## 8.1 Exploring the Logarithmic Function

### A Logarithmic Functions

*Logarithmic function* is defined as the *inverse* function of the exponential function.

So, if $y = f(x) = b^x$ then $x = f^{-1}(y) = \log_b y$.

Note. $b$ is the *base* of the exponential function and the *base* of the logarithmic function.

The following expressions are equivalent:

$$y = b^x \quad \Leftrightarrow \quad x = \log_b y$$

Reading: “log(arithm) (to the) base $b$ of $y$”

Note. $y = b^x$ is called the *exponential form* and $x = \log_b y$ is called the *logarithmic form*.

Ex 1. Convert the exponential form to the logarithmic form.

a) $8 = 2^3$

b) $10000 = 10^4$

c) $0.00001 = 10^{-5}$

d) $1024 = 2^{10}$

Ex 2. Convert the logarithmic form to the exponential form.

a) $4 = \log_2 16$

b) $3 = \log_{10} 1000$

c) $-4 = \log_{10} 0.0001$

d) $4 = \log_5 625$

### B Domain, Range and other Restrictions

The domain and the range of the exponential function:

$$b^x : (-\infty, +\infty) \to (0, +\infty)$$

are interchanged to obtain the *domain and the range* of the logarithmic function:

$$\log_b x : (0, +\infty) \to (-\infty, +\infty)$$

The base $b$ satisfies the same restrictions from the exponential function:

$$b > 0, b \neq 1$$

Ex 3. Find if the following expressions are well defined.

a) $\log_{\sqrt{2}} 1$

b) $\log_1 2$

c) $\log_{\frac{1}{2}} \sqrt{2}$

d) $\log_2 (-10)$

e) $\log_{-2} 3$

### C Basic Formulas

Ex 4. Use the exponential-logarithmic conversion to prove the following basic formulas:

a) $\log_b 1 = 0$

b) $\log_b b = 1$

c) $\log_b \frac{1}{b} = -1$

e) $\log_{\frac{1}{b}} b = -1$

e) $\log_b b^n = n$

**D Basic Equations**

Ex 5. Solve each equation by converting it to the exponential form.

a) $x = \log_5 25$

b) $x = \log_4 1$

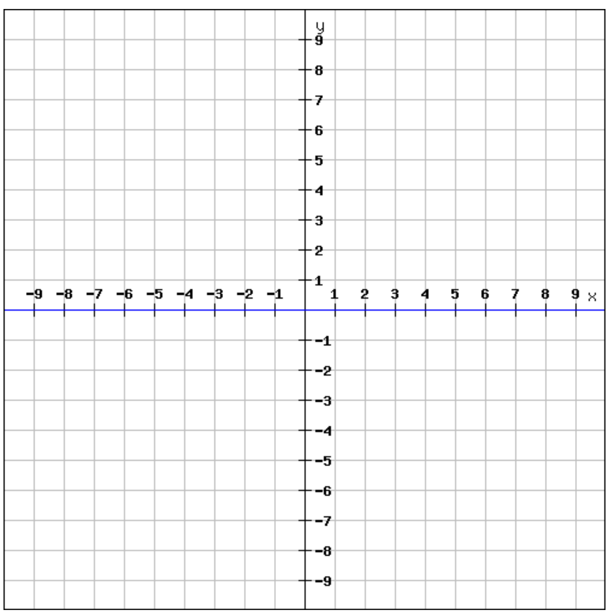
c) $\log_x 16 = 2$

d) $\log_x 3 = \frac{1}{2}$

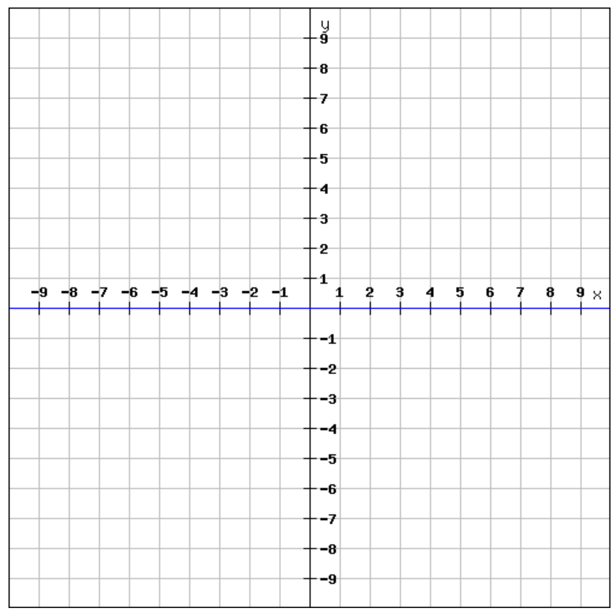
e) $\log_2 x = -2$

**E Graph of the Logarithmic Function**

Ex 6. Graph on the grid provided below both $f(x) = 2^x$ and $f^{-1}(x) = \log_2 x$.




Ex 7. Graph on the grid provided below both $f(x) = 0.5^x$ and $f^{-1}(x) = \log_{0.5} x$.




**F Characteristics of the Logarithmic Function**

Ex 8. Use the graphs obtained at example 6 and 7 to conclude about the following characteristics of the logarithmic function.

