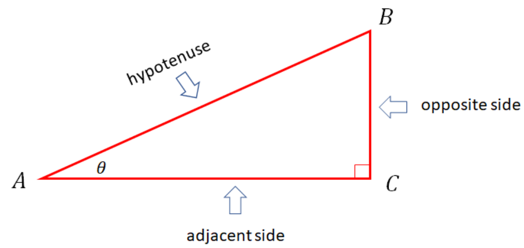
7.4 The Sine and Cosine Ratios

A Sine and Cosine Ratios

The sine and cosine ratios are defined by:

$$\sin\theta = \frac{opposite\ side}{hypotenuse} = \frac{BC}{AB}$$

$$\cos\theta = \frac{adjacent\ side}{hypotenuse} = \frac{AC}{AB}$$



Example 1. Use a scientific calculator to find the sine or the cosine of the indicated angles:

a) $\sin 0°$ b) $\cos 0°$ c) $\sin 30°$ d) $\cos 45°$ e) $\sin 90°$ f) $\cos 90°$

Note. Be sure your scientific calculator **Mode** is in **Degrees**.

B Inverse sine and cosine functions

Given the sine or cosine ratio k of an acute angle θ ($\theta < 90°$), use the inverse sine or inverse cosine functions to find the value of that angle:

$$\sin\theta = k \quad \Leftrightarrow \quad \theta = \sin^{-1}(k)$$

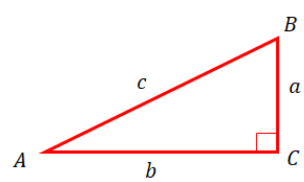
$$\cos\theta = k \quad \Leftrightarrow \quad \theta = \cos^{-1}(k)$$

Note. Press **SHIFT** and then **sin** or **cos** keys to get the inverse functions on your scientific calculator.

Example 2. For each case, find the angle θ.

a) $\sin\theta = 0.5$	b) $\sin\theta = 1$	c) $\cos\theta = 2$	d) $\cos\theta = 0.5$	e) $\sin\theta = 0.75$

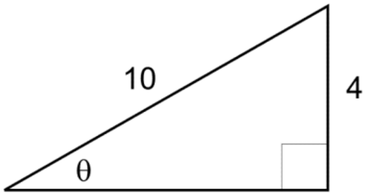
Example 3. Use the definition of the trigonometric ratios (sine, cosine and tangent) to prove the following relationships:

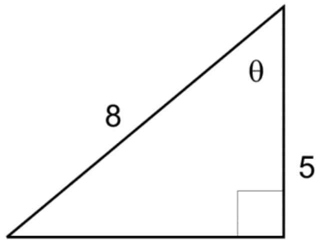


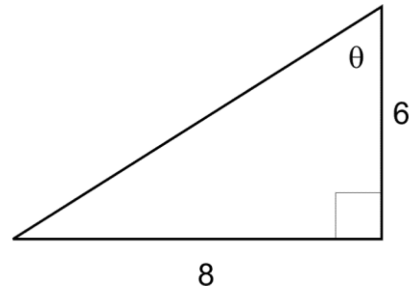
a) $\tan\theta = \frac{\sin\theta}{\cos\theta}$

b) $\sin^2\theta + \cos^2\theta = 1$

Example 4. For each case, find the value of the angle θ.





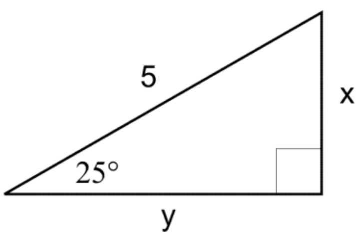


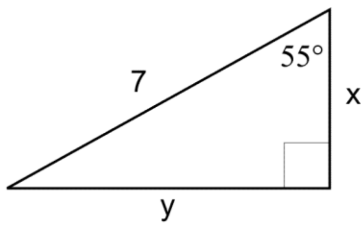
a)

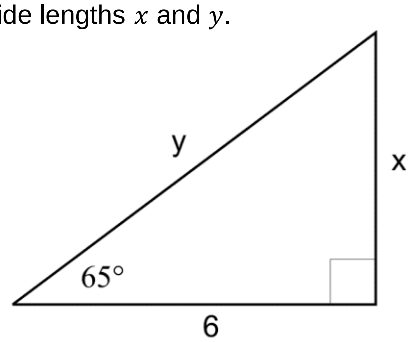
b)

c)

Example 5. For each case, use the sine or the cosine ratio to find the unknown side lengths x and y.







a)

b)

c)

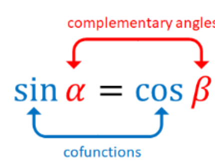
C Cofunction Identities

If $\alpha + \beta = 90°$ (α and β are called complementary angles), then the following cofunction identities are true:

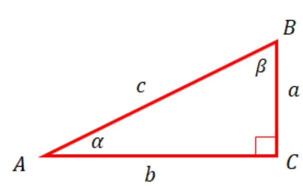
$$\sin\alpha = \cos\beta \qquad \cos\alpha = \sin\beta$$

or

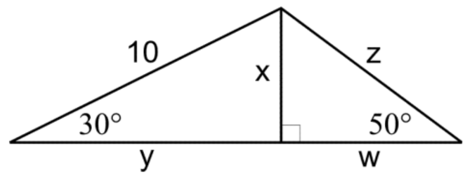
$$\sin(90° - \alpha) = \cos\alpha \qquad \cos(90° - \alpha) = \sin\alpha$$



Example 6. Prove the cofunction identities.



Example 7. Find the side lengths x, y, z, and w.

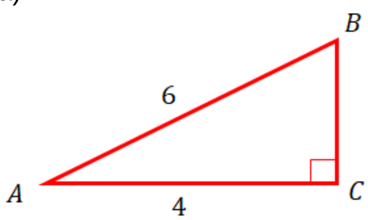


D Solving a Triangle

Solving a triangle means finding all sides and all angles

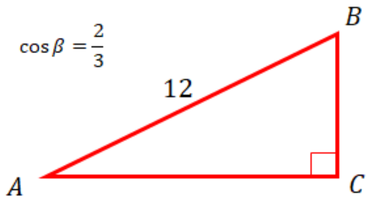
Example 8. Solve each triangle.

a)

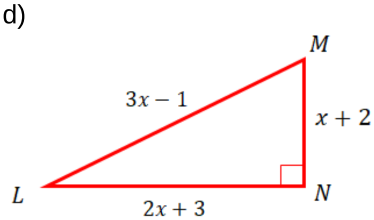


b) ΔPQR with $\angle P = 25°$, $\angle R = 90°$, $p = 10$

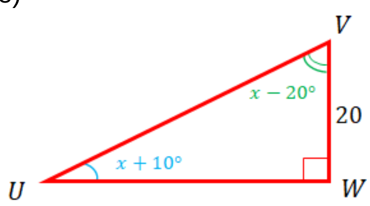
c)



d)



e)



Notes: Textbook Pages 366-371
Homework: Textbook Pages 372-377 # 1a, 2a, 6a, 7a, 8a, 9a, 10a, 11a, 12a, 13, 21, 24, 33