Questions 1-5 are Multiple-Choice Questions

1. The expression \(0.125 = 2^{-3}\) is equivalent to:
   A) \(3 = \log_2{0.125}\)  
   B) \(-3 = \log_{0.125}{2}\)  
   C) \(2 = \log_3{0.125}\)  
   D) \(-3 = \log_2{0.125}\)  
   E) \(-0.125 = \log_2{3}\)

2. The exact value of \(\frac{2}{5} = \sqrt[3]{8}\) is:
   A) \(\frac{5}{8}\)  
   B) \(3\)  
   C) \(\frac{6}{5}\)  
   D) \(\frac{8}{5}\)  
   E) \(8\)

3. The solution of the equation \(\log_2{\frac{1}{2^x}} = -3\) is:
   A) \(-30\)  
   B) \(100\)  
   C) \(1/1000\)  
   D) \(-1/1000\)  
   E) \(100\)

4. If \(\log_a{a^3} = 1\) then:
   A) \(a = 0\)  
   B) \(a = 1\)  
   C) \(a = -1\)  
   D) \(a = 2\)  
   E) \(a = -2\)

5. The approximate value of \(\log_8{0.2}\) is:
   A) \(0.20\)  
   B) \(0.10\)  
   C) \(0.70\)  
   D) \(-0.20\)  
   E) \(-0.10\)

Questions 6-10 are True-False Questions

6. The logarithmic function \(f(x) = \log_a{x}\) is always increasing.
   T  
   F

7. The exponential function \(f(x) = a^x\) has the vertical asymptote \(x = 0\).
   T  
   F

8. The expression \(0^1 = 0\) is equivalent to \(1 = \log_0{0}\).
   T  
   F

9. One key point on the graph of \(y = a^x\) is \((-1, 1/a)\).
   T*  
   F

10. The product law of logarithms states that \(\log_a{(xy)} = (\log_a{x})(\log_a{y})\).
    T  
    F

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph.

   A) \(y = \log(x + 2)\)  
   B) \(g(x) = -\log x\)  
   C) \(h(x) = \log(-x)\)  
   D) \(k(x) = \log(1 - x)\)  
   E) \(p(x) = 2\log x\)  
   F) \(q(x) = -\log(-x)\)

   I) E  
   II) D  
   III) A  
   IV) C
The following questions are long answer questions. Show your work to get full marks.

12. a) Convert the exponential notation into the logarithmic notation.

\[ 4^{3/2} = 8 \]

\[ \frac{10}{2} = \log_4 8 \]

\[ 2 \cdot x = 4 \]

\[ \therefore x = 2 \]

b) Convert the logarithmic notation into the exponential notation.

\[ \log 0.001 = -3 \]

\[ 10^{-3} = 0.001 \]

13. Solve for \( x \).

[1] a) \( 3^{x^2} = 81 \)

\[ 2^{x} = 3 \]

\[ 2 \cdot x = 2^4 \]

\[ \therefore x = 4 \]

[1] b) \( \log(x + 1) = -1 \)

\[ 10^{-1} = x + 1 \]

\[ x = \frac{1}{10} - 1 \]

\[ \therefore x = -\frac{9}{10} \>

\[ x > -1 \]

[2] c) \( 2^{x^2 - 3} = 0.25 \)

\[ 2 \cdot x^2 - 3 = \frac{1}{4} = 2^{-2} \]

\[ x^2 - 3 = -2 \]

\[ x^2 = 1 \]

\[ \therefore x = \pm 1 \]

[2] d) \( \ln x + \ln(x - 1) = 0 \)

\[ \ln x \cdot (x - 1) = \ln 1 \]

\[ x^2 - x = 1 \]

\[ x^2 - x - 1 = 0 \]

\[ x = \frac{1 \pm \sqrt{1 + 4}}{2} \]

\[ x = \frac{1 - \sqrt{5}}{2} < 0 \]

\[ x = \frac{1 + \sqrt{5}}{2} > 1 \]

\[ \therefore x = \frac{1 + \sqrt{5}}{2} \approx 1.62 \]

14. Simplify, state any restrictions. \( E = 2 \log w + 3 \log \sqrt{w} + \frac{1}{2} \log w^2 \)

\[ \log w^2 + \log w^{3/2} + \log w^{2/2} \]

\[ = \log \left( w^2 \cdot w^{3/2} \cdot w^{1/2} \right) \]

\[ = \log \left( w^{2 + 3/2 + 1/2} \right) \]

\[ = \frac{3}{2} \log w \quad \Rightarrow \]

\[ \therefore E = \frac{3}{2} \log w \quad > \quad w > 0 \]

15. Use transformations to graph the following functions.

a) \( y = (-3)2^{x-1} + 2 \)

b) \( y = 2 \log_2 (x + 2) - 3 \)
16. Solve for $x$.

