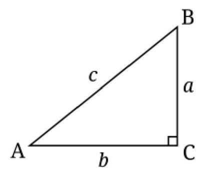
# 3.1 3.2 Properties of Triangles

## A Right Triangle

A triangle is a right triangle if either:

- one angle is right (90º)
- two sides are perpendicular to each other
- sides satisfy the Pythagorean Theorem

$$c^2 = a^2 + b^2$$




Note. The area of any triangle may be calculated by using:

$$A = \frac{base \times height}{2}$$

Ex 1. Prove that the triangle ABC where A(-3,-2), B(2,2) and C(-7,3) is a right triangle and find its area.

$AB = \sqrt{(2-(-3))^2 + (2-(-2))^2} = \sqrt{25+16} = \sqrt{41}$

$BC = \sqrt{(-7-2)^2 + (3-2)^2} = \sqrt{81+1} = \sqrt{81}$

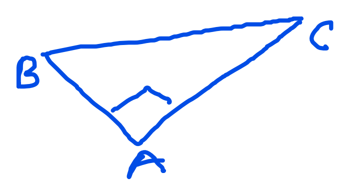
$AC = \sqrt{(-7-(-3))^2 + (3-(-2))^2} = \sqrt{16+25} = \sqrt{41}$

$AB^2 = 41$
$AC^2 = 41$
$BC^2 = 82$

$\Rightarrow BC^2 = AB^2 + AC^2$

$AB \perp AC$

$\angle A = 90^\circ$




$\therefore \triangle ABC$ is a right triangle

Method #2

$m_{AB} = \frac{2-(-2)}{2-(-3)} = \frac{4}{5} \Rightarrow$

$m_{AC} = \frac{3-(-2)}{-7-(-3)} = \frac{5}{-4}$

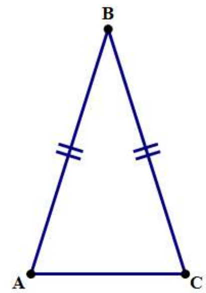
$m_{AB} = -\frac{1}{m_{AC}}$ or $m_{AB} \cdot m_{AC} = -1$

$\therefore AB \perp AC$

**B Isosceles Triangle**

A triangle is an isosceles triangle if:

- Two sides are equal
- Two angles are equal




AB = BC

Ex 2. Prove that the triangle ABC where A(-1,-2), B(-3,1) and C(2,0) is an isosceles triangle.

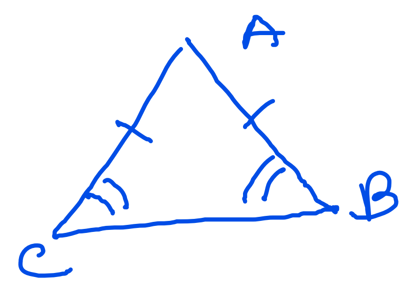
$$AB = \sqrt{(-3-(-1))^2 + (1-(-2))^2} = \sqrt{4+9} = \sqrt{13}$$

$$BC = \sqrt{(2-(-3))^2 + (0-1)^2} = \sqrt{25+1} = \sqrt{26}$$

$$AC = \sqrt{(2-(-1))^2 + (0-(-2))^2} = \sqrt{9+4} = \sqrt{13}$$

$AB = AC$

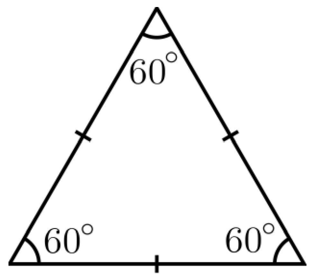
$\therefore \triangle ABC$ is isosceles



**C Equilateral Triangle**

A triangle is an equilateral triangle if:

- All sides are equal
- All angles are equal (60º)




Ex 3. Find the coordinates of vertex C of the equilateral triangle ABC where A(-4,0) and B(4,0).

