4.6 Negative and Zero Exponents

A Multiplication Notation

Multiplication notation is shortcut for *repetitive addition*.

Examples 1. Simplify.

a) $10 + 10 + 10 =$ $3(10) = 3 \times 10 = 3 \cdot 10 = (3)(10)$

b) $x + x + x + x =$ $4x = 4 \cdot x = 4 \times x$

c) ☺+☺+☺= 3☺

B Exponential Notation

Exponential notation is a shortcut for *repetitive multiplication*.

Example 2. Simplify.

a) $4 \times 4 \times 4 \times 4 =$ 4^4

b) $a \cdot a \cdot a \cdot a \cdot a \cdot a =$ a^6

c) $\Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta =$ Δ^7

C Understanding the Exponential Notation

b^e

- ✓ b is called *base*
- ✓ e is called *exponent*
- ✓ b^e is called *power*

D Multiplying powers with the same base

Example 3. Write as a single power and develop a rule.

$a^4a^3 =$ $(a \cdot a \cdot a \cdot a)(a \cdot a \cdot a) = a^7 = a^{4+3}$

Conclusion: $\boxed{a^m \times a^n = a^{m+n}}$

To multiply powers with the same base, keep the base the same and add the exponents.

Example 4. Write as a single power (simplify). Do not evaluate.

a) $10^5 \times 10^3 =$ $10^{5+3} = 10^8$

b) $x^2 \times x^6 =$ $x^{2+6} = x^8$

c) $\omega^2 \times \omega^3 \times \omega^4 =$ $\omega^{2+3+4} = \omega^9$

d) $a^3 \times a^4 \times a \times a^5 =$ $a^{3+4+1+5} = a^{13}$

Note. $\boxed{a^1 = a}$

E Zero Exponent

Example 5. Prove that $\boxed{a^0 = 1}$ only if $a \neq 0$

$a^n a^0 = a^{n+0} = a^n \Rightarrow \frac{\cancel{a^n} a^0}{\cancel{a^n}} = \frac{\cancel{a^n}}{\cancel{a^n}} \Rightarrow a^0 = 1$

Example 6. What is 0^0?

0^0 is undefined

F Dividing powers with the same base

$a \div b = \frac{a}{b}$

Example 7. Write as a single power and develop a rule.

$a^4 \div a^3 = \frac{a^4}{a^3} = \frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot a}{\cancel{a} \cdot \cancel{a} \cdot \cancel{a}} = a = a^1 = a^{4-3}$

Note. $a \div b = \frac{a}{b}$

Conclusion: $\boxed{a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}}$

To divide powers with the same base, keep the base the same and subtract the exponents.

Example 8. Write as a single power (simplify). Do not evaluate.

a) $10^5 \div 10^3 = 10^{5-3} = 10^2$

b) ♥^{9} ÷ ♥^{4} = ♥$^{9-4}$ = ♥^{5}

c) $3^{10} \div 3^2 = 3^{10-2} = 3^8$

d) $x^{10} \div x^6 = x^{10-6} = x^4$

G Power of a Power

Example 9. Write as a single power and develop a rule.

$(a^2)^3 = (a \cdot a)(a \cdot a)(a \cdot a) = a^6 = a^{2 \cdot 3} = a^{2 \times 3}$

Conclusion: $\boxed{(a^m)^n = a^{m \times n}}$

To simplify a power of a power, keep the base the same and multiply the exponents.

