8.6 Solving Logarithmic Equations

A Exponential-Logarithmic Conversion The following two expressions are equivalent: $b^x = y \quad \Leftrightarrow \quad x = \log_b y$ $b > 0, b \neq 1; \quad y > 0, x \in R$	Ex 1. Solve for x. Verify restrictions. See the solution below a) $\log x = 0$ b) $\ln x = 1$ c) $\log_2(x-1) = 0$ d) $\log(x^2+1) = 1$ e) $\ln(\log x) = 0$
B One-to-one property The logarithmic function is *one-to-one* function. So: $\log_b x = \log_b y \quad \Leftrightarrow \quad x = y$ $b > 0, b \neq 1, x > 0, y > 0$	Ex 2. Solve for x. Verify restrictions. See the solution below a) $\log(x-1) = \log(2x+1)$ b) $\ln(x+1) - \ln(x-1) = 3$ c) $\log_2(x-1) + \log_2(x+2) - \log_2(2x-1) = 1$ d) $\log x = 1 - \log(x-3)$
C Technology	Ex 3. Use technology (scientific calculator) to find the solution of the following equation to the nearest thousandth. $\ln x + \log x = 5$ Skip This

Ex 4. Solve for x.

a) $\log_2(x^2) = (\log_2 x)^2$

Skip This

b) $(\log x)^2 - \log x^2 + 1 = 0$

Skip This

c) $\log_{x-1}(4x-4) = 2$

Skip This

d) $\log_2(x-4) + \log_{\sqrt{2}}(x^3-2) + \log_{0.5}(x-4) = 20$

Skip This

e) $4\log\sqrt{x} - 5\sqrt{\log x} = 3$

Skip This

D Inequalities and Logarithms

If $b > 1$ then:

$$\log_b x > \log_b y \quad \Leftrightarrow \quad x > y$$

If $b < 1$ then:

$$\log_b x > \log_b y \quad \Leftrightarrow \quad x < y$$

Skip This

Ex 5. Solve each inequality. **Skip This**

a) $\log x > 1$

b) $\ln(x-1) < 0$

c) $\log_{0.5}(2x+1) \geq 2$

d) $\log_{0.1} x^2 \leq -1$

Reading: Nelson Textbook, Pages 487-490
Homework: Nelson Textbook, Page 491: #4acf, 5ace, 6, 7ad, 8, 10, 12, 16, 18, 19, 20

Ex1

a) $\log x = 0 \Rightarrow \log_{10} x = 0 \Rightarrow 10^0 = x$

$\boxed{\therefore x = 1}$; $x > 0$ (true)

b) $\ln x = 1 \Rightarrow \log_e x = 1 \Rightarrow e^1 = x$

$\therefore x = e$; $x > 0$ (true)

c) $\log_2 (x-1) = 0 \Rightarrow 2^0 = x - 1 \Rightarrow$

$\boxed{\therefore x = 2}$; $x - 1 > 0$ (true)

d) $\log (x^2+1) = 1 \Rightarrow \log_{10} (x^2+1) = 1 \Rightarrow$

$10^1 = x^2 + 1 \Rightarrow x^2 = 9 \Rightarrow \boxed{\therefore x = \pm 3}$

$x^2 + 1 = 9 + 1 = 10 > 0$ (true)

e) $\ln (\log (x)) = 0 \Rightarrow \log_{10} (\log x) = 0 \Rightarrow$

$10^0 = \log x \Rightarrow \log_{10} x = 1 \Rightarrow$

$10^1 = x \Rightarrow \boxed{\therefore x = 10}$

Restrictions

$x > 0$ (true)

$\log x = \log 10 = 1 > 0$ (true)

Ex 2a Solve: $\log(x-1) = \log(2x+1)$

$\Rightarrow x - 1 = 2x + 1 \Rightarrow x = -2$

Restrictions: $x - 1 > 0 \Rightarrow -2 - 1 > 0$ (false)

$\boxed{\therefore \text{ no solution}}$

Ex 2b $\ln(x+1) - \ln(x-1) = 3 \Rightarrow$

$\cancel{\ln} \frac{x+1}{x-1} = \cancel{\ln} e^3 \Rightarrow$

power law

Quotient law

$\frac{x+1}{x-1} = e^3 \Rightarrow x + 1 = xe^3 - e^3 \Rightarrow$

$1 + e^3 = x(e^3 - 1) \Rightarrow x = \underbrace{\frac{e^3+1}{e^3-1}}_{\text{exact form}} \underbrace{\simeq 1.105}_{\text{approximate form}}$

Restrictions

$x + 1 > 0 \Rightarrow 1.105 + 1 > 0$ (true)

$x - 1 > 0 \Rightarrow 1.105 - 1 > 0$ (true)

$$\boxed{\therefore \quad x = \frac{e^3+1}{e^3-1} \simeq 1.105}$$

Ex2 c

$$\log_2(x-1) + \log_2(x+2) - \log_2(2x-1) = 1$$

$$\cancel{\log_2} \frac{(x-1)(x+2)}{2x-1} = \cancel{\log_2} 2$$

Power Law

Product and Quotient Laws

$$\frac{(x-1)(x+2)}{2x-1} = 2 \Rightarrow x^2 + x - 2 = 4x - 2 \Rightarrow$$

$$x^2 - 3x = 0 \Rightarrow x(x-3) = 0 \Rightarrow$$

$x = 0$ or $x = 3$

Restrictions:

	$x=0$	$x=3$
	$x=0$	$x=3$
$x-1>0$	false	true
$x+2>0$	true	true
$2x-1>0$	false	true

reject $x=0$

$$\boxed{\therefore x = 3}$$

Ex 2d

$$\log x = 1 - \log(x-3)$$

$$\log x + \log(x-3) = 1$$

$$\log_{10}[x(x-3)] = \log_{10} 10$$

$$x(x-3) = 10$$

$$x^2 - 3x - 10 = 0$$

$$x = \frac{3 \pm \sqrt{9+40}}{2} = \frac{3 \pm 7}{2} \begin{cases} \nearrow 5 \\ \searrow -2 \end{cases}$$

$$x = 5 \text{ or } x = -2$$

Restrictions	$x=5$	$x=-2$
$x > 0$	true	false
$x - 3 > 0$	true	false

reject $x=-2$

$$\boxed{\therefore \quad x = 5}$$

