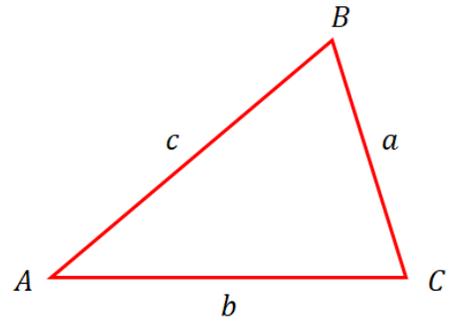# 8.2 8.3 The Cosine Law

A The Cosine Law

The Cosine Law states that in any triangle

$$c^2 = a^2 + b^2 - 2ab \cos C$$
$$a^2 = b^2 + c^2 - 2bc \cos A$$
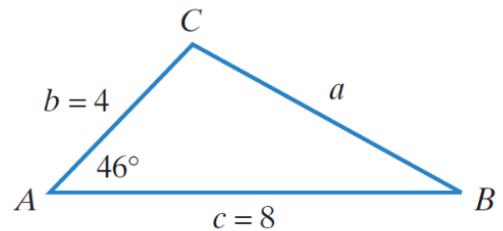$$b^2 = a^2 + c^2 - 2ac \cos B$$

Example 1. Prove the cosine law.

Example 2. Rewrite the cosine laws for a right triangle with $\angle C = 90°$.

B Using the Cosine Law to find a Side

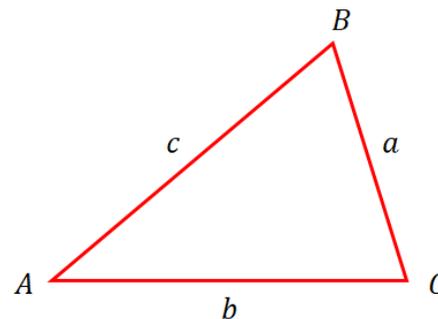Example 3. Find the unknown side $a = BC$.

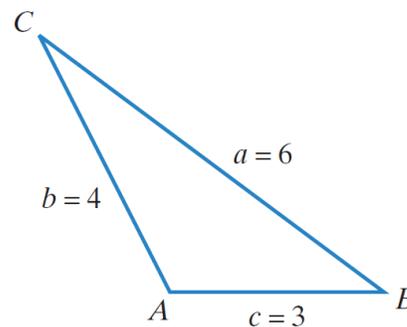**C Using the Cosine Law to find an Angle**

From the Cosine Law

$$c^2 = a^2 + b^2 - 2ab\cos C$$

By solving for the angle C we get $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$ or

$$\angle C = \cos^{-1}\dfrac{a^2 + b^2 - c^2}{2ab}$$

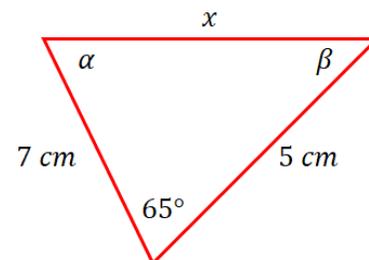Example 4. Use the cosine law to find the angles $\angle A$ and $\angle B$.

D Using the Cosine Law to solve a Triangle

Solving a triangle means finding all the side lengths and all the angle measures.
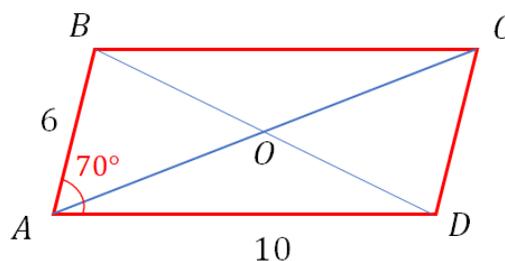
Example 5. Solve each triangle.

a)

b) $a = 6\ cm,\ b = 12\ cm,\ c = 8\ cm$

Example 6. For the parallelogram $ABCD$ on the figure below, find

a) the length of the diagonal BD

b) the length of the diagonal AC

c) the obtuse angle $\angle BOC$ between the diagonals

Notes: Textbook Pages 405-408 and 412-417
Homework: Textbook Pages 409-411 # 1a, 4a, 9, 11 and 418-419 #1a, 3a, 5a, 9, 10, 11

Applications

Example 1. Find the area of a triangle with the side lengths equal to $5m$, $7m$, and $10m$.

Example 2. The diagonals of a parallelogram are $16cm$ and $24cm$ respectively and the acute angle between the diagonals is $36°$. Find the sides of the parallelogram.

Example 3. Two ships leave a port at the same time. One sails in a direction $[N32°W]$ at 15 knots, and the other sails $[N78°E]$ at 18 knots. How far apart are the ships two hours after they have left the port? (A knot is one nautical mile per hour.)

Example 4. The hands of a clock are 4.25 in and 24 in respectively. What is the distance between the tips of the two hands at 8 o'clock?

Example 5. Two sides of a parallelogram are 65 cm and 85 cm and one of the diagonals is 45 cm. Find the angles of the parallelogram.

Example 6. A triangle has the side lengths $2m$, $3m$, and $4m$. Find the radius of the inscribed circle of the triangle.

Example 7. A triangle has the side lengths $2m$, $3m$, and $4m$. Find the radius of the circumscribed circle of the triangle.

Example 8. A triangle has two sides of 7 cm and 12 cm, respectively. If the area of the triangle is 27.6 cm², find the angle between the two sides.

Example 9. Solve ΔABC with vertices $A(2,1)$, $B(7,4)$, and $C(1,5)$.