$E = t + t + \cdots$

term

$E = f \cdot f \cdot \cdots \cdot f$

factor

5.3 Common Factors

A Factoring

Factoring is the *opposite process of expanding* and is based on the *distributive property* (see the diagram below).

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Example 1. Factor.

a) $x^2 + 2x = x \cdot x + 2 \cdot x$

$x \cdot (x + 2) = x(x+2)$

b) $2x^2 - 4y$

$= 2 \cdot x \cdot x - 2 \cdot 2 \cdot y$

$= 2(x^2 - 2y)$

c) $x^2y - 3y^2$

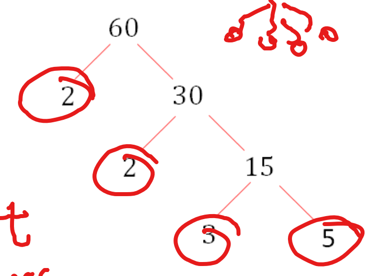
$= y(x^2 - 3y)$

TREE

B Factor Tree

✓ Used to identify all prime factors of an integer (see the diagram on the right)

prime factorization



On this example:

$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$

Any factor of 60 is a product of one or more of its prime factors.

The list of all possible positive factors of 60 is $1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60$

long list of factors

C The Greatest Common Factor

✓ GCF stand for Greatest Common Factor

✓ Greatest Common Factor (GCF) is used to factor out all common factors of the terms of a polynomial

✓ GCF is equal to the product of all *common bases* raised to the *least exponent*

Example 2. For each case, find the greatest common factor.

a) $4ax^2$, $-2axy$

$2^2 \cdot a \cdot x^2 \qquad -2 \cdot a \cdot x \cdot y$

$2ax = GCF$

b) $15y^3$, $10axy^2$, $25a^2y$

$5y = GCF$

c) $6x$, $3xy^2$, $-8y$

$GCF = 1$

D Factoring Polynomials

✓ Get the GCF of all terms and write in from of a bracket

✓ Include in the bracket all terms of the polynomial divided by the GCF

Example 3. Factor fully the following polynomials.

a) $2x + 4x^2$

$= 2x(1 + 2x)$

b) $3xy - 12x^2y^3$

$= 3xy(1 - 4xy^2)$

c) $-6ab^2 + 15bc^2$

$= 3b(-2ab + 5c^2)$

$= -3b(2ab - 5c^2)$

d) $-2x+6x^2-8x^3$

$-2x \quad -2x \quad -2x$

$=-2x(1-3x+4x^2)$

$=2x(-1+3x-4x^2)$

e) $12ab^2c-15a^2b^3c+18ab^2c$

$=3ab^2c(4-5ab+6)$

f) $abc+bcx-5ax$

$=abc+bcx-5ax$

$GCF=1$

Example 4. Factor the following expressions containing fractions.

a) $\frac{x}{2}-\frac{5x^2}{2}$

$=\frac{x}{2}(1-5x)$

b) $\frac{2}{3}ab^2+\frac{5}{3}a^2b$

$=\frac{ab}{3}(2b+5a)$

c) $\frac{2x^2y^3}{3}-\frac{8xy^4}{9}+\frac{4xy^2}{15}$

$=\frac{2xy^2}{3}\left(xy-\frac{4y^2}{3}+\frac{2}{5}\right)$

E Factorizing Binomials as GCF

✓ Binomials or other expressions could be common factors.

Example 5. Factor the following expressions using binomials as common factors.

a) $a(x-1)+b(x-1)$

$=(x-1)(a+b)$

$y-x=-(x-y)$

b) $ab^2(x-y)+a^2b(y-x)$

$=ab(x-y)(b-a)$

$\to b(c-2b)$

c) $a(bc-2b^2)+2b^2-bc$

$b(2b-c)$

$=ab(c-2b)+b(2b-c)$

$=b(c-2b)(a-b)$

F Factoring by Grouping

✓ May be applied for polynomials with four terms
✓ Make two groups of two terms each
✓ Factor each group
✓ Factor the common binomial

Example 6. Use grouping to factor fully.

a) $ac+ad+bc+bd$

$=a(c+d)+b(c+d)$

$=(c+d)(a+b)$

$=(a+b)(c+d)$

b) $x^2+x-xy-y$

$=x(x+1)-y(x+1)$

$=(x+1)(x-y)$

c) $2x^2+6xy+x+3y$

$x(2x+1)+3y(2x+1)$

$=(2x+1)(x+3y)$

d) $5x^2y-10xy^2+4y^2-2xy$

$=5xy($

e) $x^2y-xy^2z+xz-yz^2$

f) $6a^2b-2ac-9ab^2c+3bc^2$

$\vec{a}\times\vec{b}\neq\vec{b}\times\vec{a}$

Notes: Textbook Pages 228-233
Homework: Textbook Pages 234-235 1-6 (as many as you can), 9, 11, 13, 14,15

$a+b=b+a$

$a\cdot b=b\cdot a$ commutative properties