

A Domain and Range

For each function, find the domain and the range.

a) $f(x) = 2x^4 + x^2 - 3$

b) $f(x) = -2x^3 + x^2 - x + 4$

c) $f(x) = \frac{3-4x}{2x+1}$

d) $f(x) = \frac{1}{1-x^2}$

e) $f(x) = \frac{1}{(x-2)^2}$

f) $f(x) = \log \frac{x-1}{x+3}$

g) $f(x) = -3\sin(2x-\pi) + 7$

h) $f(x) = 2 \tan x + 1$

i) $f(x) = -2^{x-2} + 4$

Answers:

a) $D = (-\infty, \infty); R = [-3, \infty)$

b) $D = (-\infty, \infty); R = (-\infty, \infty)$

c) $x \neq -1/2; y \neq -2$

d) $x \neq \pm 1; R = (-\infty, 0) \cup [1, \infty)$

e) $x \neq 2; y > 0$

f) $D = (-\infty, -3) \cup (1, \infty); y \neq 0$

g) $D = (-\infty, \infty); R = [4, 10]$

h) $x \neq \pi/2 + k\pi; R = (-\infty, \infty)$

i) $D = (-\infty, \infty); R = (-\infty, 4)$

B Equations

Solve each of the following equations for x .

a) $x^3 = 7x + 6$

b) $x^4 + 2 = 3x^2$

c) $\frac{x+1}{x-1} = \frac{x-2}{2x+3}$

d) $\sqrt{3} = 2 \sin(x - \pi/2); 0 \leq x \leq 2\pi$

e) $\tan^2 x + \tan x = 0; 0 \leq x \leq 2\pi$

f) $2^x = 5^{x+1}$

g) $\log_2(x-1) = \log_2 \frac{6}{x}$

h) $\cos(3x/2) + 1 = 0; 0 \leq x \leq 2\pi$

i) $2^{1-\sin x} = 1; 0 \leq x \leq 2\pi$

Answers:

a) $x = -2, -1, 3$

b) $x = \pm 1, \pm \sqrt{2}$

c) $x = -4 \pm \sqrt{15}$

d) $x = 5\pi/6, 7\pi/6$

e) $x = 0, \pi, 3\pi/4, 7\pi/4, 2\pi$

f) $x \approx -1.756$

g) $x = 3$

h) $x = 2\pi/3, 2\pi$

i) $x = \pi/2$

C Inequalities

For each of the following inequalities, find the solution set.

a) $(x^2 - 4)(x+1) > 0$

b) $x^3(x+2)^2 < 0$

c) $x^3 \geq 3x - 2$

d) $\frac{x}{x-3} < 0$

e) $\frac{1}{x} \geq \frac{1}{x-1}$

f) $\log_3(x-1) \geq 0$

Answers:

a) $(-2, -1) \cup (2, \infty)$

b) $x < 0$

c) $x \geq -2$

d) $0 < x < 3$

e) $0 < x < 1$

f) $x \geq 2$

D x-and y-intercepts

For each case, find the x- and the y-intercept(s).

a) $f(x) = x^3 - x^2$

b) $f(x) = (x^2 - 4)^2$

c) $f(x) = \frac{2x-4}{2-3x}$

d) $f(x) = \sec x$

e) $f(x) = 3^{x+3} - 9$

f) $f(x) = 2 - \log_2(x/3)$

Answers:

a) $x_{\text{int}} = 0, 1; y_{\text{int}} = 0$

b) $x_{\text{int}} = \pm 2; y_{\text{int}} = 16$

c) $x_{\text{int}} = 2; y_{\text{int}} = -2$

d) $x_{\text{int}} = \text{undefined}; y_{\text{int}} = 1$

e) $x_{\text{int}} = -1; y_{\text{int}} = 18$

f) $x_{\text{int}} = 12; y_{\text{int}} = \text{undefined}$

E Identities

Prove each of the following identities.

a) $\frac{x^3 - 8}{x - 2} = x^2 + 2x + 4$

b) $\tan \frac{x}{2} = \frac{1 - \cos x}{\sin x}$

c) $\sec^2 x = \frac{2}{1 + \cos 2x}$

d) $\cos 4x = 8 \cos^4 x - 8 \cos^2 x + 1$

e) $\tan^2 x = \frac{1 - \cos 2x}{1 + \cos 2x}$

f) $2x + \log_2 0.25^x = 0$

g) $\log_{1/b} x + \log_b x = 0$

h) $2^x = 3^{x \log_3 2}$

i) $2x + \log_2 0.25^x = 0$

F Asymptotes

For each case, find the horizontal and the vertical asymptote (if possible).

