9.2 Combining Two Functions: Sums and Differences

A Definitions

The sum and difference of two functions are defined by

$$(f+g)(x) = f(x) + g(x)$$
$$(f-g)(x) = f(x) - g(x)$$

Ex 1. Let $f(x) = x^2 - 1$ and $g(x) = 1 + \sqrt{x+1}$. Find

a) $(f+g)(0) = f(0) + g(0)$

$= -1 + 2 = 1$

b) $(f-g)(3) = f(3) - g(3)$

$= 8 - 3 = 5$

c) $(f+g)(-2) = f(-2) + g(-2)$

$= 3 + \text{undef} = \text{undefined}$

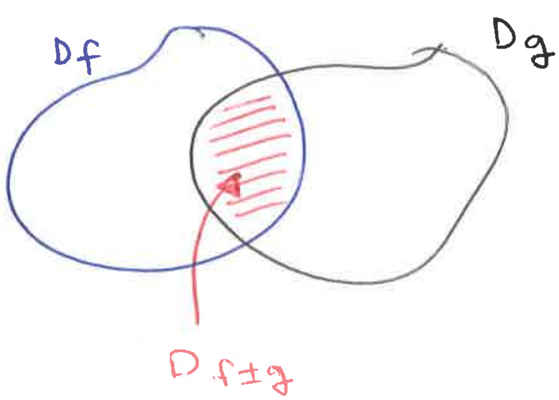
d) $(f+g)(x) = x^2 - 1 + 1 + \sqrt{x+1}$

$= x^2 + \sqrt{x+1}$

B Domain of Sum and Difference of Two Functions

The domain of the sum of two functions is the intersection of their domains.

$$D_{f\pm g} = D_f \cap D_g$$



Ex 2. For each case, find the domain of the sum of the given functions.

a) $f(x) = x$; $g(x) = 1/x^2$

$\mathbb{R}$ $x \neq 0$

$D_{f+g} = \mathbb{R} - \{0\}$

b) $f(x) = \sqrt{2-x}$; $g(x) = \log(x+1)$

$2 - x \geq 0$ $x + 1 > 0$

$x \leq 2$ $x > -1$

$D_{f+g} = (-1, 2]$

c) $f(x) = 2^{x-2}$; $g(x) = x^3 + 1$

$\mathbb{R}$ $\mathbb{R}$

$D_{f+g} = \mathbb{R}$

d) $f(x) = \sqrt{4-x^2}$; $g(x) = \dfrac{1}{x-1}$; $h(x) = \sqrt{x}$

$4 - x^2 \geq 0$ $x \neq 1$ $x > 0$

$x^2 \leq 4$

$-2 \leq x \leq 2$

$D_{f+g+h} = (0, 1) \cup (1, 2]$

$f(1) = 0$

C Point by Point

Evaluate $f \pm g$ at every possible number x.

Ex 3. Given

$f = \{(1,0), (0,-1), (-1,2)\}$
$g = \{(0,1), (2,-1), (1,3)\}$

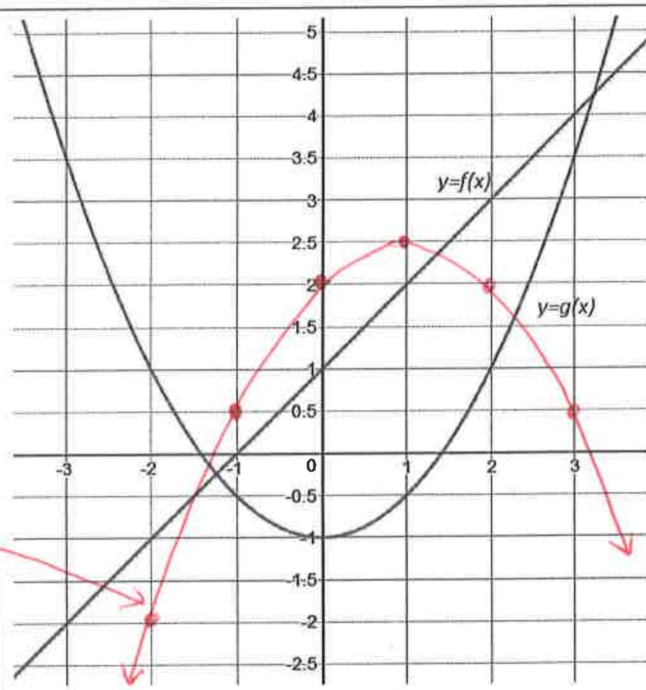
Find $f+g$.

