4.3 Investigate Transformations of Quadratics

A Compare the graphs of $y = x^2$ and $y = x^2 + k$

The graph of $y = x^2 + k$ is *translated vertically* $|k|$ units

Desmos

- upward if $k > 0$
- downward if $k < 0$

Example 1. Graph on the same grid the following quadratic relations. Then determine the vertex, minimum or maximum value, domain and the range.

a) $y = x^2$

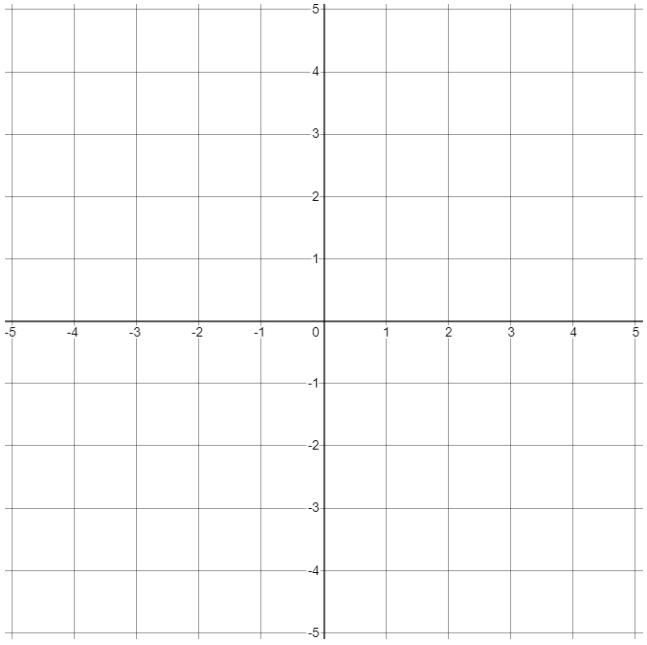
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:

b) $y = x^2 - 3$

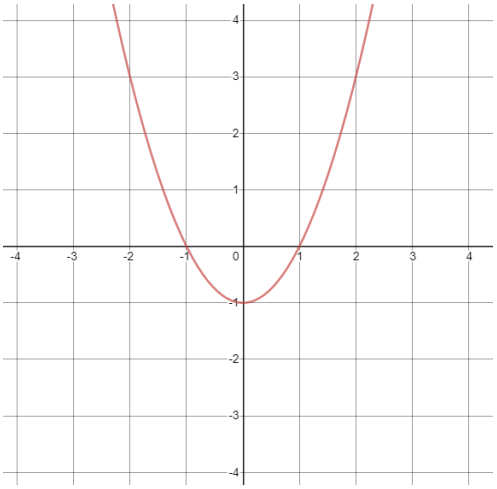
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:

c) $y = x^2 + 2$

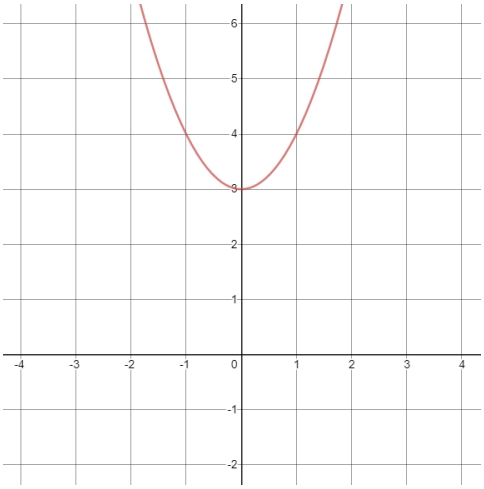
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:



Example 2. For each case, write the equation of the quadratic relation in the form $y = x^2 + k$.



a)



