4.6 Negative and Zero Exponents

A Multiplication Notation

Multiplication notation is shortcut for *repetitive addition*.

Examples 1. Simplify.

a) $10 + 10 + 10 =$

b) $x + x + x + x =$

c) ☺+☺+☺=

B Exponential Notation

Exponential notation is a shortcut for *repetitive multiplication*.

Example 2. Simplify.

a) $4 \times 4 \times 4 \times 4 =$

b) $a \cdot a \cdot a \cdot a \cdot a =$

c) $\Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta \times \Delta =$

C Understanding the Exponential Notation

$$b^e$$

- ✓ b is called *base*
- ✓ e is called *exponent*
- ✓ b^e is called *power*

D Multiplying powers with the same base

Example 3. Write as a single power and develop a rule.

$a^4 a^3 =$

Conclusion: $$a^m \times a^n = a^{m+n}$$

To multiply powers with the same base, keep the base the same and add the exponents.

Example 4. Write as a single power (simplify). Do not evaluate.

a) $10^5 \times 10^3 =$

b) $x^2 \times x^6 =$

c) $\omega^2 \times \omega^3 \times \omega^4 =$

d) $a^3 \times a^4 \times a \times a^5 =$

Note. $a^1 = a$

E Zero Exponent

Example 5. Prove that $a^0 = 1$

Example 6. What is 0^0?

F Dividing powers with the same base

Example 7. Write as a single power and develop a rule.

$a^4 \div a^3 = \frac{a^4}{a^3} =$

Note. $a \div b = \frac{a}{b}$

Conclusion: $a^m \div a^n = \frac{a^m}{a^n} = a^{m-n}$

To divide powers with the same base, keep the base the same and subtract the exponents.

Example 8. Write as a single power (simplify). Do not evaluate.

a) $10^5 \div 10^3 =$

b) $\heartsuit^9 \div \heartsuit^4 =$

c) $3^{10} \div 3^2 =$

d) $x^{10} \div x^6 =$

G Power of a Power

Example 9. Write as a single power and develop a rule.

$(a^2)^3 =$

Conclusion: $(a^m)^n = a^{m \times n}$

To simplify a power of a power, keep the base the same and multiply the exponents.

Example 10. Write as a single power (simplify). Do not evaluate.

$a)\ (10^3)^4 =$

b) $(7^2)^6 =$

c) $(\sun^5)^2 =$

d) $(x^3)^2 =$

e) $(x^2)^3 =$

Example 11. Write as a single power (simplify).

a) $5^3 \times \frac{5^6}{5^2} =$

b) $3^5 \times (3^2)^4 =$

c) $(7^4)^4 \div (7^3)^3 =$

H Negative Exponent

Example 12. Prove that $a^{-n} = \frac{1}{a^n}$

Example 13. Simplify.

a) $7^5 \times 7^{-3} =$

b) $x^{-4} \times x^2 =$

c) $2^2 \times 2^{-5} \times 2^0 =$

d) $a^3 \times a^{-5} \times a \times a^{-2} \times a^0 =$

I Multiplying powers with the same exponent

Example 14. Investigate $(ab)^3$ and develop a formula.

Conclusion: $(ab)^n = a^m b^m$

Example 14. Expand and simplify.

a) $(2x)^3 =$

b) $(ax^2)^4 =$

c) $(3x^{-3})^{-2} =$

d) $(-2x^2y^{-3})^{-4} =$

J Dividing powers with the same exponent

Example 15. Investigate $\left(\frac{a}{b}\right)^3$ and develop a formula.

Conclusion: $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$

K Exponent Rules (Review)

$(ab)^n = a^n b^n$ $a^n a^m = a^{n+m}$ $(a^n)^m = a^{nm}$ $a^{-n} = \frac{1}{a^n}$ $\frac{1}{a^{-n}} = a^n$

$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ $\frac{a^n}{a^m} = a^{n-m}$ $a^0 = 1$ $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n$ $0^0 = undefined$

Example 16. Use exponent rules to simplify.

a) $\left(\frac{-2}{3}\right)^{-2}$

b) $\left(\frac{3}{4}\right)^{-1}\left(\frac{4}{3}\right)^{-2}$

c) $\left[\left(\frac{a}{b}\right)^2\right]^{-1}$

d) $\frac{a^2 \times a^{-5}}{(a^{-2})^3}$

e) $\left(\frac{x^{-3}}{x^2}\right)^{-2}\left(\frac{x^2}{x^{-1}}\right)^3$

f) $\left(\frac{(a^2)^{-3}}{(a^{-1})^{-3}}\right)^{-2}((-a)^{-2})^3$

L Finite Differences for Exponential Relations

Example 17. Investigate finite differences for the exponential relation $y = 2^x - 1$.

x	$y = 2^x - 1$		
-2			
-1			
0			
1			
2			

Conclusion.

M Exponential Equations

Example 18. Solve for x.

a) $2^x = 1024$

b) $10^x = 0.00001$

c) $4^x = 0.125$

O Power versus Exponential Relations

The relation $y = x^n$ is a *power relation*. The relation $y = b^x$ is an *exponential relation*.

Example 19. How would you differentiate a power relation from an exponential relation?

Notes: Textbook Pages 194-198
Homework: Textbook Pages 199-200 #2, 3, 4, 8, 19