8.0 Exponential Function

### A Exponents Laws

The following relations are called exponent laws:

\[(ab)^n = a^n b^n\]
\[\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}\]
\[a^m a^n = a^{m+n}\]
\[\frac{a^m}{a^n} = a^{m-n}\]
\[(a^m)^n = a^{mn}\]
\[a^{-n} = \frac{1}{a^n}\]

Note. If the exponent is a **rational number** then:

\[\frac{p}{q} = \sqrt[q]{a^p}, \quad q \text{ is a positive integer}\]

### B Definition

**Exponential function** \(y = f(x)\) is defined by:

\[y = f(x) = b^x\]

where:

- \(b\) is the **base**
- \(x\) is the **argument**
- \(y\) is the **value** of the exponential function

Note. For each value of the base \(b\) a different exponential function is defined.

### Ex 1. Find the base of the following exponential functions.

a) \(y = 2^{3x}\)
b) \(y = \frac{1}{10^x}\)

### C Restrictions

To avoid complex number values, the base \(b\) is positive: \(b > 0\).

Indeed, if the base is negative, then the value of the exponential function is a complex number.

For example: \((-4)^{\frac{1}{2}} = \sqrt{-4} = \pm 2i\).

In order the exponential function to be a one-to-one function, the base must not be one: \(b \neq 1\).

Indeed, if the base is one, then \(1^x = 1\), which is not a one-to-one function and does not have an inverse function.

So, in conclusion the restrictions are:

\(b > 0, b \neq 1\) or \(b \in (0,1) \cup (1,\infty)\)

### Ex 2. Determine if the exponential function is well defined (satisfies the restrictions).

a) \(y = (-2)^x\)
b) \(y = -2^x\)
c) \(y = 3^x\)
d) \(y = 2^{-3x}\)
e) \(y = \sqrt{3^x}\)

### D The Graph of the Exponential Function

Ex 3. Graph on the same grid.

a) \(y = 2^x\)

b) \(y = \left(\frac{1}{2}\right)^x\)

Note. \(y = \left(\frac{1}{2}\right)^x = 2^{-x}\), so the graphs are symmetric with respect to the y-axis.
### E Characteristics of the Exponential Function

Ex 4. Use the graphs obtained at Ex 3 to find the following characteristics of the exponential function.

- **Domain:**
- **Range:**
- **x-intercepts:**
- **y-intercepts:**
- **Increasing/Decreasing:**
- **Horizontal Asymptotes:**
- **Vertical Asymptotes:**
- **Continuity:**
- **One-to-one:**
- **Key Points:**

### F Transformations

\[ g(x) = A b^{B(x-C)} + D \]

Note. The equation of the horizontal asymptote is: \( y = D \)

### Ex 5. Use transformations to graph the following exponential functions.

#### a) \( y = -3^{-x+1} + 9 \)

#### b) \( y = -4 + 2 \cdot 0.5^{x+2} \)

### G One-to-one property

The exponential function is a one-to-one function. Therefore:

\[ b^x = b^y \iff x = y \]