9.1 Exploring Combinations of Functions

A Arithmetic Combinations

Consider two functions $f(x)$ and $g(x)$. Then the sum $f+g$, difference $f-g$, product fg and the quotient f/g are defined as follows:

$$(f+g)(x) = f(x)+g(x)$$
$$(f-g)(x) = f(x)-g(x)$$
$$(fg)(x) = f(x)g(x)$$
$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, \quad g(x) \neq 0$$

Ex 1. Given $f(x) = x^2 - 1$ and $g(x) = 2\sqrt{x-1}$, find:

a) $(f+g)(1) = f(1)+g(1) = 0+0 = 0$

b) $(f-g)(2) = f(2)-g(2) = 3-2 = 1$

c) $(fg)(5) = f(5)g(5) = (24)(4) = 96$

d) $\left(\frac{f}{g}\right)(1) = \frac{f(1)}{g(1)} = \frac{0}{0}$ ∴ undefined

e) $(f+g)(x) = x^2 - 1 + 2\sqrt{x-1}$

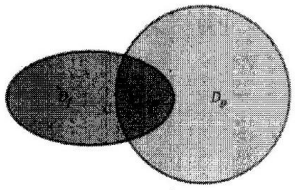
f) $(f-g)(x) = x^2 - 1 - 2\sqrt{x-1}$

g) $(fg)(x) = 2(x^2-1)\sqrt{x-1}$

h) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)} = \frac{x^2-1}{2\sqrt{x-1}}$; $x > 1$

B Domain

The domain of $f+g$, $f-g$, and fg is $D_f \cap D_g$ (the intersection between the domain of f and the domain of g) (see diagram below).



The domain of $\frac{f}{g}$ is $\{x \in D_f \cap D_g \mid g(x) \neq 0\}$.

Ex 2. Given $f(x) = x^2 - 1$ and $g(x) = \sqrt{x-2}$, find the domain of the following arithmetic combinations:

a) $(f+g)(x)$ } $D_f = \mathbb{R}$; $D_g = [2, \infty)$

b) $(f-g)(x)$ } ∴ $D = [2, \infty)$

c) $(fg)(x)$ }

d) $\left(\frac{f}{g}\right)(x)$ ∴ $D = (2, \infty)$

e) $\left(\frac{g}{f}\right)(x)$ ∴ $D = [2, \infty)$

Ex 3. The function f has the x-intercepts -1, 1, and 3 and the y-intercept is 4. The function g has the x-intercepts -2 and -1 and the y-intercept is 1. Find the x-intercepts and the y-intercept of the following arithmetic combinations:

	f	g
x-int	−1, 1, 3	−2, −1
y-int	4	1

a) $(fg)(x)$ x-int $= -1, 1, 3, -2$; y-int $= (4)(1) = 4$

b) $\left(\frac{f}{g}\right)(x)$ x-int $= 1, 3$; y-int $= 4/1 = 4$

c) $\left(\frac{g}{f}\right)(x)$ x-int $= -2$; y-int $= \frac{1}{4}$

Reading: Nelson Textbook, Pages 518-520
Homework: Nelson Textbook, Page 520#2,3

