

MHF4U

Combining Functions

1. Complete the following table

x	$f(x)$	$g(x)$	$(f + g)(x)$	$(f - g)(x)$	$(fg)(x)$	$\left(\frac{f}{g}\right)(x)$	$(f \circ g)(x)$	$(g \circ f)(x)$	$(f \circ f)(x)$	$(g \circ g)(x)$
-1	0	-1								
0	1	0								
1	3	-2								
2	0	3								

2. Given

$$f(x) = x + 1 \text{ and } g(x) = x^2$$

Find

a) $(ff^{-1})(1)$

b) $(f^{-1} \circ g \circ f)(1)$

c) $\left(\left(\frac{f}{g}\right) \circ \left(\frac{g}{f}\right)\right)(1)$

d) $(f \circ (fg) \circ g)(1)$

e) $((f + g) \circ (f - g))(1)$

3. Find two functions f and g such that $h(x) = (f \circ g)(x)$ where $h(x) = \frac{x-1}{x^2-x}$.

Combining Functions

1. Complete the following table

 $f(g(x))$ $g(f(x))$ $f(f(x))$ $g(g(x))$

x	$f(x)$	$g(x)$	$(f+g)(x)$	$(f-g)(x)$	$(fg)(x)$	$(\frac{f}{g})(x)$	$(f \circ g)(x)$	$(g \circ f)(x)$	$(f \circ f)(x)$	$(g \circ g)(x)$
-1	0	-1	-1	1	0	0	0	0	1	-1
0	1	0	1	1	0	DNE	1	-2	3	0
1	3	-2	1	5	-6	-3/2	DNE	DNE	DNE	DNE
2	0	3	3	-3	0	0	DNE	0	1	DNE

2. Given

$$f(x) = x + 1 \text{ and } g(x) = x^2 \longrightarrow f(x) = x + 1 \Rightarrow f^{-1}(x) = x - 1$$

Find

$$a) (ff^{-1})(1) = f(1) \cdot f^{-1}(1) = (2)(0) = 0$$

$$b) (f^{-1} \circ g \circ f)(1) = f^{-1}(g(f(1))) = f^{-1}(g(2)) = f^{-1}(4) = 3$$

$$c) \left(\frac{f}{g} \circ \frac{g}{f}\right)(1) = \frac{f}{g}\left(\frac{g}{f}(1)\right) = \frac{f}{g}\left(\frac{g(1)}{f(1)}\right) = \frac{f}{g}\left(\frac{1}{2}\right) = \frac{f(1/2)}{g(1/2)} = \frac{3/2}{1/4} = 6$$

$$d) (f \circ (fg) \circ g)(1) = f((fg)(g(1))) = f((fg)(1)) = f(f(1)g(1)) = f(2 \cdot 1) = f(2) = 3$$

$$e) ((f+g) \circ (f-g))(1) = (f+g)((f-g)(1)) = (f+g)(f(1)-g(1)) = (f+g)(2-1) = (f+g)(1) = f(1)+g(1) = 2+1 = 3$$

3. Find two functions f and g such that $h(x) = (f \circ g)(x)$ where $h(x) = \frac{x-1}{x^2-x}$.

Many solutions are possible

$$\left\{ \begin{aligned} h(x) &= \frac{x-1}{x(x-1)} = \frac{1}{x} \end{aligned} \right. ;$$

$$\left. \begin{aligned} g(x) &= x \\ f(x) &= \frac{1}{x} \end{aligned} \right\} \Rightarrow (f \circ g)(x) = h(x)$$