

6.4 Quadratic Formula

A Quadratic Equation

The equation

$$ax^2 + bx + c = 0 \quad a \neq 0$$

is called the *quadratic equation*.

B Quadratic Formula

The *roots (solutions)* of the quadratic equation may be found by using the *quadratic formula*:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 1. Prove the quadratic formula by following these steps.

- ✓ Move c on the right side
- ✓ Divide both sides by a
- ✓ Add to both sides $\left(\frac{b}{2a}\right)^2$ to form a perfect square on the left side
- ✓ Write the left side as a square and get the common denominator on the right side
- ✓ Square root both sides
- ✓ Isolate the variable x

Example 2. Solve the following (non-factorable) quadratic equations given in standard form by using the algorithm presented at example 1. For each case, find the exact and the approximate value rounded to the nearest hundredths.

a) $2x^2 + 5x - 3 = 0$

b) $x^2 - 4x + 2 = 0$

Example 3. Solve each quadratic equation by using the quadratic formula.

a) $x^2 - 9 = 0$

b) $4x^2 - 4x + 1 = 0$

c) $x^2 - x - 20 = 0$

d) $6x^2 - 13x + 6 = 0$

e) $x^2 - 2x - 4 = 0$

f) $x^2 + x + 1 = 0$

C Vieta's Formulas

The roots x_1 and x_2 of the quadratic equation $ax^2 + bx + c = 0$ satisfy Vieta's formulas:

$$x_1 + x_2 = -\frac{b}{a} \quad \text{and} \quad x_1x_2 = \frac{c}{a}$$

Example 4. Prove Vieta's formulas.

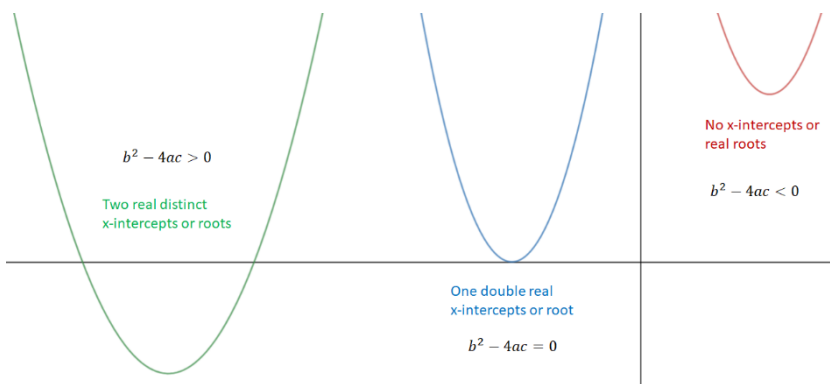
D Quadratic Relations and Equations

The *quadratic relation* $y = ax^2 + bx + c$ and the *quadratic equation* $ax^2 + bx + c = 0$ are close related to each other.

The *zeros* or the *x-intercepts* of the quadratic relation are identical to the *roots (solutions)* of the quadratic equation.

The quantity $D = b^2 - 4ac$ is called *discriminant*.

- ✓ If $D = b^2 - 4ac > 0$, then the quadratic equation has *two real distinct roots* (solutions). The corresponding quadratic function has *two distinct x-intercepts* (zeros).
- ✓ If $D = b^2 - 4ac = 0$, then the quadratic equation has *one double real root* (solution) (or *two coincident roots*). The corresponding quadratic function has *one double x-intercept* (zero) (or *two coincident x-intercepts or zeros*).
- ✓ If $D = b^2 - 4ac < 0$, then the quadratic equation has *no real roots* (solutions). In this case the quadratic equation has *two distinct complex roots*. The corresponding quadratic function has *no x-intercept*.



Example 5. For each case, find how many real distinct roots each quadratic equation has.

a) $x^2 + x + 1 = 0$

b) $x^2 + x - 2 = 0$

c) $x^2 + 2x + 1 = 0$

Example 6. Find the parameter k , such that the following quadratic equation has one double real root.

$$2x^2 + kx + k + 6 = 0$$

E Technology

Example 7. Use technology to solve the following quadratic equations. Find the exact values of the roots and the approximate values to the nearest hundredths.

a) $0.1x^2 - x + 0.25 = 0$

(use a scientific calculator)

b) $\pi x^2 - 5x + \frac{1}{3} = 0$

(use [Wolfram Alpha](#))

Notes: Textbook Pages 292-300

Homework: Textbook Pages 300-303 # 1ab, 2ab, 3ab, 6, 9ab, 11, 16