- Domain:
- Range:
- x-intercepts
- y-intercepts:
- Increasing/Decreasing:
- Horizontal Asymptotes:
- Vertical Asymptotes:
- Continuity:
- One-to-one:
- Key Points:

**Reading**: Nelson Textbook, Pages 448-450
**Homework**: Nelson Textbook, Page 451: #1ac 3, 4, 6, 8, 9

Ex 1. Convert the exponential form to the logarithmic form.

a) $8 = 2^3$ $\quad 3 = \log_2 8$

b) $10000 = 10^4$ $\quad 4 = \log_{10} 10{,}000$

c) $0.00001 = 10^{-5}$ $\quad -5 = \log_{10} 0.00001$

d) $1024 = 2^{10}$ $\quad 10 = \log_2 1024$

Ex 2. Convert the logarithmic form to the exponential form.

a) $4 = \log_2 16$ $\quad 2^4 = 16$

b) $3 = \log_{10} 1000$ $\quad 10^3 = 1000$

c) $-4 = \log_{10} 0.0001$ $\quad 10^{-4} = 0.0001$

d) $4 = \log_5 625$ $\quad 5^4 = 625$

Ex 3. Find if the following expressions are well defined.

a) $\log_{\sqrt{2}} 1$ ✓

b) $\log_1 2$ $\quad b \neq 1$ ✗

c) $\log_{\frac{1}{2}} \sqrt{2}$ ✓

d) $\log_2(-10)$ $\quad x > 0$ ✗

e) $\log_{-2} 3$ $\quad b > 0$ ✗

**C Basic Formulas**

Ex 4. Use the exponential-logarithmic conversion to prove the following basic formulas:

a) $\boxed{\log_b 1 = 0}$ $\Longleftrightarrow$ $b^0 = 1$

b) $\boxed{\log_b b = 1}$ $\Longleftrightarrow$ $b^1 = b$

c) $\boxed{\log_b \frac{1}{b} = -1}$ ? $\Longleftrightarrow$ $b^{-1} = \frac{1}{b}$

e) $\boxed{\log_{\frac{1}{b}} b = -1}$ ? $\Longleftrightarrow$ $\left(\frac{1}{b}\right)^{-1} = b$

e) $\boxed{\log_b b^n = n}$ $\quad b^n = b^n$

**D Basic Equations**

Ex 5. Solve each equation by converting it to the exponential form.

a) $x = \log_5 25$ $\Longleftrightarrow$ $5^x = 25 = 5^2$

$\therefore x = 2$

b) $x = \log_4 1$ $\Longleftrightarrow$ $4^x = 1 = 4^0$

$\therefore x = 0$

c) $\log_x 16 = 2$ $\Longleftrightarrow$ $x^2 = 16 \Rightarrow x = \pm 4;\ x > 0$

$\therefore x = 4$

d) $\log_x 3 = \frac{1}{2}$ $\quad (x)^{1/2} = 3 \Rightarrow x = 9$

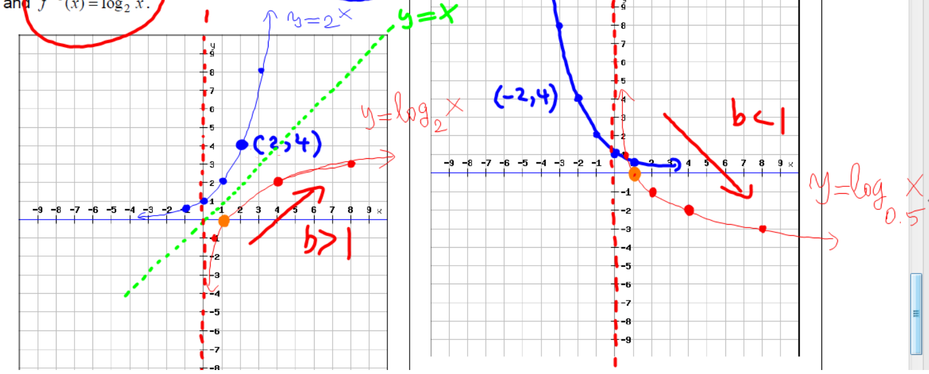
$\therefore x = 9$

e) $\log_2 x = -2$ $\quad 2^{-2} = x$

$\therefore x = \frac{1}{4}$

Ex 6. Graph on the grid provided below both $f(x)=2^x$ and $f^{-1}(x)=\log_2 x$.

$y=0.5^x$




**F Characteristics of the Logarithmic Function**

Ex 8. Use the graphs obtained at example 6 and 7 to conclude about the following characteristics of the logarithmic function.

- Domain: $(0,\infty)$
- Range: $\mathbb{R}$
- x-intercepts $=1$
- y-intercepts: none

b>1 b<1

- Increasing/Decreasing:
- Horizontal Asymptotes: none
- Vertical Asymptotes: $x=0$ (y-axis)
- Continuity: yes
- One-to-one: yes
- Key Points: $(1,0)$; $(b,1)$; $\left(\frac{1}{b},-1\right)$

**Reading**: Nelson Textbook, Pages 448-450
**Homework**: Nelson Textbook, Page 451: #1ac 3, 4, 6, 8, 9