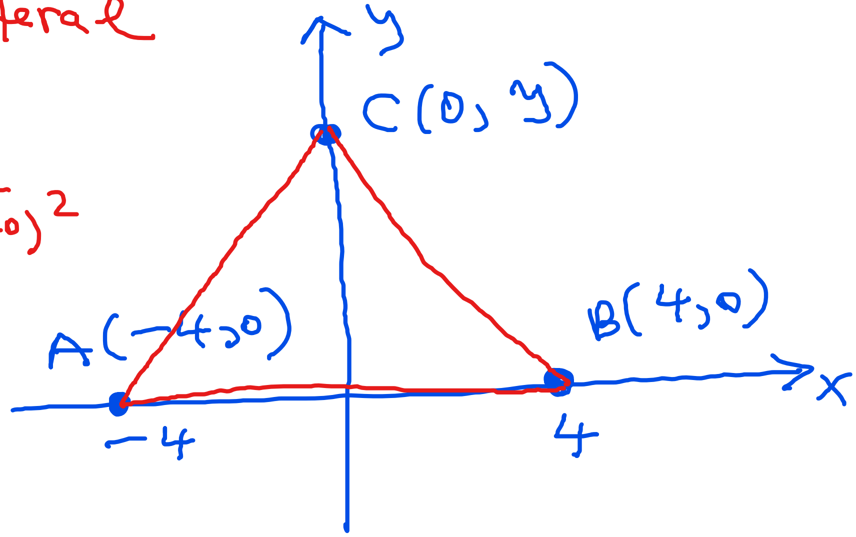
$\triangle ABC$ is equilateral

$AB = BC = CA$

$AB = 8$

$BC = \sqrt{(4-0)^2 + (y-0)^2}$

$= \sqrt{16 + y^2}$




$AB = BC$

$8 = \sqrt{16 + y^2}$

$64 = 16 + y^2$

$y^2 = 48$

$y = \pm\sqrt{48}$

$= \pm 4\sqrt{3}$

$\therefore$ $C(0, 4\sqrt{3})$

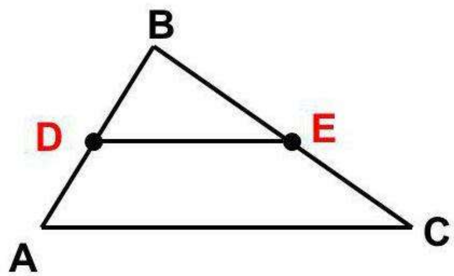
or

$C(0, -4\sqrt{3})$

(two solutions)

## D Mid-segment

The mid-segment is the line segment that joins the midpoints of two sides of a triangle.




The following two facts are true about the mid-segment:

$$m_{DE} = m_{AC}$$

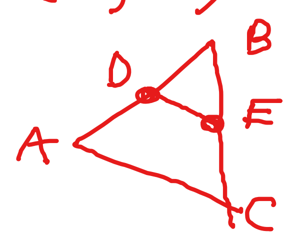
$$DE = \frac{1}{2}AC$$

Ex 4. Let consider the triangle ABC where A(-4,-5) and B(8,3), and C(6,-1).

a) Find the midpoint D of the side AB

$x_D = \frac{x_A + x_B}{2} = \frac{-4+8}{2} = 2$ $\therefore D(2,-1)$

$y_D = \frac{y_A + y_B}{2} = \frac{-5+3}{2} = -1$




b) Find the midpoint E of the side BC

$x_E = \frac{8+6}{2} = 7$ $\therefore E(7,1)$

$y_E = \frac{3-1}{2} = 1$

c) Compare the slopes of DE and AC

$m_{DE} = \frac{1-(-1)}{7-2} = \frac{2}{5}$ $\therefore m_{DE} = m_{AC}$, $DE \parallel AC$

$m_{AC} = \frac{-1-(-5)}{6-(-4)} = \frac{4}{10} = \frac{2}{5}$

d) Compare the lengths of DE and AC

$DE = \sqrt{(7-2)^2 + (1-(-1))^2} = \sqrt{25+4} = \sqrt{29}$

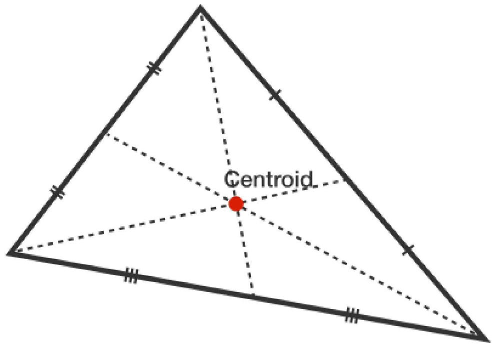
$AC = \sqrt{(6-(-4))^2 + (-1-(-5))^2} = \sqrt{100+16} = \sqrt{116}$

$\frac{AC}{DE} = \frac{\sqrt{116}}{\sqrt{29}} = \sqrt{\frac{116}{29}} = \sqrt{4} = 2$ $\therefore AC = 2DE$

or $DE = \frac{AC}{2}$

**E Centroid**

The centroid of a triangle is the point of intersection of all medians.




The coordinates of the centroid of a triangle $\Delta ABC$ are given by:

$$x = \frac{x_A + x_B + x_C}{3}$$
$$y = \frac{y_A + y_B + y_C}{3}$$

Ex 5. Find the centroid D of the triangle ABC where A(-4,-5) and B(8,3), and C(5,-1).

$$x_D = \frac{-4+8+5}{3} = 3$$

$$y_D = \frac{-5+3-1}{3} = -1$$

$\therefore D(3, -1)$

Notes: Textbook Pages 110-113 and 117-123
Homework: Textbook Pages 124 #2, 3, 4, 6, 7, 13,