

## 8.1 Exploring the Logarithmic Function

<p><b>A Logarithmic Functions</b></p> <p><i>Logarithmic function</i> is defined as the <i>inverse</i> function of the exponential function.</p> <p>So, if <math>y = f(x) = b^x</math> then <math>x = f^{-1}(y) = \log_b y</math>.</p> <p>Note. <math>b</math> is the <i>base</i> of the exponential function and the <i>base</i> of the logarithmic function.</p> <p>The following expressions are equivalent:</p> $y = b^x \Leftrightarrow x = \log_b y$ <p>Reading: "log(arithm) (to the) base <math>b</math> of <math>y</math> "</p> <p>Note. <math>y = b^x</math> is called the <i>exponential form</i> and <math>x = \log_b y</math> is called the <i>logarithmic form</i>.</p>	<p>Ex 1. Convert the exponential form to the logarithmic form.</p> <p>a) <math>8 = 2^3</math></p> <p>b) <math>10000 = 10^4</math></p> <p>c) <math>0.00001 = 10^{-5}</math></p> <p>d) <math>1024 = 2^{10}</math></p> <p>Ex 2. Convert the logarithmic form to the exponential form.</p> <p>a) <math>4 = \log_2 16</math></p> <p>b) <math>3 = \log_{10} 1000</math></p> <p>c) <math>-4 = \log_{10} 0.0001</math></p> <p>d) <math>4 = \log_5 625</math></p>
<p><b>B Domain, Range and other Restrictions</b></p> <p>The domain and the range of the exponential function:</p> $b^x : (-\infty, +\infty) \rightarrow (0, +\infty)$ <p>are interchanged to obtain the <i>domain and the range</i> of the logarithmic function:</p> $\log_b x : (0, +\infty) \rightarrow (-\infty, +\infty)$ <p>The base <math>b</math> satisfies the same restrictions from the exponential function:</p> $b > 0, b \neq 1$	<p>Ex 3. Find if the following expressions are well defined.</p> <p>a) <math>\log_{\sqrt{2}} 1</math></p> <p>b) <math>\log_1 2</math></p> <p>c) <math>\log_{\frac{1}{2}} \sqrt{2}</math></p> <p>d) <math>\log_2 (-10)</math></p> <p>e) <math>\log_{-2} 3</math></p>
<p><b>C Basic Formulas</b></p> <p>Ex 4. Use the exponential-logarithmic conversion to prove the following basic formulas:</p> <p>a) <math>\log_b 1 = 0</math></p> <p>b) <math>\log_b b = 1</math></p>	<p>c) <math>\log_b \frac{1}{b} = -1</math></p> <p>e) <math>\log_{\frac{1}{b}} b = -1</math></p> <p>e) <math>\log_b b^n = n</math></p>

**D Basic Equations**

Ex 5. Solve each equation by converting it to the exponential form.

a)  $x = \log_5 25$

b)  $x = \log_4 1$

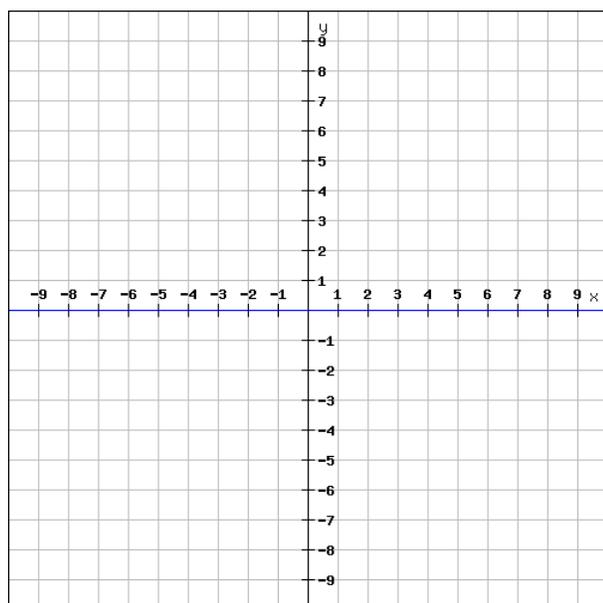
c)  $\log_x 16 = 2$

d)  $\log_x 3 = \frac{1}{2}$

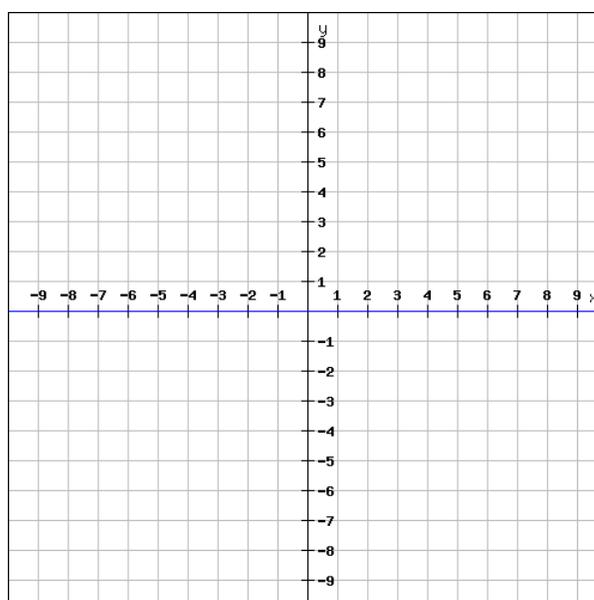
e)  $\log_2 x = -2$

**E Graph of the Logarithmic Function**

Ex 6. Graph on the grid provided below both  $f(x) = 2^x$  and  $f^{-1}(x) = \log_2 x$ .



Ex 7. Graph on the grid provided below both  $f(x) = 0.5^x$  and  $f^{-1}(x) = \log_{0.5} x$ .

**F Characteristics of the Logarithmic Function**

Ex 8. Use the graphs obtained at example 6 and 7 to conclude about the following characteristics of the logarithmic function.

- Domain:
- Range:
- x-intercepts
- y-intercepts:
- Increasing/Decreasing:
- Horizontal Asymptotes:
- Vertical Asymptotes:
- Continuity:
- One-to-one:
- Key Points:

**Reading:** Nelson Textbook, Pages 448-450

**Homework:** Nelson Textbook, Page 451: #1ac 3, 4, 6, 8, 9