### A Exponential Growth and Decay

Exponential Growth and Decay may be modelled by a function of the form:

\[ A(t) = A_0 b^{kt} \]

where
- \( t \) is time
- \( A_0 \) is the initial amount
- \( A(t) \) is the amount at time \( t \)
- \( b \) is the base
- \( k \) is a constant depending on the application

#### Ex 1.

Let \( f(x) = 4(2^{4x-1}) \).

a) Write this relation in the form \( f(x) = Ab^x \).

b) Write this relation in the form \( f(x) = Ab^x \).

c) Write this relation in the form \( f(x) = A(10^Bx) \).

d) Write this relation in the form \( f(x) = A(3^Bx) \).

### B Common Ratio

The values of the exponential growth function form a geometric sequence:

\[ \frac{y_2}{y_1} = \frac{y_3}{y_2} = \ldots = \frac{y_{n+1}}{y_n} \]

where
- \( y_1 = f(x_1), \ y_2 = f(x_2), \ldots \)

and \( x_1, \ x_2, \ldots \) are in arithmetic sequence.

#### Ex 2.

Show that the following relation is exponential.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
</tr>
<tr>
<td>4</td>
<td>54</td>
</tr>
<tr>
<td>5</td>
<td>162</td>
</tr>
</tbody>
</table>

### C Developing Exponential Growth Formula

If \( r \) is the increasing rate per year, use:

\[ A(t) = A_0 (1 + r)^t \]

Indeed \( A(1) = A_0 (1 + r)^1 = A_0 + rA_0 \).

If, over a period \( T \), the amount is increasing \( b \) times, use:

\[ A(T) = A_0 b^T \]

Indeed \( A(T) = A_0 b^T = bA_0 \).

#### Ex 3.

For each case, find an exponential function that model best the situation.

a) The value of a house is increasing by 7% per year.

b) The number of bacteria is triple every two hours.

c) The number of bacteria is double every five hours.
# D Developing Exponential Decay Formula

Exponential Decay may be modelled by a function of the form:

\[ A(t) = A_0 b^{kt} \]

or by

\[ A(t) = A_0 \left( \frac{1}{2} \right)^{\frac{t}{H}} \]

where \( H \) is half-life

\[ A(H) = \frac{A_0}{2} \]

or by

\[ A(t) = A_0 (1-r)^t \]

where \( r \) is the decreasing rate per year

\[ A(1) = A_0 (1-r)^1 = A_0 - rA_0 \]

# Ex 4. For each case, find an exponential function that model best the situation.

a) The value of a car is decreasing by 5% per year.

b) The half-life of a radioactive source is 81 years.

c) The luminosity decreases 3 times for each 10cm of depth.

# E Sound Level

\[ L = 10\log \left( \frac{I}{I_0} \right) \]

where

- \( L \) is the soundness (sound level) in decibels
- \( I \) is the intensity of the sound
- \( I_0 = 10^{-12} W/m^2 \) is a constant (intensity of the sound at the threshold of hearing)

Note. \( L_2 - L_1 = 10\log(I_2 / I_1) \)

# Ex 5. A whisper has a sound level of 15 dB and a rock concert has a sound level of 120 dB. How many more intense is the rock concert in comparison to a whisper?

# F Earthquake Magnitude

\[ M = \log \left( \frac{A}{A_0} \right) \]

where

- \( M \) is the magnitude of the earthquake
- \( A \) is the amplitude (intensity) of the earthquake
- \( A_0 \) is a constant

Note. \( M_2 - M_1 = \log(A_2 / A_1) \)

# Ex 6. In 2017, in Mexico, two earthquakes happened with a magnitude more than 7. One happened on September 7 and had a magnitude of 8.2 and the other happened on September 19 and had a magnitude of 7.1. How many times was the amplitude of the September 7 earthquake greater in comparison to the amplitude of the September 19 earthquake?

# G pH Scale

\[ pH = -\log n \]

where

- \( pH \) is a number measuring acidity/alkalinity of a substance
- \( n = [H^+] \) is the concentration of hydrogen ions

# Ex 7. Lemon juice has a pH of 2.5 and milk has a pH of 9. How many times the hydrogen ions are more concentrated in lemon juice than in milk.

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**Reading:** Nelson Textbook, Pages 493-499

**Homework:** Nelson Textbook, Page 499 #1-5, 8, 10, 14, 15, 17, 18