1.3 Investigate Equivalent Linear Relations and Equivalent Linear Systems

A Equivalent Linear Relations

- ✓ Two linear relations are equivalent if their graphs are coincident.
- ✓ The following transformations applied to a linear relation lead to equivalent relations:
  - adding or subtracting a quantity from the left and the right side of the equation
  - moving terms from one side to the other side
  - dividing or multiplying by the same quantity both the left and the right side of the equation

Ex 1. Consider the following linear relation: $y = 2x + 1$. Apply the required transformations:

a) add 5 to both sides	b) subtract 1 from both sides	c) move $2x$ to the left side	d) divide by 2 both sides	e) multiply by 3 both sides
$y+5=2x+1+5$ $\boxed{y+5=2x+6}$	$y-1=2x+1-1$ $\boxed{y-1=2x}$	$y-2x=1$	$\frac{y}{2}=\frac{2x+1}{2}$ $\frac{y}{2}=x+\frac{1}{2}$	$3\cdot y=3\cdot(2x+1)$ $3y=6x+3$

Ex 2. Show that the following two linear relations are equivalent.

① $y - 3x + 2 = 0$ ① $\Rightarrow \boxed{y=3x-2}$ ①

② $3y = 9x - 6$

$\frac{3y}{3}=\frac{9x-6}{3} \Rightarrow y=\frac{9x}{3}-\frac{6}{3}$

$\boxed{y=3x-2}$ ②

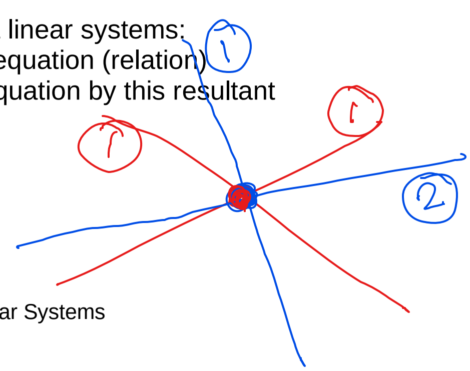
same

∴ ① and ② are two equivalent linear relations.

B Equivalent Linear Systems of Equations

- ✓ Two linear systems are equivalent if:
  - their solution(s) are identical
  - the corresponding lines have the same point(s) of intersection
- ✓ The following transformations applied to a linear system lead to equivalent linear systems:
  - replacing any (linear) equation (relation) by an equivalent (linear) equation (relation)
  - adding or subtracting any two equations and then replacing any equation by this resultant equation

① $\{$ ② $\{$



Ex 3. Do the required transformation:

$$\begin{cases} y + x = 3 & (1) \\ y - x = -1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} (1) + (2) \\ (1) - (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} (1) \div 2 \\ (2) \div 2 \end{cases}$$

$$\begin{cases} 2y = 2 \\ 2x = 4 \end{cases} \Rightarrow \begin{cases} \frac{\cancel{2}y}{\cancel{2}} = \frac{2}{2} \\ \frac{\cancel{2}x}{\cancel{2}} = \frac{4}{2} \end{cases} \Rightarrow \boxed{\begin{cases} y = 1 \\ x = 2 \end{cases}}$$

(Solution)

(1) $y + x = 3$

(2) $y - x = -1$

$(+)$

(1) + (2) $y + \cancel{x} + y - \cancel{x} = 3 + (-1)$

$2y = 2$

(1) $y + x = 3$

(2) $y - x = -1$

$(-)$

(1) − (2) $\Rightarrow y + x - (y - x) = 3 - (-1)$

$\cancel{y} + x - \cancel{y} + x = 4$

$2x = 4$

✓ Note that the solution of this system can be obtained by transforming the original linear system in equivalent linear systems.

Ex 4. Convert the original system of linear equations by doing the required transformations:

$$\begin{cases} 2x + 3y = 8 & (1) \\ 5x - 2y = 1 & (2) \end{cases} \quad \Leftrightarrow \quad \begin{cases} 5 \times (1) - 2 \times (2) \\ 2 \times (1) + 3 \times (2) \end{cases}$$

Ex 5. Prove that the following systems of linear equations are equivalent by solving each of the systems by substitution.

① $\begin{cases} x - 2y = -7 & (a) \\ 2y + 3x = 3 & (b) \end{cases}$ $\Leftrightarrow$ ② $\begin{cases} 2x + 3y = 7 \\ y - 2x = 5 \end{cases}$

(a) $x = 2y - 7$ (c)

$2y + 3(2y - 7) = 3$

$2y + 6y - 21 = 3$

$8y = 21 + 3$

$\frac{8y}{8} = \frac{24}{8}$

$\boxed{y = 3}$

(c) $x = 2(3) - 7$

$x = 6 - 7$

$\boxed{x = -1}$

Check (skip)

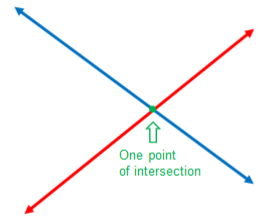
Ex 6. Are the following systems of linear equations equivalent? Explain. Use Desmos to visualize the case.

① $\begin{cases} 2x + 3y = 6 \\ y - 2x = 10 \end{cases}$ $\Leftrightarrow$ ② $\begin{cases} x = -3 \\ y = 4 \end{cases}$

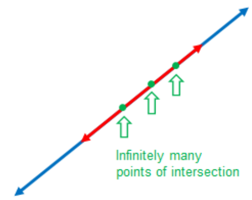
C Consistent and Inconsistent Systems of Linear Equations

A system of linear equation may have:

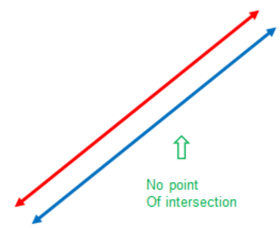
- ✓ Exactly one solution (consistent system)
- ✓ The corresponding lines are not parallel
- ✓ $m_1 \neq m_2$ (slopes are different)
- ✓ The solution is related to the point of intersection of the corresponding lines



- ✓ Infinitely many solutions (consistent system)
- ✓ One equation is a transformation of the other equation
- ✓ The corresponding lines are identical
- ✓ $m_1 = m_2$ and $b_1 = b_2$ (slopes and y-intercepts are the same)
- ✓ The solutions correspond to any point of the common line



- ✓ No solution (inconsistent system)
- ✓ The corresponding lines are parallel but not identical
- ✓ $m_1 = m_2$ and $b_1 \neq b_2$ (same slopes, different y-intercepts)
- ✓ No solution is related to no point of intersection of the corresponding lines



Example 4. For each case, find how many solutions the system of linear equations has. Do not solve.

a) $\begin{cases} y = 2x + 3 \\ 4x - 2y + 6 = 0 \end{cases}$

b) $\begin{cases} x - y + 1 = 0 \\ 2x = 2y - 4 \end{cases}$

c) $\begin{cases} x = 2 - y \\ y = x + 3 \end{cases}$

Reading: Textbook Pages 29-32
Homework: Textbook Page 32-33 # 1, 2, 6, 8