

Factor over integers: all coefficients are integers

$(ax+b)(cx+d) \Rightarrow a, b, c, d$ are all integers

5.5 Factor Quadratic Expressions of the Form $ax^2 + bx + c$

A Factoring $ax^2 + bx + c$

To factor the trinomial $ax^2 + bx + c$ $a \neq 0, \pm 1$

- Find two integers p and q so:

$pq = ac$ and $p + q = b$

- break up the middle term bx into $px + qx$
- factor by grouping

Example 1. Factor fully, if possible.

a) $6x^2 + x - 2$
 $pq = (6)(-2) = -12$
 $p + q = 1$
 $p, q = 4, -3$
 $6x^2 + 4x - 3x - 2$
 $2x(3x+2) - 1(3x+2)$
 $\therefore (2x-1)(3x+2)$

b) $3x^2 - 11x + 6$
 $pq = (3)(6) = 18$
 $p + q = -11$
 $p, q = -9, -2$
 $3x^2 - 9x - 2x + 6$
 $3x(x-3) - 2(x-3)$
 $\therefore (3x-2)(x-3)$

c) $15x^2 + 16x + 4$
 $pq = (15)(4) = 60$
 $p + q = 16$
 $p, q = 6, 10$ $16x$
 $15x^2 + 6x + 10x + 4$
 $= 3x(5x+2) + 2(5x+2)$
 $\therefore (3x+2)(5x+2)$

Example 2. Factor fully, if possible.

a) $9x^2 - 6xy - 8y^2$
 $pq = 9(-8) = -72$
 $p + q = -6$
 $p, q = -12, 6$
 $9x^2 - 12xy + 6xy - 8y^2$
 $3x(3x-4y) + 2y(3x-4y)$
 $= (3x-4y)(3x+2y)$

b) $10x^2 + 17xy + 3y^2$
 $pq = (10)(3) = 30$
 $p + q = 17$
 $p, q = 15, 2$
 $10x^2 + 15xy + 2xy + 3y^2$
 $= 5x(2x+3y) + y(2x+3y)$
 $= (2x+3y)(5x+y)$

c) $6x^2 - 23xy + 20y^2$
 $pq = 6(20) = 120$
 $p + q = -23$
 $p, q = -15, -8$
 $6x^2 - 15xy - 8xy + 20y^2$
 $= 3x(2x-5y) - 4y(2x-5y)$
 $= (2x-5y)(3x-4y)$

Example 3. Factor fully by factoring first the GCF.

a) $-6x^3y - 9x^2y + 6xy$
 $= -3xy(2x^2 + 3x - 2)$
 $pq = (2)(-2) = -4$
 $p + q = 3$
 $p, q = 4, -1$
 $2x^2 + 4x - x - 2$
 $2x(x+2)$

b) $12x^3y^2 + 14x^2y^3 + 4xy^3$

c) $-30x^3yz + 52x^2y^2z - 16xy^3z$

Example 4. Factor fully.

a) $(x + a)^2 + 3(x + a) + 2$

b) $(x - 2)^2 + 4(x - 2) - 5$

c) $3(x + 1)^3 - 7(x + 1)^2 + 2(x + 1)$

C Technology

- ✓ Use technology to find x_1 and x_2 for the equation $ax^2 + bx + c = 0$
- ✓ *Substitute* x_1 and x_2 into $a(x - x_1)(x - x_2)$ to get the *factored form*
- ✓ Distribute the factors of a between the two brackets according to specific needs

Example 5. Use technology (scientific calculator, [Wolfram Alpha](#), [Symbolab](#)) to factor.

a) $5x^2 - 5x - 6$

b) $12x^2 - 23x + 5$