a) $f(x) = 2x^3 - x + 1$

b) $f(x) = \frac{1}{x^2 - x - 2}$

c) $f(x) = \frac{3-x}{2x+6}$

d) $f(x) = 2 - 3^{x-1}$

e) $f(x) = 3 + 4 \log(x+2)$

f) $f(x) = \log \frac{x}{x+1}$

Answers:

a) $HA : \text{undef}; VA : \text{undef}$

b) $HA : y = 0; VA : x = 2, -1$

c) $HA : y = -1/2; VA : x = -3$

d) $HA : y = 2; VA : \text{undef}$

e) $HA : \text{undef}; VA : x = -2$

f) $HA : y = 0; VA : x = -1, 0$

G Graphing

For each case, sketch the graph of the given function.

a) $f(x) = x(x-1)^2(x+2)$

b) $f(x) = x(x^2 - 4)$

c) $f(x) = \frac{-2}{x^2 + 2x + 1}$

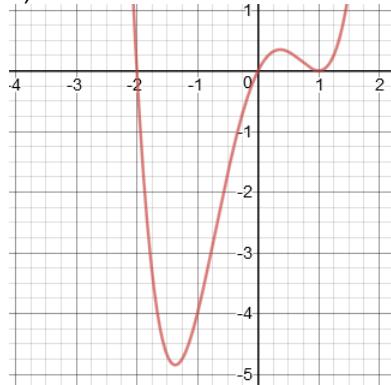
d) $f(x) = -\cos(x/2) + 3$

e) $f(x) = 6 \sin x \cos x$

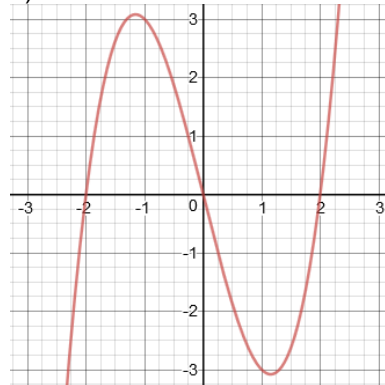
f) $f(x) = -\log_2(x+3)$

Answers:

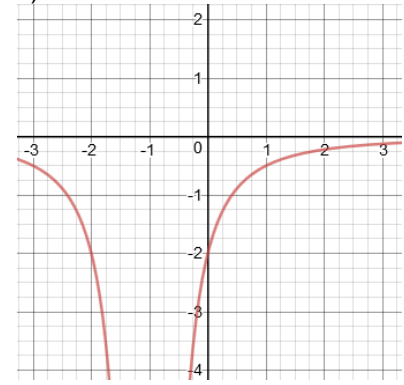
a)



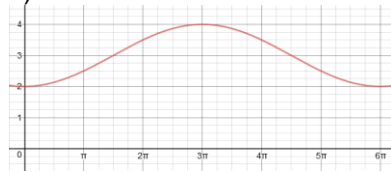
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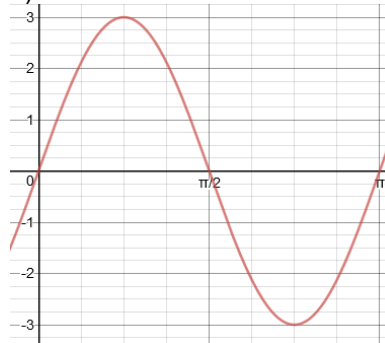
c)



d)



e)



f)



H Real Life Applications

1. A species of bacteria triple each twenty minutes. Find the time (in minutes) after which the bacteria population is one thousand times the initial number.
2. A sample of plutonium-238 has a half-life of 88 years. Find the time (in years) after which the mass of the radioactive source will be 1% of the initial value.
3. How many more intense is the noise made by a vacuum cleaner (80 dB) in comparison to a normal conversation (60 dB)?
4. Which has a greater concentration of hydrogen ions, orange juice (pH=4) or tomato juice (pH=3) and by how much?
5. The London Eye (see figure on the right) has the maximum point at 135 m and a diameter of 120 m and rotates at 26 cm/s. If a passenger is initially at the lowest point, find the height (in meters) of the passenger as a function of time (in seconds).
6. On September 26, 2001, an earthquake in North Bay measured 5.2 on the Richter scale. What is the magnitude of an earthquake 15 times as intense as North Bay's earthquake?



Answers:

1. 125.75 minutes
2. 584.66 years
3. 100 times
4. 10 times more in tomato juice
5. $f(t) = 75 - 60 \cos(13t / 3000)$
6. 6.38