### 9.4 Exploring Quotients of Functions

#### A Definitions

The quotient of two functions is defined by

\[
\frac{f}{g}(x) = \frac{f(x)}{g(x)}
\]

\[
(f + g)(x) = f(x) + g(x)
\]

\[
\left( \frac{f}{g} \right)(x) = \frac{f(x)}{g(x)}
\]

### Ex 1. Let \( f(x) = 1 + x^2 \) and \( g(x) = \sqrt{x-1} \). Find

a) \( \frac{f}{g}(1) = \frac{f(1)}{g(1)} = \frac{2}{\sqrt{0}} = \text{undefined} \)

b) \( (f + g)(2) = \frac{f(2) + g(2)}{g(2)} = \frac{5}{1} = 5 \)

c) \( \left( \frac{g}{f} \right)(1) = \frac{g(1)}{f(1)} = \frac{0}{2} = 0 \)

#### B Domain of the Quotient of Two Functions

The domain of the quotient of two functions is the given by

\[
D_{f/g} = \{ x \in R | x \in D_f \cap D_g \text{ and } g(x) \neq 0 \}
\]

Note. Division by zero is not allowed.

\[
D_f = [0, \infty) \quad D_g = (-\infty, \infty) \]

\[
D_{f/g} = (1, \infty)
\]

### Ex 2. For each case, find the domain of the quotients \( f/g \) and \( g/f \).

a) \( f(x) = 2^x \); \( g(x) = \log x \)

\[
D_f = \mathbb{R} \quad D_g = (0, \infty) \quad D_{f/g} = (0, \infty)
\]

b) \( f(x) = x^2 - 4 \); \( g(x) = \sqrt{x-1} \)

\[
f(2) = 0 \quad \text{at} \quad x = \pm 2
\]

\[
D_{g/f} = [1, 2) \cup (2, \infty)
\]

### Ex 3. The functions \( f \) and \( g \) are given by their graphs. Graph the function \( f/g \).

![Graph of the function \( f/g \)](image)
Ex 4. The functions $f$ and $g$ are given graphically on the right figure. Match each graph given below with one of the following combinations:

- a) $f + g$
- b) $f - g$
- c) $g - f$
- d) $fg$
- e) $f/g$
- f) $g/f$

Reading: Nelson Textbook, Pages 540-542
Homework: Nelson Textbook, Page 542 #1,2