trinomial*, then use the following identities to factor.

$$a^2 + 2ab + b^2 = (a + b)^2 \qquad a^2 - 2ab + b^2 = (a - b)^2$$

Example 1. Verify if the following trinomials are perfect square trinomials. Then, factor.

a) $1 + 2x + x^2$

$= (x+1)^2$

$= (x+1)(x+1)$

b) $4 - 4x + x^2$

$= (2-x)^2$

$= (x-2)^2$

c) $9x^2 - 24xy + 16y^2$

$= (3x-4y)^2$

$-2(3x)(4y)$

d) $4a^2x^2 + 4abxy + b^2y^2$

$= (2ax+by)^2$

e) $\frac{9x^2}{16} - 2xy + \frac{16y^2}{9}$

$= \left(\frac{3x}{4} - \frac{4y}{3}\right)^2$

$-2\left(\frac{\cancel{3}x}{\cancel{4}}\right)\left(\frac{\cancel{4}y}{\cancel{3}}\right)$

$= -2xy$

(OK)

f) $4a^2 - 12ab + 9b^2$

$= (2a-3b)^2$

Example 2. Factor fully by factoring first the GCF.

a) $12x^2 + 60x + 75$

$= 3(4x^2+20x+25)$

$= 3(2x+5)^2$

b) $32x^3 - 160x^2 + 200x$

$= 8x(4x^2-20x+25)$

$= 8x(2x-5)^2$

c) $72x^3y - 120x^2y^2 + 50xy^3$

$= 2xy(36x^2-60xy+25y^2)$

$= 2xy(6x-5y)^2$

B Factor a Difference of Squares

To factor a *difference of squares*, use the following identity

$$a^2 - b^2 = (a - b)(a + b)$$

Example 3. Verify if the following binomials are difference of squares. Then, factor.

a) $1 - 9x^2$

$= 1^2 - (3x)^2$

$= (1 - 3x)(1 + 3x)$

b) $4a^2 - 25b^2$

$= (2a)^2 - (5b)^2$

$= (2a - 5b)(2a + 5b)$

c) $16a^2x^2 - 25b^2y^2$

$= (4ax)^2 - (5by)^2$

$= (4ax - 5by)(4ax + 5by)$

d) $(x + 1)^2 - 4$

$= (x+1)^2 - 2^2$

$= [(x+1) - 2][(x+1) + 2]$

$= (x - 1)(x + 3)$

e) $(2x + 1)^2 - (2x - 1)^2$

$\cancel{4x^2} + 4x \cancel{+1} - (\cancel{4x^2} - 4x \cancel{+1})$

$= 8x$

f) $\frac{4a^2}{9} - \frac{9b^2}{16}$

$= \left(\frac{2a}{3}\right)^2 - \left(\frac{3b}{4}\right)^2$

$= \left(\frac{2a}{3} - \frac{3b}{4}\right)\left(\frac{2a}{3} + \frac{3b}{4}\right)$

Example 4. Factor fully by factoring first the GCF.

a) $8x^2 - 32y^2$

$= 8(x^2 - 4y^2)$

$= 8(x + 2y)(x - 2y)$

b) $18x^2y^2 - 50y^4$

$= 2y^2(9x^2 - 25y^2)$

$= 2y^2(3x + 5y)(3x - 5y)$

c) $128x^2y - 50y^3$

$= 2y(64x^2 - 25y^2)$

$= 2y(8x - 5y)(8x + 5y)$

Example 5. Factor fully.

a) $a^4 - b^4$

$= (a^2 - b^2)(a^2 + b^2)$

$= (a - b)(a + b)(a^2 + b^2)$

b) $x^8 - y^8$

$= (x^4 - y^4)(x^4 + y^4)$

$= (x - y)(x + y)(x^2 + y^2)(x^4 + y^4)$

c) $a^{16} - b^8$

$= (a^8 - b^4)(a^8 + b^4)$

$= (a^4 - b^2)(a^4 + b^2)(a^8 + b^4)$

$= (a^2 - b)(a^2 + b)(a^4 + b^2)(a^8 + b^4)$

Example 6. Find the value of the parameter k, such that the following expressions may be factored over the integers. (You may skip this!)

a) $4x^2 + kx + 25$

$pq = (4)(25) = 100 > 0$

$p + q = k = ?$

$p \cdot q$	$p + q$
$1 \cdot 100$	
$2 \cdot 5$	

b) $kx^2 - 12xy + 4y^2$

Example 7. Prove the following identity (challenge).

$$(a + b + c)(a + b - c)(a - b + c)(-a + b + c) = 2a^2b^2 + 2a^2c^2 + 2b^2c^2 - a^4 - b^4 - c^4$$

Notes: Textbook Pages 248-253

Homework: Textbook Pages 253-255 #1ab, 2ab, 3ab, 4ab, 5ab, 6ab, 8ad, 9a, 10, 14, 19, 20, 21