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9.3 Combining Two Functions: Products

A Definitions

The product of two functions is defined by

$$(fg)(x) = f(x)g(x)$$
$$(f \times g)(x) = f(x) \times g(x)$$

Ex 1. Let $f(x) = x^2$ and $g(x) = \log x$. Find

a) $(fg)(1) = f(1)g(1) = (1)(0) = 0$

b) $(f \times g)(0) = f(0)g(0) = (0)(\text{und}) = \text{undefined}$

B Domain of the Product of Two Functions

The domain of the product of two functions is the intersection of their domains.

$$D_{fg} = D_{f \times g} = D_f \cap D_g$$

Ex 2. For each case, find the domain of the product of the given functions.

a) $f(x) = 2^x$; $g(x) = \sin x$

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a) $f(x) = 2^x$; $g(x) = \sin x$

$D_f = \mathbb{R}$; $D_g = \mathbb{R} \Rightarrow D_{fg} = \mathbb{R}$

b) $f(x) = \dfrac{1}{x-2}$; $g(x) = \log x$

$x \neq 2$; $x > 0 \Rightarrow D_{fg} = (0,2) \cup (2,\infty)$

c) $f(x) = \sqrt{3-x}$; $g(x) = \log(x+1)$

$x \leq 3$; $x > -1 \Rightarrow D_{fg} = (-1,3]$

C Point by Point

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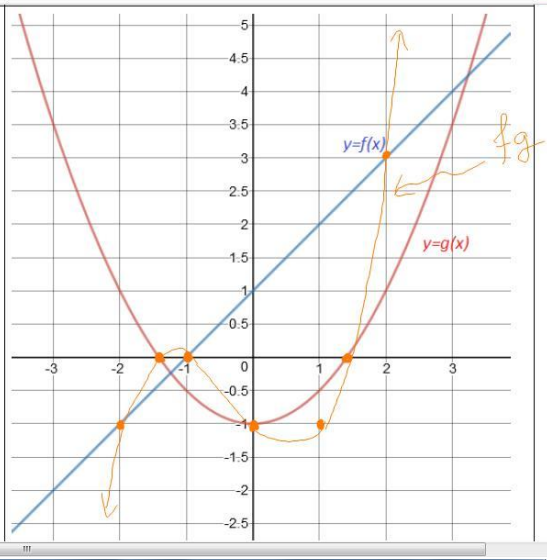
C Point by Point

Evaluate fg at every possible number x.



Ex 3. The functions f and g are given by their graphs on the right figure. Graph the function fg.

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Ex 4. Prove that the product of two odd functions is an even function.

$f(-x) = -f(x)$

$g(-x) = -g(x)$

$\therefore fg$ is even

$(fg)(-x) = f(-x)g(-x)$

$= (-f(x))(-g(x)) = f(x)g(x)$

$= (fg)(x)$

Ex 5. Complete the following table. Justify your reasoning.

	fg	even	odd
g	even	even	odd
	odd	odd	even

it depends

Ex 6. For each case, justify your answer.

a) Is the product of two polynomial functions a polynomial function?

Ex 7. For each case, graph on the same grid the functions f and g, and then the graph of the product fg. Use technology (Desmos) to check your answer.

a) $f(x) = x \quad ; \quad g(x) = \sin x$

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a) Is the product of two polynomial functions a polynomial function?

yes

b) Is the product of two rational functions a rational function?

yes

c) Is the product of two sine functions a sine function?

yes if $T_1 = T_2$

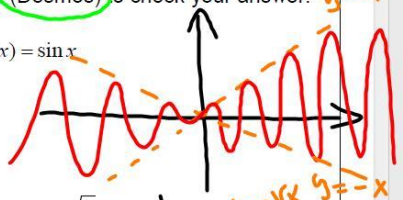
d) Is the product of two periodic functions a periodic function?

yes if $\frac{T_1}{T_2} \in \mathbb{Q} \setminus \{0\}$

fg. Use technology (Desmos) to check your answer.