Example 10. Write as a single power (simplify). Do not evaluate.

a) $(10^3)^4 = 10^{3 \cdot 4} = 10^{12}$

b) $(7^2)^6 = 7^{2 \cdot 6} = 7^{12}$

c) (☼^{5})^2 = ☼$^{5 \cdot 2}$ = ☼^{10}

d) $(x^3)^2 = x^{3 \cdot 2} = x^6$

$3 \cdot 2 = 2 \cdot 3$

e) $(x^2)^3 = x^{2 \cdot 3} = x^6$

Example 11. Write as a single power (simplify). Do not evaluate

a) $5^3 \times \frac{5^6}{5^2} = 5^3 \cdot 5^4 = 5^7$

b) $3^5 \times (3^2)^4 = 3^5 \cdot 3^8 = 3^{13}$

c) $(7^4)^4 \div (7^3)^3 = 7^{4\cdot4} \div 7^{3\cdot3} = \frac{7^{16}}{7^9} = 7^{16-9} = 7^7$

H Negative Exponent

Example 12. Prove that $\boxed{a^{-n} = \frac{1}{a^n}}$

$a^n \cdot a^{-n} = a^{n-n} = a^0 = 1$

$\rightarrow \frac{\cancel{a^n} \cdot a^{-n}}{\cancel{a^n}} = \frac{1}{a^n}$

Example 13. Simplify. Write the answer by using a positive exponent

a) $7^5 \times 7^{-3} = 7^{5-3} = 7^2$

b) $x^{-4} \times x^2 = x^{-4+2} = x^{-2} = \frac{1}{x^2}$

c) $2^2 \times 2^{-5} \times 2^0 = 2^{2-5+0} = 2^{-3} = \frac{1}{2^3}$

d) $a^3 \times a^{-5} \times a \times a^{-2} \times a^0 = a^{3-5+1-2+0} = a^{-3} = \frac{1}{a^3}$

I Multiplying powers with the same exponent

Example 14. Investigate $(ab)^3$ and develop a formula.

$(ab)^3 = (ab)(ab)(ab) = a^3 b^3$

Conclusion: $\boxed{(ab)^n = a^n b^n}$

Example 14. Expand and simplify. → remove brackets

a) $(2x)^3 = 2^3 \cdot x^3 = 8x^3$

b) $(ax^2)^4 = a^4 (x^2)^4 = a^4$

c) $(3x^{-3})^{-2} =$

d) $(-2x^2y^{-3})^{-4} =$

J Dividing powers with the same exponent

Example 15. Investigate $\left(\frac{a}{b}\right)^3$ and develop a formula.

Conclusion: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

K Exponent Rules (Review)

$(ab)^n = a^n b^n$ $\quad a^n a^m = a^{n+m}$ $\quad (a^n)^m = a^{nm}$ $\quad a^{-n} = \frac{1}{a^n}$ $\quad \frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\quad \frac{a^n}{a^m} = a^{n-m}$ $\quad a^0 = 1$ $\quad \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $\quad 0^0 = undefined$

Example 16. Use exponent rules to simplify.

a) $\left(\frac{-2}{3}\right)^{-2}$

b) $\left(\frac{3}{4}\right)^{-1}\left(\frac{4}{3}\right)^{-2}$

c) $\left[\left(\frac{a}{b}\right)^2\right]^{-1}$

d) $\frac{a^2 \times a^{-5}}{(a^{-2})^3}$

e) $\left(\frac{x^{-3}}{x^2}\right)^{-2}\left(\frac{x^2}{x^{-1}}\right)^3$

f) $\left(\frac{(a^2)^{-3}}{(a^{-1})^{-3}}\right)^{-2}((-a)^{-2})^3$

L Finite Differences for Exponential Relations

Example 17. Investigate finite differences for the exponential relation $y = 2^x - 1$.

x	$y = 2^x - 1$		
-2			
-1			
0			
1			
2			

Conclusion.

M Exponential Equations

Example 18. Solve for x.

a) $2^x = 1024$

b) $10^x = 0.00001$

c) $4^x = 0.125$

O Power versus Exponential Relations

The relation $y = x^n$ is a *power relation*. The relation $y = b^x$ is an *exponential relation*.

Example 19. How would you differentiate a power relation from an exponential relation?

Notes: Textbook Pages 194-198
Homework: Textbook Pages 199-200 #2, 3, 4, 8, 19