5.4 Factor Quadratic Expressions of the Form $x^2 + bx + c$

$a = 1$ ← standard ← factored

$(x+p)(x+q)$

A Factoring $x^2 + bx + c$

Example 1. Expand $(x + p)(x + q)$. Simplify the answer and make connection to $x^2 + bx + c$.

$(x+p)(x+q) = x^2 + xq + px + pq = x^2 + (p+q)x + pq$

$b = p + q$ $\quad$ $c = pq$

Conclusion: To factor the trinomial $x^2 + bx + c$

- ✓ Find two integers p and q so:

$$pq = c \qquad \text{and} \qquad p + q = b$$

- ✓ Replace the trinomial $x^2 + bx + c$ by the equivalent factored expression $(x + p)(x + q)$

Example 2. For each case, find two integers p and q satisfying the required conditions.

a) $pq = 6$, $p + q = 5$

$p \times q$	$p+q$
1×6	7
$-1 \times (-6)$	-7
2×3	5
$(-2) \times (-3)$	-5

$\therefore$ p and q are 2 and 3

b) $pq = -6$, $p + q = 1$

$pq = -6$	$p+q$
$1 \times (-6)$	-5
-1×6	5
$2 \times (-3)$	-1
-2×3	1

$\therefore$ p, q are -2 and 3

c) $pq = -6$, $p + q = -1$

$\therefore$ p, q are 2 and -3

d) $pq = -6$, $p + q = -5$

$\therefore$ p, q are 1 and -6

e) $pq = 42$, $p + q = -13$

p, q are negative

pq	$p+q$
$(-1)(-42)$	
$(-2)(-21)$	
$(-3)(-14)$	
$(-6)(-7)$	-13

$\therefore$ p, q are $-6, -7$

f) $pq = -12$, $p + q = 6$

pq	$p+q$
$1 \times (-12)$	± 11
$2 \times (-6)$	± 4
$3 \times (-4)$	± 1

$\therefore$ no solution if p and q are integers

Example 3. Factor fully, if possible.

a) $x^2 + 7x + 12$

$p + q$ ↑ ↳ pq

$p, q = 4, 3$

$\therefore (x+4)(x+3)$

b) $x^2 + x - 20$

$p + q = 1$

$pq = -20$

$p, q = 5, -4$

$\therefore (x+5)(x-4)$

c) $x^2 - 2x - 35$

$p + q = -2$

$pq = -35$

$p, q = 5, -7$

$\therefore (x+5)(x-7)$

∴ solution is possible over complex numbers

d) $x^2 - 9x - 36$

$p, q = -12, 3$

$\therefore (x-12)(x+3)$

e) $-x^2 + 11x - 28$

$= -(x^2 - 11x + 28)$

$p+q = -11$

$pq = 28$

$p, q = -7, -4$

$\therefore -(x-7)(x-4)$

f) $x^2 + x + 1$

$p+q = +1$

$pq = 1$

$(1)(1) \rightarrow 2$ ✗ $p+q$

$(-1)(-1) \rightarrow -2$ ✗

∴ no solution over integers

∴ no solutions over real numbers

Example 4. Factor fully, if possible.

a) $x^2 - 2xy + 48y^2$

b) $x^2 + 14xy + 45y^2$

c) $x^2 + 5xy - 12y^2$

Example 5. Factor fully by factoring first the GCF.

a) $-2x^3y + 4x^2y + 48xy$

b) $5ax^2y + 20axy + 60ay$

c) $-\frac{x^3y^2}{2} + \frac{11x^2y^3}{2} + 30xy^4$

Example 6. Find the value of the parameter k, such that the following trinomial may be factored over the integers.

a) $x^2 + kx - 6$

b) $x^2 + 6x + k$

B Technology (scientific calculator)

- ✓ Use a scientific calculator to find x_1 and x_2 for the equation $x^2 + bx + c = 0$
- ✓ Substitute x_1 and x_2 into $(x - x_1)(x - x_2)$ to get the factored form

Example 7. Use a scientific calculator to factor. Watch a Loom Video with Casio fx-991 ES PLUS

a) $x^2 + 5x - 750$

b) $x^2 + 0.5x - 1$ (optional)

c) $x^2 + \pi x - 2$ (optional)

d) $x^2 + x + 1$ (optional)

C Technology (Wolfram Alpha)

- ✓ Search the Internet for Wolfram Alpha or use this link: https://www.wolframalpha.com/
- ✓ Enter: factor x^2-x-2 or zeros of x^2-x-2 (see the figure below)

- ✓ Click on the button to get the answer

Example 8. Use Wolfram Alpha to factor.

a) $x^2 + 6x - 391$

b) $x^2 + \pi x - 2$ (optional)

c) $x^2 + x + 1$ (optional)

Notes: Textbook Pages 236-240
Homework: Textbook Pages 240-241 #1ab, 2ab, 3ab, 4ab, 5ab, 7ab, 8ab, 9ab, 11, 13, 15, 16cd