Questions 1-5 are Multiple-Choice questions [K/U 1 mark each]

1. The expression $10^{-4} = 0.0001$ is equivalent to:
   A) $-1 = \log 0.0001$
   B) $-4 = \log 0.0001$
   C) $10 = \log 0.0001$
   D) $-10000 = \log 4$
   E) $-4 = \log 0.0004$

2. The exact value of $\log_{2} \sqrt{2}$ is:
   A) 2
   B) 3
   C) $1/2$
   D) $1/3$
   E) 8

3. The solution of the equation $\log x = -2$ is:
   A) $-20$
   B) $-100$
   C) $1/100$
   D) $-1/100$
   E) 100

4. If $\log_{1/a}(1/b) = -1$ then:
   A) $a = b$
   B) $a = 1/b$
   C) $ab = -1$
   D) $a + b = 0$
   E) $a^b = 1$

5. The approximate value of $\log_{4} 5$ is:
   A) -1.61
   B) -0.54
   C) 1.39
   D) 1.61
   E) 1.16

Questions 6-10 are True-False questions [K/U 1 mark each]

6. The exponential function is a one-to-one function. [T] (F)

7. The inverse function of the logarithmic function is the exponential function.. [T] (F)

8. The relation $a^0 = 1$ is true if $a \neq 0$. [T] (F)

9. The logarithmic function does not have a vertical asymptote. [T] (F)

10. The graph of the function $f(x) = a^x$ does not intersect the x-axis. [T] (F)

11. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [A 4 marks]

   A) [ ]
   \[ f(x) = -2^x \]

   B) [ ]
   \[ g(x) = 2^{x+2} \]

   C) [ ]
   \[ h(x) = 3^x \]

   D) [ ]
   \[ k(x) = 2^{x-2} \]

   E) [ ]
   \[ p(x) = 0.5^x + 2 \]

   F) [ ]
   \[ q(x) = 0.5^x - 1 \]
The following questions are long answer questions. Show your work to get full marks.

12. a) Convert the exponential notation into the logarithmic notation.
\[ 2^x = 16 \quad \therefore x = \log_2 16 \]

a) Convert the logarithmic notation into the exponential notation.
\[ \log_2 81 = 4 \quad \therefore 2^4 = 81 \]

13. Solve for \( x \).

\[ 1 \text{ a) } 2^x = 65536 \]
\[ \log_2 2 = \log_2 65536 \]
\[ x = \frac{\log 65536}{\log 2} \]
\[ \therefore x = 16 \]

\[ 1 \text{ b) } \log x = -5 \]
\[ 10^{-5} = x \]
\[ x = 10^{-5} = 0.00001 \]
\[ \text{Restrictions: } x > 0 \quad (\text{true}) \]

\[ 2 \text{ c) } 5x^2 - 2 = 25 \]
\[ 5x^2 - 2 = 25 \]
\[ x^2 - 2 = 5 \]
\[ x^2 = 7 \]
\[ \therefore x = \pm \sqrt{7} \]

\[ 2 \text{ d) } \ln(x^2 + x) = 0 \]
\[ \ln(x^2 + x) = \ln 1 \]
\[ x^2 + x = 1 \]
\[ x^2 + x - 1 = 0 \]
\[ x = \frac{-1 \pm \sqrt{1 + 4}}{2} \]
\[ \text{Restrictions: } x > 0 \]
\[ x > -1 \]
\[ x \in (0, \infty) \]

14. Find the domain of the following functions:

\[ 2 \text{ a) } f(x) = \frac{1}{10^x - 1} \]
\[ 10^x - 1 \neq 0 \]
\[ 10^x \neq 1 \]
\[ \therefore x \neq 0 \]

\[ 2 \text{ b) } g(x) = \log(1 - \sqrt{x}) \]
\[ 1 - \sqrt{x} > 0 \]
\[ 1 > \sqrt{x} \]
\[ \sqrt{x} < 1 \implies x < 1 \]
\[ x > 0 \]
\[ \therefore x \in (0, 1) \]

15. Use transformations to graph the following functions.

\[ \text{a) } y = -2x^2 - 1 \]

\[ \text{b) } y = -2\log_2(x - 1) \]
16. Find the exact value of:
\[
\log_a \frac{1}{\sqrt{a}} + \log_a \sqrt{a} - \frac{1}{3} \log_a a^2
\]
\[
= -\frac{1}{2} + \frac{1}{3} - \frac{2}{3} = -\frac{1}{2} - \frac{1}{3} = -\frac{5}{6}
\]

