

5.3 Common Factors

A Factoring

Factoring is the *opposite process of expanding* and is based on the *distributive property* (see the diagram below).

$$a \cdot b + a \cdot c = a \cdot (b + c)$$

Example 1. Factor.

a) $x^2 + 2x$

b) $2x^2 - 4y$

c) $x^2y - 3y^2$

B Factor Tree

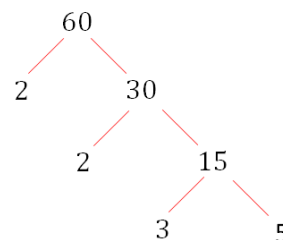
✓ Used to identify all prime factors of an integer (see the diagram on the right)

On this example:

$$60 = 2 \cdot 2 \cdot 3 \cdot 5 = 2^2 \cdot 3 \cdot 5$$

Any factor of 60 is a product of one or more of its prime factors.

The list of all possible positive factors of 60 is 1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60



C The Greatest Common Factor

- ✓ GCF stand for Greatest Common Factor
- ✓ Greatest Common Factor (GCF) is used to factor out all common factors of the terms of a polynomial
- ✓ GCF is equal to the product of all *common bases* raised to the *least exponent*

Example 2. For each case, find the greatest common factor.

a) $4ax^2, -2axy$

b) $15y^3, 10axy^2, 25a^2y$

c) $6x, 3xy^2, -8y$

D Factoring Polynomials

- ✓ Get the GCF of all terms and write in from of a bracket
- ✓ Include in the bracket all terms of the polynomial divided by the GCF

Example 3. Factor fully the following polynomials.

a) $2x + 4x^2$

b) $3xy - 12x^2y^3$

c) $-6ab^2 + 15bc^2$

d) $-2x + 6x^2 - 8x^3$

e) $12ab^2c - 15a^2b^3c + 18ab^2c$

f) $abc + bcx - 5ax$

Example 4. Factor the following expressions containing fractions.

a) $\frac{x}{2} - \frac{5x^2}{2}$

b) $\frac{2}{3}ab^2 + \frac{5}{3}a^2b$

c) $\frac{2x^2y^3}{3} - \frac{8xy^4}{9} + \frac{4xy^2}{15}$

E Factorizing Binomials as GCF

✓ Binomials or other expressions could be common factors.

Example 5. Factor the following expressions using binomials as common factors.

a) $a(x - 1) + b(x - 1)$

b) $ab^2(x - y) + a^2b(y - x)$

c) $a(bc - 2b^2) + 2b^2 - bc$

F Factoring by Grouping

- ✓ May be applied for polynomials with four terms
- ✓ Make two groups of two terms each
- ✓ Factor each group
- ✓ Factor the common binomial

Example 6. Use grouping to factor fully.

a) $ac + ad + bc + bd$

b) $x^2 + x - xy - y$

c) $2x^2 + 6xy + x + 3y$

d) $5x^2y - 10xy^2 + 4y^2 - 2xy$

e) $x^2y - xy^2z + xz - yz^2$

f) $6a^2b - 2ac - 9ab^2c + 3bc^2$

Notes: Textbook Pages 228-233

Homework: Textbook Pages 234-235 1-6 (as many as you can), 9, 11, 13, 14, 15