8.4 Laws of Logarithms

A Power Law $\log_b x^n = n\log_b x, \quad x > 0$	Ex 1. Prove the power law. See the proof below.
Ex 2. Use the power law to simplify. a) $\log_2 64 = \log_2 2^6 = 6$ b) $\log 0.0001 = \log_{10} 10^{-4} = -4$ c) $\ln\sqrt[3]{e^2} = \log_e e^{2/3} = \frac{2}{3}$	Ex 3. Simplify. State restrictions. a) $\log x^2 = 2\log x \quad ; \quad x > 0$ b) $\ln\sqrt[5]{x^4} = \ln x^{4/5} = \frac{4}{5}\ln x \quad ; \quad x > 0$ c) $\log_2 x^{1/3} = \frac{1}{3}\log_2 x \quad ; \quad x > 0$
B Product Law $\log_b(xy) = \log_b x + \log_b y; \quad x, y > 0$	Ex 4. Prove the product law. See the proof below.
Ex 5. Use the power law and the product law to expand. State restrictions. a) $\log(10xy) = \log 10 + \log(xy) = 1 + \log x + \log y$ $x, y > 0$ b) $\log_2(16a^2b^3) = \log_2 16 + \log_2 a^2 + \log_2 b^3$ $= 4 + 2\log_2 a + 3\log_2 b$ $a, b > 0$	Ex 6. Write as a single logarithm. Evaluate, if possible. State restrictions. a) $\log 20 + \log 50 + \log 0.1 = \log(20\times 50\times 0.1)$ $= \log 100 = \log_{10} 10^2 = 2$ b) $2\log_5 10 + \frac{1}{2}\log_5 \frac{1}{16} = \log_5 10^2 + \log_5\left(\frac{1}{16}\right)^{1/2}$ $= \log_5 100 + \log_5 \frac{1}{4} = \log_5 \frac{100}{4} = \log_5 25 = \log_5 5^2 = 5$ c) $2\ln x + 3\ln y + \frac{1}{3}\ln z = \ln x^2 + \ln y^3 + \ln z^{1/3}$ $= \ln(x^2y^3z^{1/3}) = \ln(x^2y^3\sqrt[3]{z})$ $x, y, z > 0$
C Quotient Law $\log_b \frac{x}{y} = \log_b x - \log_b y; \quad x, y > 0$	Ex 7. Prove the quotient law. See the proof below.

Ex 8. Expand using the logarithms laws. State restrictions. a) $\log\frac{2}{3} = \log 2 - \log 3$ $\ln\frac{a^2\sqrt{b}}{c^3} = \ln(a^2\sqrt{b}) - \ln c^3 = \ln a^2 + \ln\sqrt{b} - 3\ln c$ b) $= 2\ln a + \frac{1}{2}\ln b - 3\ln c$ $a, b, c > 0$	Ex 9. Write as a single logarithm. Evaluate, if possible. State restrictions. a) $\log_3 18 - \log_3 2 = \log_3\frac{18}{2} = \log_3 9 = \log_3 3^2 = 2$ $\frac{1}{3}\ln a - \frac{2}{3}\ln b + 2\ln 3 = \ln a^{1/2} - \ln b^{2/3} + \ln 3^2$ b) $= \ln\frac{9\sqrt{a}}{\sqrt[3]{b^2}}$; $a, b > 0$
D Change of Base Law $\log_a x = \frac{\log_b x}{\log_b a}$ Skip this.	Ex 10. Prove the change of base law. Skip this.
Ex 11. Prove the following formulas. Skip this. a) $\ln 10 = \frac{1}{\log e}$ b) $\log_b x = \frac{\log x}{\log b} = \frac{\ln x}{\ln b}$	Ex 12. Use technology to evaluate. Skip this. a) $\log_2 3$ b) $\log_{\sqrt{3}}\sqrt[5]{2}$
E Change of Base Formula for Exponential Function $a^x = b^{x\log_b a}$ Skip this.	Ex 13. Prove the change of base for exponential function law. Skip this.
Ex 14. Prove the following relations. Skip this. a) $x = e^{\ln x}$ b) $x = \ln e^x$ c) $a^x = e^{x\ln a}$ d) $x = b^{\log_b x}$	Ex 15. Change to the base 10. Skip this. a) $f(x) = 2^x$ b) $f(x) = 3e^{2x} - 1$

Reading: Nelson Textbook, Pages 469-474
Homework: Nelson Textbook, Page 475: #4ab, 5, 6abc, 7d, 8, 9af, 10af, 11af, 12, 15, 17

Ex1. Prove the power law.

$$\underbrace{\log_b x^n}_{\alpha} = n \underbrace{\log_b x}_{\beta} \quad ; x > 0$$

a) Let $\alpha = \log_b x^n$ and $\beta = \log_b x$

b) change to exponential form

$$x^n = b^\alpha \quad (1) \qquad x = b^\beta \quad (2)$$

c) Substitute (2) into (1)

$$(b^\beta)^n = b^\alpha$$

d) use the exponent rules

$$b^{\beta n} = b^\alpha$$

e) use one-to-one property

$$\alpha = n\beta$$

f) substitute α and β

$$\log_b x^n = n \log_b x$$

Ex 4. Prove the product law.

$$\underbrace{\log_b(xy)}_{\alpha} = \underbrace{\log_b x}_{\beta} + \underbrace{\log_b y}_{r} \quad ; x, y > 0$$

a) Let $\alpha = \log_b(xy)$; $\beta = \log_b x$; $r = \log_b y$

b) change to exponential form

$$\underset{①}{xy = b^{\alpha}} \; ; \; \underset{②}{x = b^{\beta}} \; ; \; \underset{③}{y = b^{r}}$$

c) Substitute ② and ③ into ①

$$(b^{\beta})(b^{r}) = b^{\alpha}$$

d) Use the exponent rules

$$b^{\beta + r} = b^{\alpha}$$

e) Use one-to-one property

$$\alpha = \beta + r$$

f) substitute α, β and r

$$\log_b(xy) = \log_b x + \log_b y$$

Ex 7. Prove the quotient law.

$$\underbrace{\log_b \frac{x}{y}}_{\alpha} = \underbrace{\log_b x}_{\beta} - \underbrace{\log_b y}_{r} \quad ; x, y > 0$$

a) Let $\alpha = \log_b \frac{x}{y}$; $\beta = \log_b x$; $r = \log_b y$

b) change to exponential form

$$\frac{x}{y} = b^{\alpha} \ (1) \quad ; \quad x = b^{\beta} \ (2) \quad ; \quad y = b^{r} \ (3)$$

c) Substitute (2) and (3) into (1)

$$\frac{b^{\beta}}{b^{r}} = b^{\alpha}$$

d) Use the exponent rules

$$b^{\beta - r} = b^{\alpha}$$

e) Use one-to-one property

$$\alpha = \beta - r$$

f) substitute α, β, and r

$$\log_b \frac{x}{y} = \log_b x - \log_b y$$