17. Solve for \(x\):

\[3 \text{ a)} \ 64^x - 10(8^x) + 16 = 0 \]
\[
g^x = \frac{4}{3}
\]
\[
3^x - 10 \cdot 2^y + 16 = 0
\]
\[
y = \frac{10 + 5}{2} = 5
\]
\[
g^x = 8 \implies x = 1
\]
\[
g^x = 2^x \implies 3^x = 2^x \implies x = \frac{1}{3}
\]
\[
\therefore x = 1 \text{ or } x = \frac{1}{3}
\]

\[3 \text{ a)} \ -\ln 3 + \ln(2x - 1) = -\ln 4 + \ln(x + 1)
\]
\[
\ln \frac{2x - 1}{3} = \ln \frac{x + 1}{4}
\]
\[
2x - 1 = \frac{x + 1}{4}
\]
\[
+ (2x - 1) = 7(x + 1)
\]
\[
x - 4 = 3x + 2
\]
\[
5x = 7
\]
\[
x = \frac{7}{5} = 1.4
\]
\[
\therefore x = \frac{7}{5} = 1.4
\]

18. Find the domain, range, x- and y-intercept for the following functions. Do not graph.

\[3 \text{ a)} \ f(x) = (3x - 2)^2 + 4
\]
\[
D = \mathbb{R}
\]
\[
R = (-\infty, 4]
\]
\[
y - y_0 = (-3) \cdot 2^x + 4 = -\frac{3}{2} + 4 = \frac{5}{2}
\]
\[
x - y_0 \rightarrow 0 = (-2) \cdot 2^x + 4
\]
\[
(3) \cdot 2^x = 4
\]
\[
2^x = \frac{4}{3}
\]
\[
(x - 1) = \ln \left( \frac{4}{3} \right)
\]
\[
x = 1 + \frac{\ln \left( \frac{4}{3} \right)}{\ln 2}
\]

\[3 \text{ a)} \ g(x) = (-0.5)(\log_2 (x + 1)) + 3
\]
\[
D = ?
\]
\[
x + 1 > 0 \implies x > -1
\]
\[
\therefore D = (-1, \infty)
\]
\[
R = \mathbb{R}
\]
\[
y - y_0 = g(0) = -0.5 \cdot \log_2 1 + 3
\]
\[
= 3
\]
\[
x - x_0 \rightarrow 0 = (-0.5) \cdot \log_2 (x + 1) + 3
\]
\[
0.5 \cdot \log_2 (x + 1) = 3
\]
\[
\log_2 (x + 1) = 6
\]
\[
2^6 = x + 1
\]
\[
\therefore x = 63
A model for the number of bacteria in a culture after \( t \) hours is given by \( P(t) = P_0 e^{kt} \). After 3 hours it is observed that 300 bacteria are present. After 10 hours it is observed that 1000 bacteria are present. 

[1] a) What is the initial number of bacteria?

\[
\begin{align*}
300 &= P_0 e^{3kh} \\
1000 &= P_0 e^{10kh} \\
\frac{1000}{300} &= e^{7kh} \\
\frac{10}{3} &= e^{7kh} \\
e^{kh} &= \left(\frac{10}{3}\right)^{1/7} \\
k &= \frac{\ln(10/3)}{7} \\
P_0 &= \frac{300}{e^{3kh}} = \frac{300}{\left(\frac{10}{3}\right)^{3/7}} = 173
\end{align*}
\]

The initial number of bacteria is 173.

[1] b) What is the doubling period (the time after which the number of bacteria is doubled) of these bacteria?

\[
\begin{align*}
2P_0 &= P_0 e^{D} \\
2 &= e^{D} \\
\ln 2 &= D \\
kD &= \ln 2 \\
k &= \frac{\ln 2}{D} \\
k &= 4.03 \text{ hours}
\end{align*}
\]

The doubling period is 4.03 hours.

[1] c) How many bacteria will be after 15 hours?

\[
\begin{align*}
t &= 15h \\
P &= P_0 e^{15kh} = 173 \times \left(\frac{10}{3}\right)^{15/7} \\
P &= 2362
\end{align*}
\]

After 15 hours there will be 2362 bacteria.

[1] d) What is the time after which the number of bacteria is 2000?

\[
\begin{align*}
2000 &= 173 \times e^{kh} \\
\ln \frac{2000}{173} &= kh \\
t &= \frac{\ln \left(\frac{2000}{173}\right)}{\frac{1}{7} \ln(10/3)} \\
t &= 14.03 \text{ hours}
\end{align*}
\]

After 14.03 hours there will be 2000 bacteria.

[1] e) What is the growth rate (percentage per hour) of these bacteria?

\[
\begin{align*}
\eta &= \frac{P(4h) - P_0}{P_0} = \frac{P_0 e^{4kh} - P_0}{P_0} \\
&= e^{4kh} - 1 = \left(\frac{10}{3}\right)^{4/7} - 1 = 18.77\% \\
\eta &= 18.77\% \\
The growth rate is 18.77% per hour.