9.3 Combining Two Functions: Products

A Definitions

The product of two functions is defined by

\[(fg)(x) = f(x)g(x)\]
\[(f \times g)(x) = f(x) \times g(x)\]

Ex 1. Let \(f(x) = x^2\) and \(g(x) = \log x\). Find
a) \((fg)(1) = f(1)g(1) = (1)(0) = 0\)
b) \((f \times g)(0) = f(0)g(0) = (0)(\text{undefined}) = \text{undefined}\)

B Domain of the Product of Two Functions

The domain of the product of two functions is the intersection of their domains.

\[D_{fg} = D_f \cap D_g\]

Ex 2. For each case, find the domain of the product of the given functions.

a) \(f(x) = 2^x\); \(g(x) = \sin x\)

C Point by Point

\[D_f = \mathbb{R}; D_g = \mathbb{R} \Rightarrow D_{fg} = \mathbb{R}\]
\[f(x) = \frac{1}{x-2}; g(x) = \log x\]
\[x \neq 2; x > 0 \Rightarrow D_{fg} = (0,2) \cup (2,\infty)\]
\[f(x) = \sqrt{5-x}; g(x) = \log(x+1)\]
\[x \leq 3; x > -1 \Rightarrow D_{fg} = (-1,3)\]
C Point by Point

Evaluate $f \cdot g$ at every possible number $x$.

Ex 3. The functions $f$ and $g$ are given by their graphs on the right figure. Graph the function $f \cdot g$.

Ex 4. Prove that the product of two odd functions is an even function.

\[ f(-x) = -f(x) \]
\[ g(-x) = -g(x) \]
\[ (f \cdot g)(-x) = f(-x)g(-x) = (-f(x))(-g(x)) = f(x)g(x) = (f \cdot g)(x) \]

$\therefore f \cdot g$ is even

Ex 5. Complete the following table. Justify your reasoning.

<table>
<thead>
<tr>
<th>$f \cdot g$</th>
<th>even</th>
<th>odd</th>
</tr>
</thead>
<tbody>
<tr>
<td>even</td>
<td>even</td>
<td>odd</td>
</tr>
</tbody>
</table>

Ex 6. For each case, justify your answer.

a) Is the product of two polynomial functions a polynomial function?

Ex 7. For each case, graph on the same grid the functions $f$ and $g$, and then the graph of the product $f \cdot g$. Use technology (Desmos) to check your answer.

a) $f(x) = x$ ; $g(x) = \sin x$
f, Use technology (Desmos) to check your answer.

a) \( f(x) = x \quad g(x) = \sin x \)

X: \( -\sin x \)

b) \( f(x) = \cos x \quad g(x) = \sqrt{x} \)

\( \sqrt{x} \cos x \) \( X \)

Yes if \( t_1 = t_2 \)

Readln Text: Nelson Textbook, Pages 531-537

Ex 8. Find the zeroes of the product \( fg \) where

\( f(x) = \sqrt{x^2 - 4x} \quad g(x) = \log(2 - x) \)

\( Z_{fg} = \{ 0, 4, 1 \} = \{ 0 \} \)

Reading: Nelson Textbook, Pages 531-537

Homework: Nelson Textbook, Pages 537 #3, 5, 8, 12, 17
Ex 8. Find the zeroes of the product $f \cdot g$ where

$$f(x) = \sqrt{x^2 - 4} \quad \text{and} \quad g(x) = \log(2 - x)$$

$$Z_{fg} = \{0, 4, 1\} = \{0\}$$

Ex 9. If $D_f$ and $D_g$ are the domains of the functions $f$ and $g$ and $Z_f$ and $Z_g$ are the sets of zeroes of the functions $f$ and $g$, make a statement about the zeroes of the product $f \cdot g$.

$$Z_{fg} = (Z_f \cap D_g) \cup (Z_g \cap D_f)$$

Reading: Nelson Textbook, Pages 531-537
Homework: Nelson Textbook, Page 537 #3, 5, 8, 12, 17