Questions 1-5 are Multiple-Choice questions

1. The expression $\log_2 32 = ?$ is equivalent to:
   A) $2^5 = 32$  B) $5^2 = 25$  C) $3^2 = 9$  D) $2^3 = 32$  E) $2^3 = 8$

2. The exact value of $\log_3 \sqrt[3]{27}$ is:
   A) 2  B) 3  C) $3/2$  D) 2/3  E) 9

3. The solution of the equation $\log_3 x = -1$ is:
   A) -9  B) -10  C) 1/3  D) -1/3  E) 3

4. If $\log_a (-2 / b) = -1$ then:
   A) $a + b = 0$  B) $a = 1/b$  C) $ab = -2$  D) $2a + b = 0$  E) $a^b = 1$

5. The approximate value of $\log_3 7$ is:
   A) -1.61  B) -0.54  C) 0.39  D) 1.77  E) 3.16

Questions 6-10 are True-False questions

6. The exponential and logarithmic functions are one-to-one functions.  
   T  F

7. The inverse function of the exponential function is the logarithmic function.  
   T  F

8. The relation $0^0 = 0$ is equivalent to $\log_0 0 = 1$.  
   T  F

9. The exponential $y = e^{-x}$ function has the range $(-\infty, 0)$.  
   T  F

10. The graph of the function $f(x) = \log_2 \frac{1}{x}$ does not intersect the x-axis.  
    $\frac{1}{x} = -\log_2 x$  T  F

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [A 4 marks]
The following questions are long answer questions. Show your work to get full marks.

12. a) Convert the exponential notation into the logarithmic notation.

\[ 10^{-3} = 0.001 \]
\[ -3 = \log_{10} 0.001 \]

b) Convert the logarithmic notation into the exponential notation.

\[ \log_{5} 625 = \]
\[ 5^4 = 625 \]

13. Solve for \( x \). State any restriction.

[1] a) \( 3^x = 243 \)
\[ \log_3 243 = x \]
\[ x = \log_3 243 = 5 \]

[2] a) \( 3^{1-x} = 27 \)
\[ 3^{1-x} = 3^3 \]
\[ 1-x = 3 \]
\[ x = -2 \]

14. Find the domain of the following functions:

[2] a) \( f(x) = \frac{1}{2^x - 16} \)
\[ 2^x \neq 16 = 2^4 \]
\[ x \neq 4 \]
\[ x \neq \pm 2 \]

15. Use transformations to graph the following functions.

[2] a) \( y = \frac{1}{2^x - 16} \)
\[ D = \mathbb{R} \]
\[ \mathbb{R} = (3, \infty) \]

b) \( y = 1 - \log_{2}(x + 1) \)
\[ y - \log_{2}(x + 1) = 1 \]
\[ x - \log_{2} = \text{none} \]
\[
-2 \log(x+1) + 3 \log(2x) - \frac{1}{2} \log(x-1)
\]
\[
E = \log \left( \frac{(2x)^3}{(x+1)^2 \sqrt{x-1}} \right)
\]

[A 3 marks]

\[
\begin{align*}
x &> -1 \\
x &> 0 \\
x &> 1
\end{align*}
\]
\[
\Rightarrow x > 1
\]

17. Solve for \(x\):

\[3 \text{ a) } 2^{-3} = 8^{x+1}\]

\[
\begin{align*}
x - 3 &= 2 \cdot (x + 1) \\
2x &= 2 \\
x &= -1
\end{align*}
\]

[A 6 marks]

\[3 \text{ a) } \ln 2 + \ln(4x - 1) = \ln(2x + 5)\]

\[
\begin{align*}
\ln \left[ 2(4x-1) \right] &= \ln (2x+5) \\
8x - 2 &= 2x + 5 \\
x &= \frac{7}{6}
\end{align*}
\]

\[
\begin{align*}
D &= \{ x \in \mathbb{R} | 4x - 1 > 0 \} \\
&\Rightarrow x > -\frac{1}{4} \\
&\Rightarrow D = (\frac{1}{4}, \infty)
\end{align*}
\]

18. Find the domain, range, x- and y-intercept for the following functions. Do not graph.

\[3 \text{ a) } f(x) = -2^{x+1} + 1\]

\[
\begin{align*}
D &= \mathbb{R} \\
R &= (-\infty, 1) \\
y - \text{int } &= -2^3 + 1 = -8 + 1 = -7 \\
x - \text{int } &= -2^2 + 1 \\
&= -2 \\
x + 3 &= 0 \\
&= -3
\end{align*}
\]

[A 6 marks]

\[3 \text{ a) } g(x) = -2 \log_3 (x - 1) + 2\]

\[
\begin{align*}
x - 1 &= 0 \\
D &= (1, \infty) \\
R &= \mathbb{R} \\
y - \text{int } &= -2 \log_3 (-1) + 2 \\
&= -2 \times \text{Does Not Exist} \\
&= x - \text{int } \\
&= -2 \log_3 (x - 1) + 2 \\
&= \log_3 \frac{1}{x - 1} = 1 = \log_3 3 \\
&= x - 1 = 3 \\
&= x - \text{int } = 4
\end{align*}
\]
19. A model for the number of bacteria in a culture after $t$ hours is given by $P(t) = P_0 e^{kt}$. After 1 hour it is observed that 100 bacteria are present. After 10 hours it is observed that 10000 bacteria are present. [A 5 marks]

1) a) What is the initial number of bacteria?

$100 = P_0 e^{k(1h)}$

$10000 = P_0 e^{k(10h)}$ 

$\frac{10000}{100} = e^{9k}$

$\ln 100 = k(9h)$

$k = \frac{\ln 100}{9h}$

$p_0 = \frac{100}{e^{k(1h)}} = \frac{100}{e^{\frac{\ln 100}{9}}} = \boxed{59.95 \approx 60}$

The initial population is about 60 bacteria.

b) What is the doubling period (the time after which the number of bacteria is doubled) of these bacteria?

$2P_0 = P_0 e^{kD}$

$ln 2 = kD$

$D = \frac{ln 2}{k} = \frac{(ln 2)(9h)}{ln 100} = \boxed{1.354 \text{ hours}}$

The doubling period is about 1.354 hours.

c) How many bacteria will be after one day?

$t = 24h$

$P(24h) = p_0 e^{k(24h)}$

$= 60 \cdot e^{\frac{\ln 100}{9} \cdot 24}$

$= 12,926,680 = P(24h)$

After one day, there will be about 12,926,680 bacteria.

d) What is the time after which the number of bacteria is 5000?

$5000 = 60 \cdot e^{kt}$

$\ln 5000 = kt$

$t = \frac{\ln 5000}{\frac{\ln 100}{9h}} = \boxed{8.64 \text{ hours}}$

The number of bacteria will be 5000 after 8.64 hours.

e) What is the growth rate (percentage per hour) of these bacteria?

$r = \frac{P(1h) - P(0)}{P(0)} = \frac{P_0 e^{k(1h)} - P_0}{P_0}$

$= e^k - 1 = e^\frac{\ln 100}{9} - 1 = 0.668 \approx 66.8\% \text{ per hour}$

$r = 66.8\% \text{ per hour}$

The growth rate is about 66.8% per hour.