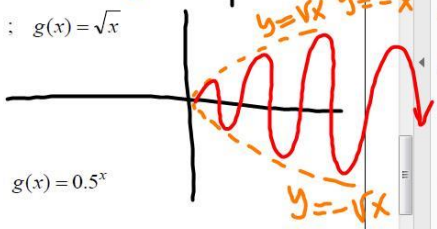
a) $f(x) = x$; $g(x) = \sin x$

$x \cdot \sin x$



b) $f(x) = \cos x$; $g(x) = \sqrt{x}$

$\sqrt{x}\cos x$



c) $f(x) = x^3$; $g(x) = 0.5^x$

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$(fg)(x) = \sqrt{x(x-4)} \cdot \log(2-x)$

Ex 8. Find the zeroes of the product fg where

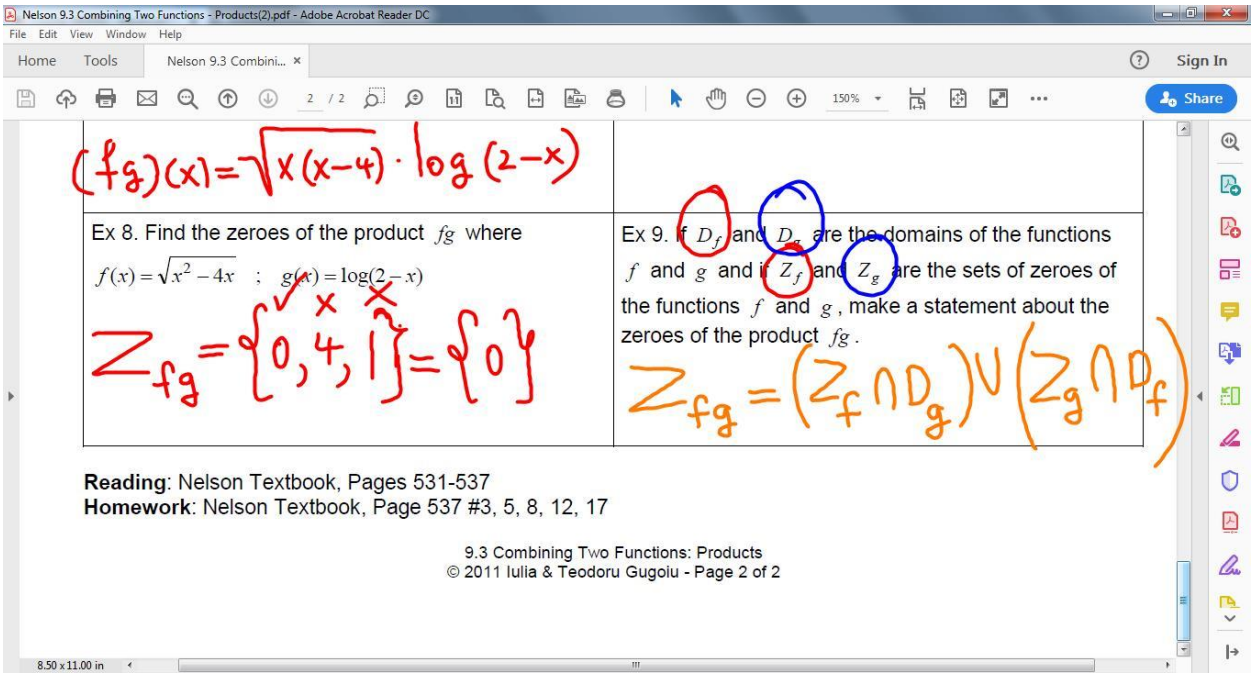
$f(x) = \sqrt{x^2 - 4x}$; $g(x) = \log(2-x)$

$Z_{fg} = \{0, 4, 1\} = \{0\}$

Ex 9. If D_f and D_g are the domains of the functions f and g and if Z_f and Z_g are the sets of zeroes of the functions f and g, make a statement about the zeroes of the product fg.

$Z_{fg} = (Z_f \cap D_g) \cup (Z_g \cap D_f)$

Reading: Nelson Textbook, Pages 531-537
Homework: Nelson Textbook, Page 537 #3, 5, 8, 12, 17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