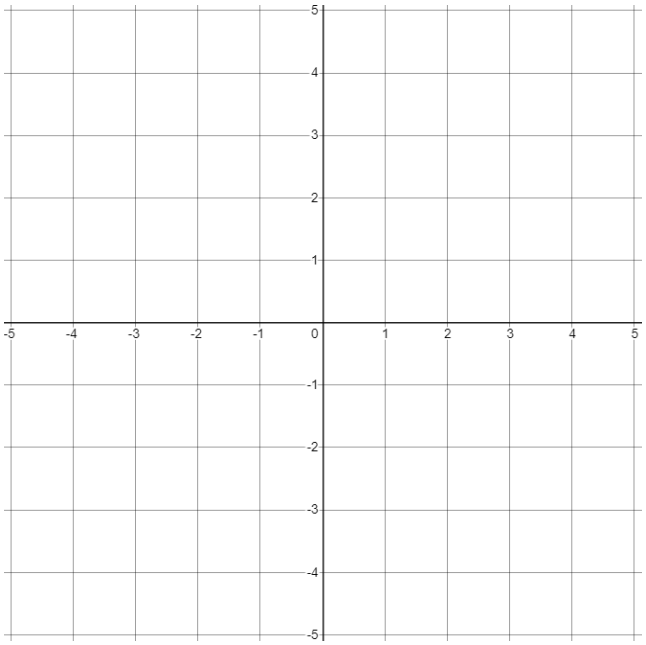
b)

2. Compare the graphs of $y = x^2$ and $y = (x-h)^2$ Desmos

The graph of $y = (x-h)^2$ is *translated horizontally* h units

- to the right if $h > 0$
- to the left if $h < 0$

Example 3. Graph on the same grid the following quadratic relations. Then determine the vertex, minimum or maximum value, domain and the range.

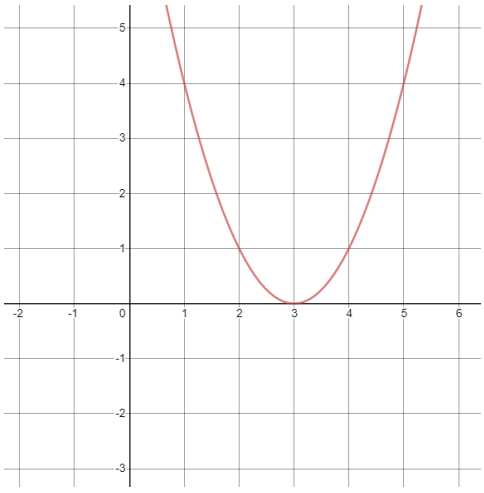


a) $y = x^2$
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:

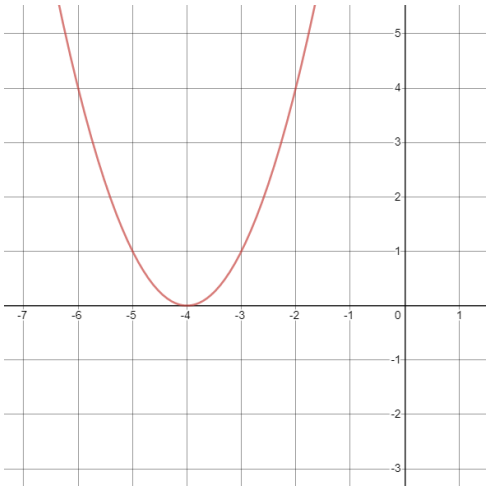
b) $\boldsymbol{y = (x-1)^2}$
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:

c) $y = (x+2)^2$
Vertex:
Minimum Value:
Maximum Value:
Domain:
Range:

Example 4. For each case, write the equation of the quadratic relation in the form $y = (x-h)^2$.



a)



b)

2. Compare the graphs of $y = x^2$ and $y = ax^2$ Desmos

The graph of $y = ax^2$ is

- *stretched (expanded) vertically* if $|a| > 1$
- *compressed vertically* if $|a| < 1$

by a scale factor of $|a|$ and

- *reflected in the x-axis* if $a < 0$

Note. The *scale factor* is defined by = $\frac{size\ of\ the\ copy\ (image)}{size\ of\ the\ original}$

