### 8.6 Solving Logarithmic Equations

#### A Exponential-Logarithmic Conversion
The following two expressions are equivalent:

\[ b^x = y \quad \Leftrightarrow \quad x = \log_b y \]

\[ b > 0, b \neq 1; \quad y > 0, x \in R \]

**Ex 1.** Solve for \( x \). Verify restrictions.

| a) \( \log x = 0 \) |
| b) \( \ln x = 1 \) |
| c) \( \log_2 (x-1) = 0 \) |
| d) \( \log(x^2 + 1) = 1 \) |
| e) \( \ln(\log x) = 0 \) |

#### B One-to-one property
The logarithmic function is a one-to-one function. So:

\[ \log_b x = \log_b y \quad \Leftrightarrow \quad x = y \]

\[ b > 0, b \neq 1, x > 0, y > 0 \]

**Ex 2.** Solve for \( x \). Verify restrictions.

| a) \( \log(x-1) = \log(2x+1) \) |
| b) \( \ln(x+1) - \ln(x-1) = 3 \) |
| c) \( \log_2 (x-1) + \log_2 (x+2) - \log_2 (2x-1) = 1 \) |
| d) \( \log x = 1 - \log(x-3) \) |

#### C Technology
**Ex 3.** Use technology (scientific calculator) to find the solution of the following equation to the nearest thousandth.

\[ \ln x + \log x = 5 \]
Ex 4. Solve for $x$.

a) $\log_2(x^2) = (\log_2 x)^2$

b) $(\log x)^2 - \log x^2 + 1 = 0$

c) $\log_{x-1}(4x-4) = 2$

d) $\log_2(x-4) + \log_2(x^2-2) + \log_{0.5}(x-4) = 20$

e) $4\log\sqrt{x} - 5\sqrt{\log x} = 3$

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**D Inequalities and Logarithms**

If $b > 1$ then:

$$\log_b x > \log_b y \iff x > y$$

If $b < 1$ then:

$$\log_b x > \log_b y \iff x < y$$

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Ex 5. Solve each inequality.

a) $\log x > 1$

b) $\ln(x+1) < 0$

c) $\log_{0.5} (2x+1) \geq 2$

d) $\log_{0.1} x^2 \leq -1$