5.2 Special Products

A Perfect Square Identities

Example 1. Use the distributive property to develop a formula for:

a) $(a+b)^2$

b) $(a-b)^2$

B Algebraic Identities

An *algebraic identity* is a *true statement* regardless the values assigned to each variable.

The following algebraic identities are called *perfect square identities*.

$$(a+b)^2 = a^2 + 2ab + b^2 \qquad (a-b)^2 = a^2 - 2ab + b^2$$

In any context, the left side may be replaced by the right side, and the right side may be replaced by the left side.

Example 2. Use the algebraic identities to quickly expand.

a) $(1+2x)^2$	b) $(2x+3y)^2$	c) $\left(2xy+\frac{x^2}{2}\right)^2$
d) $(2a-b)^2$	e) $(-2x+5xy)^2$	f) $\left(\frac{x}{2}-\frac{xy}{3}\right)^2$

C Perfect Square Trinomials

- ✓ A trinomial is a polynomial with *three terms*
- ✓ A *perfect square trinomial* is an expression like $a^2+2ab+b^2$ or $a^2-2ab+b^2$ that is the result of squaring a binomial
- ✓ A perfect square trinomial can be f*actored* or condensed (reduced) by using the perfect square identities:

$$a^2 + 2ab + b^2 = (a+b)^2 \qquad a^2 - 2ab + b^2 = (a-b)^2$$

Example 3. Use the algebraic identities to factor the following perfect square trinomials.

a) $4x^2+12xy+9y^2$	b) $1-4x^2+4x^4$	c) $4a^2+2ab+\frac{b^2}{4}$
d) $4x^2y^2-16xy^2z^2+16y^2z^4$	e) $25a^4+40a^2b^3+16b^6$	f) $\frac{4x^2}{9}-\frac{4xy^2}{5}+\frac{9y^4}{25}$

Example 4. Find the missing term to make each expression a perfect square trinomial, then factor.

a) $\ldots\ldots\ldots\ldots + 2xy + y^2$

b) $4a^2 \ldots\ldots\ldots\ldots + 9b^2$

c) $9a^2b^4 - 30a^2b^2x + \ldots\ldots\ldots\ldots$

d) $\ldots\ldots\ldots\ldots + 4abx^2 + \ldots\ldots$

E Difference of Squares

Example 5. Expand and develop a formula for:

$(a+b)(a-b)$

The following identity is called the difference of squares identity:

$$(a+b)(a-b) = a^2 - b^2$$

Example 6. Use the difference of squares identity to quickly multiply the following binomials.

a) $(1+2x)(1-2x)$	b) $(2x+3y)(2x+3y)$	c) $\left(2x+\frac{y}{2}\right)\left(2x-\frac{y}{2}\right)$
d) $(ab-xyz)(ab+xyz)$	e) $(3ax^2-5by)(3ax^2+5by)$	f) $\left(\frac{2x}{3}+\frac{3y}{4}\right)\left(\frac{2x}{3}-\frac{3y}{4}\right)$

Example 7. Use the difference of squares identity to quickly factor (reduce) the following expressions.

a) $25x^2-16y^2$	b) $4a^4-9b^2$	c) $\frac{y^2}{4}-4x^2$
d) x^4-y^4	e) $4x^2y^2-9z^2$	f) $\frac{4x^2}{9}-\frac{9y^2}{25}$

Example 8. Prove the following algebraic identities.

a) $(a+b)^3 = a^3+3a^2b+3ab^2+b^3$
b) $(a-b)^3 = a^3-3a^2b+3ab^2-b^3$
c) $a^3+b^3=(a+b)(a^2-ab+b^2)$
d) $a^3-b^3=(a-b)(a^2+ab+b^2)$
e) $a^4-b^4=(a+b)(a-b)(a^2+b^2)$
f) $(a+b+c)^2=a^2+b^2+c^2+2ab+2bc+2ac$
g) $(a+b+c)(a+b-c)(a-b+c)(-a+b+c)=2a^2b^2+2a^2c^2+2b^2c^2-a^4-b^4-c^4$

Notes: Textbook Pages 220-224
Homework: Textbook Pages 225-227 # 2-6 (as many as you can), 8, 11, 12, 13 (optional), 19