Original

Image (Copy)
Scale Factor is 0.5

Image (Copy)
Scale Factor is 1.5

Example 3. Graph on the same grid the following quadratic relations. Then determine the vertex, minimum or maximum value, domain and the range.

a) $y = -x^2$
Vertex:

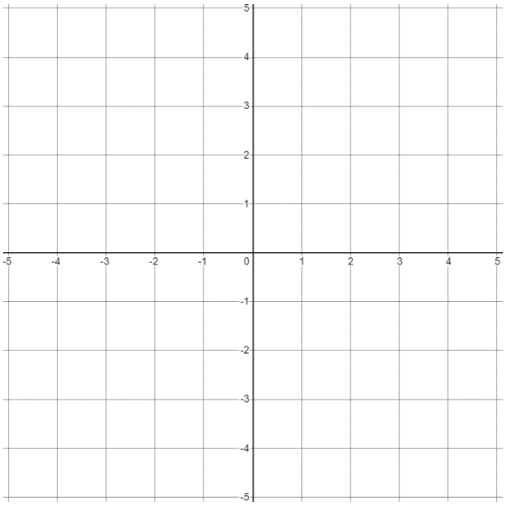
Range:

b) $y = 2x^2$
Vertex:

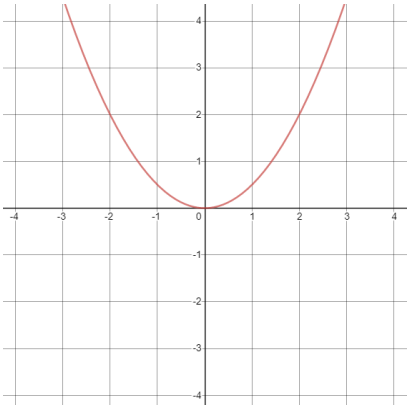
Range:

c) $y = -0.5x^2$
Vertex:

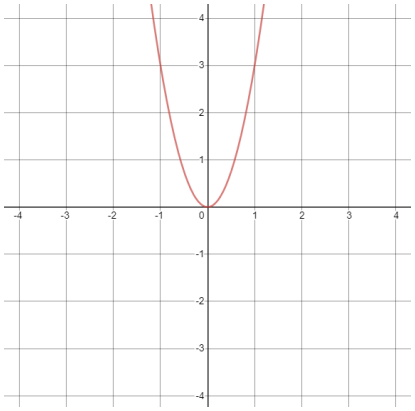
Range:



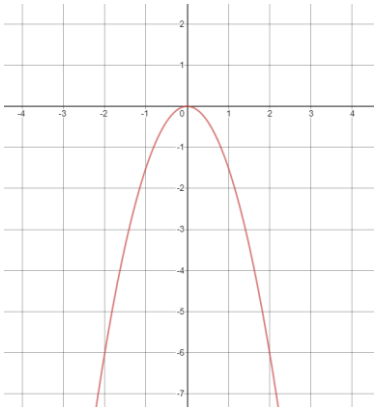
Example 4. For each case, write the equation of the quadratic relation in the form $y = ax^2$.



a)



b)



c)

Example 5. For each case, describe the transformations from the graph of $y = x^2$.

a) $y = x^2 + 5$

b) $y = (x - 5)^2$

c) $y = 3x^2$

c) $y = -(x + 1)^2$

d) $y = -\frac{1}{2}x^2$

e) $y = 2 - 3x^2$

Example 6. For each case, write an equation for the quadratic relation that results from $y = x^2$ by performing the following transformations:

a) translation upward by 3 units

b) translation to the left by 2 units

c) reflection in the x-axis

d) vertical expansion by a factor of 5

e) reflection in the x-axis and vertical compression be a factor of 1/2

Example 7. For each case, write an equation for the quadratic relation that results from $y = x^2$ by performing all transformations one by one.

a)

- reflection in the x-axis
- vertical stretch by a factor of 2

b)

- translation to the left by 3 units
- vertical compression by a factor of ½
- reflection in the x-axis

Notes: Textbook Pages 174-177
Homework: Textbook Pages 178 #4, 6, 8, 12, 13