

1.2 The Method of Substitution

A The Method of Substitution

The method of substitution requires 5 steps:

1. Use one equation (1) to write one variable (x) in terms of the other variable (y)
2. Substitute that variable (x) into the other equation (2)
3. Solve the other equation (2) for the other variable (y)
4. Use the relation gotten at step 1. to find the first variable (x)
5. Check if the solution is correct

Ex 1. Solve the following linear system by substitution.

$$\begin{cases} y+1=2x & \textcircled{1} \\ y-x=2 & \textcircled{2} \end{cases}$$

Step 1. $\textcircled{1} \Rightarrow y=2x-1$ $\textcircled{3}$

Step 2. $\textcircled{3} \rightarrow \textcircled{2}$

$$\textcircled{2} \Rightarrow (2x-1)-x=2$$

$$\text{Step 3. } 2x-1-x=2$$

$$2x-x=2+1$$

$$x=3 \quad \textcircled{4}$$

Step 4. $\textcircled{4} \rightarrow \textcircled{3}$

$$\textcircled{3} \Rightarrow y=2(3)-1$$

$$y=6-1$$

$$y=5$$

Step 5. (check)

$$x=3, y=5$$

$$\textcircled{1} \Rightarrow 5+1=2(3)$$

$$6=6 \text{ (true)}$$

$$\textcircled{2} \Rightarrow 5-3=2$$

$$2=2 \text{ (true)}$$

$$\therefore (3, 5)$$

$$\therefore x=3 \text{ and } y=5$$

B Tips

Look for an equation where the coefficient of x or y is 1 or -1.

Ex 2. Solve the following linear system by substitution.

$$\begin{cases} 2x+3y=4 & \textcircled{1} \\ 2y-x-5=0 & \textcircled{2} \end{cases}$$

isolate x from $\textcircled{2}$

$$\textcircled{2} \Rightarrow 2y-5=x$$

$$x=2y-5 \quad \textcircled{3}$$

$$\textcircled{3} \Rightarrow x=2(2)-5$$

$$x=4-5$$

$$x=-1$$

check

$$x=-1, y=2$$

$$\textcircled{1} \Rightarrow 2(-1)+3(2)=4$$

$$-2+6=4$$

$$4=4 \text{ (true)}$$

$$\textcircled{2} \Rightarrow 2(2)-(-1)-5=0$$

$$4+1-5=0$$

$$0=0 \text{ (true)}$$

$$\textcircled{1} \Rightarrow 2(2y-5)+3y=4$$

Solve this equation for y

$$4y-10+3y=4$$

$$4y+3y=4+10$$

$$7y=14$$

$$y=2$$

$$y=2$$

$$\therefore (-1, 2)$$

$$\therefore x=-1 \text{ and } y=2$$

C Expression substitution

If a variable has the same coefficient in both equations, substitute the whole expression (the variable and the coefficient).

Ex 3. Solve the following linear system by substitution.

$$\begin{cases} 3x - 2y + 3 = 0 & (1) \\ 5x = 2y - 1 & (2) \end{cases}$$

Handwritten solution for Ex 3:

(2) $\Rightarrow 5x + 1 = 2y$
 $2y = 5x + 1$ (3)

(3) \rightarrow (1)

(1) $\Rightarrow 3x - \underbrace{(5x + 1)}_{2y} + 3 = 0$
 $3x - 5x - 1 + 3 = 0$
 $-2x + 2 = 0$
 $\underline{-2x} + 2 = 0$
 $2 = 2x$

(3) $\Rightarrow 2y = 5(1) + 1$
 $2y = 6$
 $\frac{2y}{2} = \frac{6}{2}$
 $y = 3$

(4) \rightarrow (3)

(4) $\Rightarrow x = 1$

D Fractions (Challenge)

Sometimes, the substitution method leads to working with fractions.

Ex 4. Solve the following linear system by substitution.

$$\begin{cases} 2x + 3y = 2 \\ 8x + 9y = 7 \end{cases}$$

Ex 5. Solve the following linear system by substitution.

$$\textcircled{1} \quad \frac{x-1}{2} + \frac{y+2}{3} = 3$$

$$\textcircled{2} \quad 2(x + 1) - 3(y - 3) - 5 = 0$$

E Applications

Ex 6. Marie went to the convenience store to buy pencils and pens. A pencil cost \$1 and a pen costs \$2. Altogether Marie bought 8 items and paid 10\$ in total. How many pencils and pens did Marie buy?

Reading: Textbook Pages 20-26

Homework: Textbook Page 26-28 # 4b, 5d, 7, 12, 13, 14, 17, 20, 21