6.1 Maxima and Minima (by completing the square)

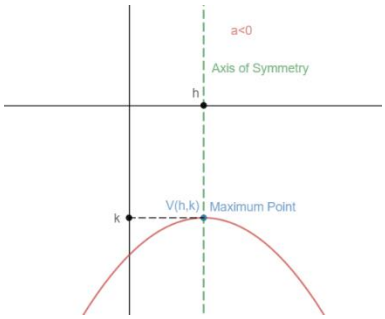
A Terminology

- ✓ The *maximum* element of a set is the *largest* element in that set
- ✓ The *minimum* element of a set is the *smallest* element in that set
- ✓ An *extremum* is either maximum or a minimum
- ✓ *Maxima* is the plural of maximum
- ✓ *Minima* is the plural of minimum
- ✓ *Extrema* is the plural of extremum

B Maximum for a quadratic relation

If $a < 0$ for a quadratic relation, then

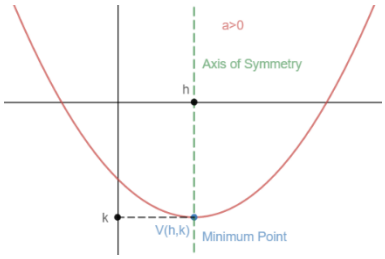
- ✓ The vertex point $V(h,k)$ is a *maximum point*
- ✓ The y-coordinate of the vertex point k is the *maximum value*
- ✓ The *range* of the quadratic relation is $y \leq k$



C Minimum for a quadratic relation

If $a > 0$ for a quadratic relation, then

- ✓ The vertex point $V(h,k)$ is a *minimum point*
- ✓ The y-coordinate of the vertex point k is the *minimum value*
- ✓ The *range* of the quadratic relation is $y \geq k$



D Vertex Form

If the quadratic relation is given in the vertex form

- ✓ The vertex point $V(h,k)$ is an extremum point (either a maximum or a minim point)

Example 1. Complete the following table.

Quadratic Relation	$y = 7 - 2(x+3)^2$	$y = 0.1(x-5)^2$
Vertex Point		
Minimum/Maximum Point		
Minimum/Maximum Value for the variable y		
Range (the set of all y-values)		

E Completing the Square (Method #1)

To convert the *standard form* into the *vertex form* by *completing the square*:

- ✓ Factor out the coefficient a from the first two terms $y = a\left(x^2 + \frac{b}{a}x\right) + c$
- ✓ Add and subtract the square of the half of the coefficient of x $y = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$
- ✓ Write the first three terms as a square $y = a\left(\left(x + \frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right) + c$
- ✓ Remove the bracket $y = a\left(x + \frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c$

Example 2. Find the vertex form by completing the square (non-factorable trinomials).

Quadratic Relation	$y = x^2 + 2x + 3$	$y = -2x^2 + 4x + 1$	$y = 2x^2 + 3x - 1$ (challenge)
Factor out the coefficient a from the first two terms			
Add and subtract the square of the half of the coefficient of x			
Write the first three terms as a square			
Remove the bracket			
Vertex Form			
Minimum/Maximum Value for the variable y			

F Convert Standard Form into Vertex Form (Method #2)

To convert the *standard form* into the *vertex form*:

- ✓ Find h by using $h = \frac{-b}{2a}$
- ✓ Find k by substitution $x = h$ into the standard form
- ✓ Write the vertex form $y = a(x - h)^2 + k$

Example 3. Prove that the x-coordinate of the vertex point is given by $h = \frac{-b}{2a}$

Example 3. Complete the following table. Note that all these trinomials are non-factorable.

Quadratic Relation	$y = x^2 + 4x + 1$	$y = -2x^2 + 3x - 4$ (challenge)
Find h		
Find k		
Vertex Form		
Minimum/Maximum Value for the variable y		

Example 4. The rate of fuel consumption of an aircraft, f, in litres per hour, is given approximately by: $f = 0.01v^2 - 5v + 1000$, where v is the speed in kilometres per hour. At what speed is the rate of fuel consumption a minimum?

G Technology

Example 5. Use Wolfram Alpha to find the vertex form for $y = -15x^2 + 27x - 11$.

Hint. Go to Wolfram Alpha and enter:
`vertex form -15x^2+27x-11` or `complete the square -15x^2+27x-11`

Notes: Textbook Pages 264-269
Homework: Textbook Pages 270-273 # 3ag, 4a, 5, 7cd, 8ad, 9e, 10ae, 11a, 17a, 22 (use Desmos or Wolfram Alpha)