9.4 Exploring Quotients of Functions

A Definitions

The quotient of two functions is defined by

$$(f/g)(x) = f(x)/g(x)$$
$$(f \div g)(x) = f(x) \div g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

Ex 1. Let $f(x) = 1 + x^2$ and $g(x) = \sqrt{x-1}$. Find

a) $(f/g)(1) = \frac{f(1)}{g(1)} = \frac{2}{0}$ undefined

b) $(f \div g)(2) = \frac{f(2)}{g(2)} = \frac{5}{1} = 5$

c) $\left(\frac{g}{f}\right)(1) = \frac{g(1)}{f(1)} = \frac{0}{2} = 0$

B Domain of the Quotient of Two Functions

The domain of the quotient of two functions is the given by

$$D_{f/g} = \{x \in R \mid x \in D_f \cap D_g \text{ and } g(x) \neq 0\}$$

Note. Division by zero is not allowed.

$D_f = \mathbb{R}$

$D_g = [1, \infty)$ $\Rightarrow D_f \cap D_g = [1, \infty)$

$g(x) = 0$ at $x = 1$

$D_{f/g} = (1, \infty)$

Ex 2. For each case, find the domain of the quotients f/g and g/f.

$g(x) \neq 0 \Rightarrow x \neq 1$

a) $f(x) = 2^x$; $g(x) = \log x$

$D_f = \mathbb{R}$ $\quad D_g = (0, \infty)$

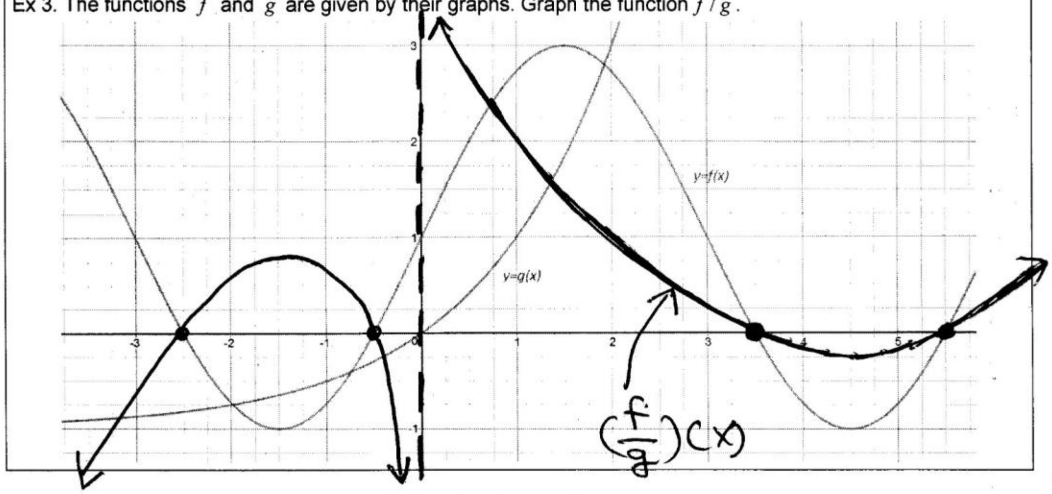
$D_{f/g} = (0, 1) \cup (1, \infty)$; $D_{g/f} = (0, \infty)$

b) $f(x) = x^2 - 4$; $g(x) = \sqrt{x-1}$

$f(x) = 0$ at $x = \pm 2$

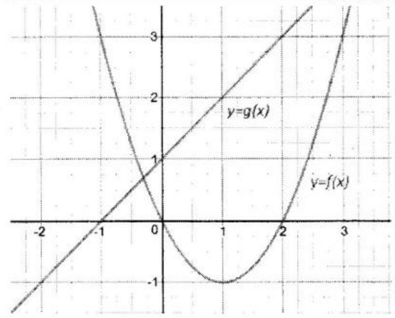
$D_{g/f} = [1, 2) \cup (2, \infty)$

Ex 3. The functions f and g are given by their graphs. Graph the function f/g.



Ex 4. The functions f and g are given graphically on the right figure. Match each graph given below with one of the following combinations:

a) $f+g$
b) $f-g$
c) $g-f$
d) fg
e) f/g
f) g/f



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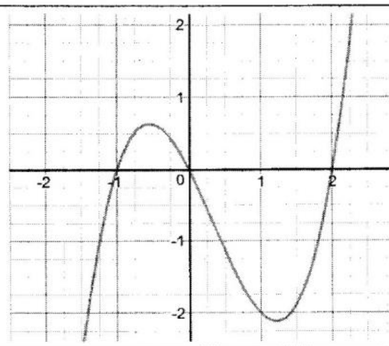
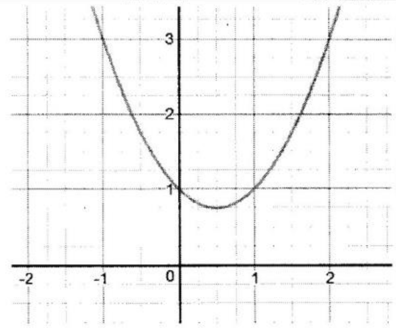
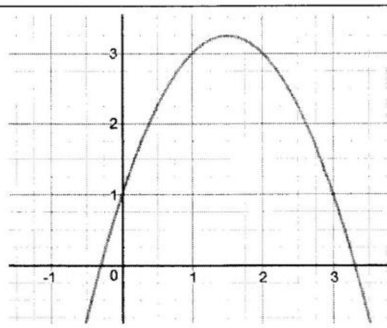


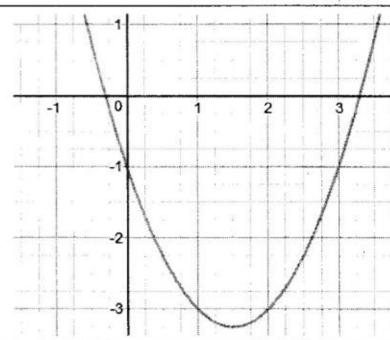
f+g



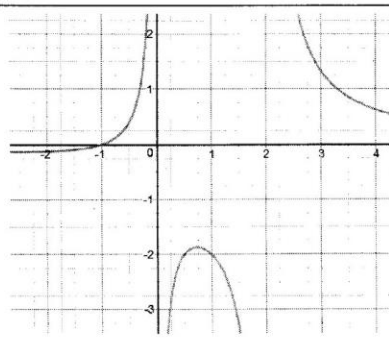
g−f



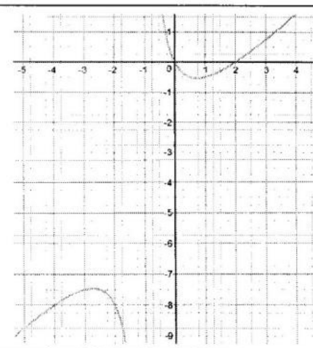
f−g



$\frac{g}{f}$



$\frac{f}{g}$



Reading: Nelson Textbook, Pages 540-542
Homework: Nelson Textbook, Page 542 #1,2