



5.1 Multiply Polynomials

$$p = t + t - t + \dots - t$$

A Polynomials

A *polynomial* is an algebraic expression formed by *adding or subtracting terms*. Each *term* of the polynomial contains:

- a *numeric coefficient* multiplied by
- one or more *variables raised to non-negative integral exponent*

integer (noun)
integral (adj)

Example 1. Find if each of the following expression is or is not a polynomial.

- $-2x^0 = -2$ ✓
- a) -2 ✓ b) x ✓ c) $-x^2$ ✓ d) $3xy$ ✓ e) $-2x^{-1}$ ✗
- f) $1 + \frac{1}{3}x$ ✓ g) $-2x^2 + 3x - 5$ ✓ h) $\frac{2}{x+1}$ ✗ (rational expression) i) $x + xy - 2x^2y^5$ ✓ j) $-2x + 2^x$ ✗ (exponential expression)

B Monomials

Monomials are polynomials with *one single term*.

Example 2. Find if each of the following expression is or is not a monomial.

- a) x ✓ b) xy ✓ c) $-2x$ ✓ d) $x + y$ ✗ e) $-3x^2$ ✓
- f) $-2xy^2$ ✓ g) $-2xyz$ ✓ h) $1 - x + 4y - z$ ✗ i) 10 ✓ j) 2^x ✗

C Multiply monomials

- ✓ The product of two or more monomials is also a monomial
- ✓ The *numeric coefficient* is equal to the *product* of all numeric coefficients
- ✓ For each variable, apply the exponent rule $x^n x^m = x^{n+m}$

Example 3. Multiply the following monomials.

- a) $(-2)(+3x) = -6x$ b) $(-2x)(-y) = 2xy$ c) $(-3x)(-5x) = 15x^2$
- d) $(-4x)(\frac{1}{2}xy) = -2x^2y$ e) $(-xy^2)(3x^3) = -3x^4y^2$ f) $(-\frac{2x}{3})(\frac{-3xy}{4}) = \frac{1}{2}x^2y$

D Binomials

- ✓ *Binomials* are polynomials with *two terms*
- ✓ By adding or subtracting two monomials, a binomial is formed
- ✓ Binomials may be *simplified* if they contain *like terms*

binomial

Example 4. Find if each of the following expression is or is not a ~~monomial~~.

- a) x ✗ b) xy ✗ c) $-2x$ ✗ d) $x + y$ ✓ e) $-3x^2$ ✓
- f) $-2xy^2$ ✗ g) $-2xyz$ ✗ h) $1 - x + 4y - z$ ✗ i) 10 ✗ j) 2^x ✗

E Distributive Property

The distributive property states that for any numbers a , b , and c :

$$a(b + c) = ab + ac$$

F Multiplying a monomial by a binomial

- ✓ In order to multiply a monomial by a binomial, use the distributive property.

Example 5. Multiply the following monomials and binomials.

$$\begin{aligned} \text{a) } 2(x - y) &= 2(x) + 2(-y) \\ &= 2x - 2y \end{aligned}$$

$$\begin{aligned} \text{b) } (-x)(1 - 2y) &= (-x)(1) + (-x)(-2y) \\ &= -x + 2xy \end{aligned}$$

$$\begin{aligned} \text{c) } (-3x)(2x - 3y) \\ &= -6x^2 + 9xy \end{aligned}$$

$$\begin{aligned} \text{d) } (2x^2y)(xy^2 - 3x) \\ &= 2x^3y^3 - 6x^3y \end{aligned}$$

G Combining like terms

Like Terms:

- ✓ differ eventually by the *numeric coefficient*
- ✓ have *identical variables raised to identical exponents*
- ✓ may be combined by using the *distributive property*:

$$ax + bx = (a + b)x$$

Example 6. For each case, combine (collect) like terms.

$$\text{a) } 1 - x + 2 - 4x = 3 - 5x$$

$$\text{b) } x - xy - 2x + 3xy = -x + 2xy$$

$$\begin{aligned} \text{c) } -2 + x - xy + 4 - 5x + 3xy \\ &= 2 - 4x + 2xy \end{aligned}$$

$$\begin{aligned} \text{d) } x + 1 - x^2 - 2x - 3x^2 + 5 \\ &= -x + 6 - 4x^2 \\ &= 6 - x - 4x^2 \\ &= -4x^2 - x + 6 \end{aligned}$$

H Multiplying binomials (FOIL Method)

FOIL:

- ✓ stands for First, Outside, Inside, and Last
- ✓ is used to multiply two binomials (see the diagram below)

$$(a + b)(c + d) = ac + ad + bc + bd$$

First Last
Inside
Outside

Example 7. Use the FOIL method to multiply the following binomials.

$$\begin{aligned} \text{a) } (1 - 2x)(x - 3) \\ &= \underline{1} \cdot \underline{x} - \underline{3} - \underline{2x} \cdot \underline{x} + \underline{6x} \\ &= x - 3 - 2x^2 + 6x = -2x^2 + 7x - 3 \end{aligned}$$

$$\begin{aligned} \text{b) } (3x - 2y)(-2x + 4y) \\ &= -6x^2 + 16xy - 8y^2 \end{aligned}$$

$$\begin{aligned} &= (3x)(-2x) \\ &+ (3x)(4y) \\ &+ (-2y)(-2x) \\ &+ (-2y)(4y) \\ &= -6x^2 + 12xy \\ &+ 4xy - 8y^2 \end{aligned}$$

$$\text{F: } (1)(x) = x$$

$$\text{O: } (1)(-3) = -3$$

$$\text{I: } (-2x)(x) = -2x^2$$

I Multiplying binomials (Grid Method)

The *Grid Method* is used to multiply *two or more* polynomials (see the diagram below).

$$(x - 2)(2x + 3) = \begin{array}{r|rr} & x & -2 \\ \hline 2x & 2x^2 & -4x \\ 3 & 3x & -6 \\ \hline \end{array} = 2x^2 - x - 6$$

Example 8. Use the grid method to multiply binomials.

a) $(1 + x)(x + x^2)$

b) $(2x + 3y)(2x - 3y)$

J Multiplying polynomials (Each by Each Method)

✓ Multiply *each term* of the first expression by *each term* of the second expression (see the diagram below)

$$(a + b)(c + d + e) = ac + ad + ae + bc + bd + be$$

Example 9. Multiply (expand brackets). Simplify the answer.

a) $(1 + x + xy)(x - y)$

b) $(1 + x - x^2)(3 - x + 2x^2)$

c) $(a + b + c)(a + b - c)(a - b + c)(-a + b + c)$