\[ 2^{x-1} \geq 0.25 \]

\[ 2^{x-1} \geq 2^{-2} \]

\[ x-1 \geq -2 \]

\[ \therefore x \geq -1 \]

\[ 17. \text{Solve for } x: \]

\[ [2] \text{a)} \quad 2^x - 2^{-x} = 4 \]

\[ 2^x = y \]

\[ y - \frac{1}{y} = 4 \]

\[ y^2 - 4y - 1 = 0 \]

\[ y = \frac{4 \pm \sqrt{16 + 4}}{2} = 2 \pm \sqrt{5} \]

\[ 2^x = 2 + \sqrt{5} \]

\[ x \ln 2 = \ln (2 + \sqrt{5}) \]

\[ \therefore x = \frac{\ln (2 + \sqrt{5})}{\ln 2} \approx 2.083 \]

\[ [2] \text{a)} \quad \log_{0.1}(x-1) \leq -1 \]

\[ \log_{0.1}(x-1) \leq -\log_{0.1}0.1 \]

\[ \log_{0.1}(x-1) \leq \log_{0.1}(0.1)^{-1} \]

\[ x-1 \leq (0.1)^{-1} \]

\[ x \geq 1 + \frac{1}{0.1} \]

\[ \therefore x > 11 \]

\[ [3] \text{a)} \quad \log_2(x+5) - \log_2(2x) = \frac{3}{2} \]

\[ x > -5 \quad \land \quad x > 0 \]

\[ \log_2 \frac{x+5}{2x} = \log_2 8 \]

\[ \frac{x+5}{2x} = 8 \]

\[ 2x = 8 \]

\[ x+5 = 16x \]

\[ 5 = 15x \]

\[ \therefore x = \frac{1}{3} \]

18. Find the domain, range, x- and y-intercept for the following functions. Do not graph.

\[ [3] \text{a)} \quad f(x) = \frac{3^x + 3^{-x}}{3^x - 3^{-x}} \]

\[ 3^x - 3^{-x} \neq 0 \Rightarrow x \neq 0 \]

\[ D = \{ x \in \mathbb{R} \mid x \neq 0 \} \]

\[ y - \lfloor y \rfloor = \frac{1}{2} \Rightarrow \text{not defined} \]

\[ f(x) = 0 \Rightarrow 3^x + 3^{-x} = 0 \]

\[ \text{(no solution)} \]

\[ \therefore \text{no x-intercept} \]

\[ \therefore \text{no y-intercept} \]

\[ R = (-\infty, 1) \cup (1, \infty) \]

\[ [3] \text{a)} \quad g(x) = 1 - 2 \log \frac{x-1}{x+1} \]

\[ \begin{array}{c|c|c|c|c}
\hline
x & -1 & -0 & 0 & 1 \\
\hline
x-1 & -2 & -1 & 0 & 1 \\
\hline
x+1 & 0 & 1 & 2 & 3 \\
\hline
\end{array} \]

\[ D = (-\infty, 3) \cup (3, \infty) \]

\[ g(0) = 1 - 2 \log \frac{0-1}{0+1} \quad \text{not defined} \]

\[ \therefore \text{no y-intercept} \]

\[ g(x) = 0 \Rightarrow 1 = 2 \log \frac{x-1}{x+1} \]

\[ \sqrt{10} = \frac{x-1}{x+1} \Rightarrow \sqrt{10} x + \sqrt{10} = x - 1 \]

\[ x - \text{left} \quad = \frac{10}{10-1} \approx -1.93 \]
19. A computer, originally purchased for $2000, loses value according to the exponential function \( V = 2000 \left( \frac{1}{2} \right)^{\frac{t}{H}} \), where \( V \) is the value, in dollars, of the computer at any time, \( t \), in years, after purchase and \( H \) represents the half-life, in years, of the value of the computer. After one year, the computer has a value of approximately $1516.

[1] a) What is the half-life \( H \) of the value of the computer?

\[
\begin{align*}
1516 &= 2000 \left( \frac{1}{2} \right)^{\frac{1}{H}} \\
\frac{1516}{2000} &= \left( \frac{1}{2} \right)^{\frac{1}{H}} \\
\frac{1}{H} &= \ln \frac{1}{2} = \ln \frac{1516}{2000} \\
H &= \frac{\ln \frac{1}{2}}{\ln \frac{1516}{2000}} \\&= 2.5 \text{ years}
\end{align*}
\]

[1] b) What is the value of the computer after three years?

\[
V = 2000 \left( \frac{1}{2} \right)^{\frac{3}{2.5}} \approx \$ 870.55
\]

[1] c) How long will it take for the computer to be worth 10% of its purchase price?

\[
(0.10) \times 2000 = 2000 \left( \frac{1}{2} \right)^{\frac{t}{2.5}} \\
\ln 0.1 &= \frac{t}{2.5} \ln 0.5 \\
t &= 2.5 \frac{\ln 0.1}{\ln 0.5} \approx 8.3 \text{ years}
\]

[2] How much will be the depreciation of the value of the computer during the fourth year?

\[
\text{Depreciation} = V(5) - V(4) \\
= 2000 \left[ \left( \frac{1}{2} \right)^{\frac{3}{2.5}} - \left( \frac{1}{2} \right)^{\frac{4}{2.5}} \right] \\
\approx \$ 210.8
\]