x	-1	0	1	2
$f(x)$	2	-1	0	$-$
$g(x)$	$-$	1	3	-1
$(f+g)(x)$	$-$	0	3	$-$

$f+g = \{(0,0), (1,3)\}$

Ex 4. The functions f and g are given by their graphs on the right figure. Graph the function $f-g$.

$y = (f-g)(x)$



Ex 5. Complete the following table. Justify your reasoning.

	$f+g$	f: even	f: odd	f: neither
g	even	even	neither	neither
	odd	neither	odd	neither
	neither	neither	neither	neither

Ex 6. For each case, justify your answer.

a) Is the sum of two polynomial functions a polynomial function?

Yes

b) Is the difference of two rational functions a rational function?

Yes

c) Is the sum of two sine functions a sine function?

Yes only if their periods are equal.
No if their periods are not equal

Ex 7. Write the following functions as a sum or a difference of two other functions.

a) $f(x)=\frac{1}{x^2-9}$ $= \frac{A}{x-3} + \frac{B}{x+3}$

$= \frac{A(x+3)+B(x-3)}{x^2-9}$

$0 = A+B \Rightarrow B = -A$

$1 = 3A-3B \Rightarrow 1 = 6A$

$f(x) = \frac{1}{6(x-3)} - \frac{1}{6(x+3)}$

b) $f(x)=\log\frac{x}{x+1}$ $= \log x - \log(x+1)$

c) $f(x)=\sin(x-\pi/4)$ $= (\sin x)\cos\frac{\pi}{4} - (\cos x)\sin\frac{\pi}{4}$

$= \frac{\sqrt{2}}{2}\sin x - \frac{\sqrt{2}}{2}\cos x$

Ex 8. Is the sum of two increasing functions, increasing, decreasing or neither? Give examples to justify your answer.

Increasing

$f(x) = x$ (increasing)

$g(x) = x^3$ (increasing)

$(f+g)(x) = x+x^3$ is increasing

Ex 9. If $Z_f=\{1,2,3\}$ is the set of all zeros of the function f and $Z_g=\{0,1,2\}$ is the set of all zeros of the function g, what could you say about the set of all zeros of the function $f+g$. Explain your reasoning.

$Z_f \cap Z_g \subset Z_{f+g}$

$\{1,2\} \subset Z_{f+g}$

At least 1 and 2 are zeros of $f+g$.
More are possible.

Ex 10. Let $f(x)=\sin x$ and $g(x)=\cos x$. Write $f+g$ and $f-g$ as:

a) a single sine function

$(f+g)(x) = \sin x + \cos x = \sqrt{2}\sin\left(x+\frac{\pi}{4}\right)$

$(f-g)(x) = \sin x - \cos x = \sqrt{2}\left(\sin x - \frac{\pi}{4}\right)$

b) a single cosine function

subtract $\frac{\pi}{2}$ from previous

$\sin x + \cos x = \sqrt{2}\cos(x-\pi/4)$

$\sin x - \cos x = \sqrt{2}\cos\left(x-\frac{3\pi}{4}\right)$

Ex 11. The rational function $y=f(x)$ has a horizontal asymptote $y=5$ and the rational function $y=g(x)$ has a horizontal asymptote $y=-3$. What could you say about the horizontal asymptote of the functions:

a) $f+g$

HA: $y = 5+(-3) = 2$

b) $f-g$

HA: $y = 5-(-3) = 8$

Reading: Nelson Textbook, Pages 521-528
Homework: Nelson Textbook, Page 528 #3, 4, 5, 7, 10, 13, 16

