MATHEMATICS GRADE 12 MHF4U, ADVANCED FUNCTIONS **TEST Chapter 4** **TRIGONOMETRIC FUNCTIONS** Teacher: Mr Gugoiu	Date Oct 12, 2017 Name SOLUTIONS	$\overline{\quad}$ = 43

1. Match the functions from the left side with a graph from the right side. Some functions have no corresponding graph. [K/U 4 marks]

A) [—]
$q(x) = 2\cos x$

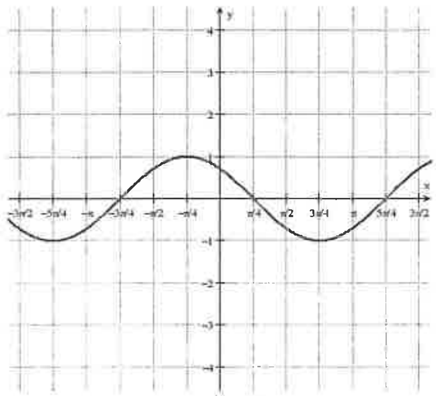
B) [III]
$g(x) = 2 - \sin x$

C) [—]
$h(x) = 2\sin(0.5x)$

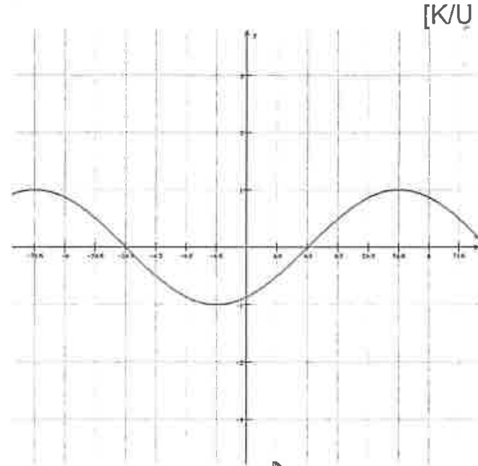
D) [II]
$p(x) = \sin(x - \pi/3)$

E) [I]
$k(x) = \cos(x + \pi/4)$

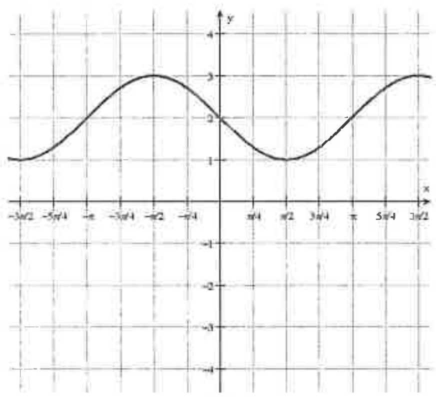
F) [IV]
$f(x) = \cos(2x)$



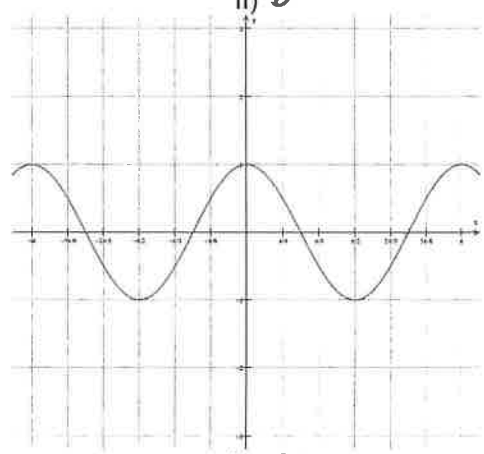
I) E



II) D



III) B



IV) F

2. Do the required conversions: [K/U 2 marks]

a) $35° = ?rad$ $= 35\frac{\pi}{180} \approx 0.61 = \frac{7\pi}{6}$

b) $2.5rad = ?°$ $= 2.5\frac{180°}{\pi} \cong 143.24° = \frac{450°}{\pi}$

3. Find the exact value. Show your work. [K/U 4 marks]

[2] a) $\sin\frac{43\pi}{6} = \sin\left(8\pi - \frac{5\pi}{6}\right) = \sin\left(-\frac{5\pi}{6}\right) = -\sin\frac{5\pi}{6}$

$= -\sin\frac{\pi}{6} = -\frac{1}{2}$

[2] b) $\sin\frac{5\pi}{8} > 0$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\sin\frac{5\pi}{8} = \sqrt{\frac{1-\cos\frac{5\pi}{4}}{2}} = \sqrt{\frac{1+\cos\frac{\pi}{4}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2}}{2}}{2}}$

$\therefore \sin\frac{5\pi}{8} = \frac{\sqrt{2+\sqrt{2}}}{2}$

4. Let $y = -2\cos\left(2x + \frac{\pi}{2}\right) + 3$.

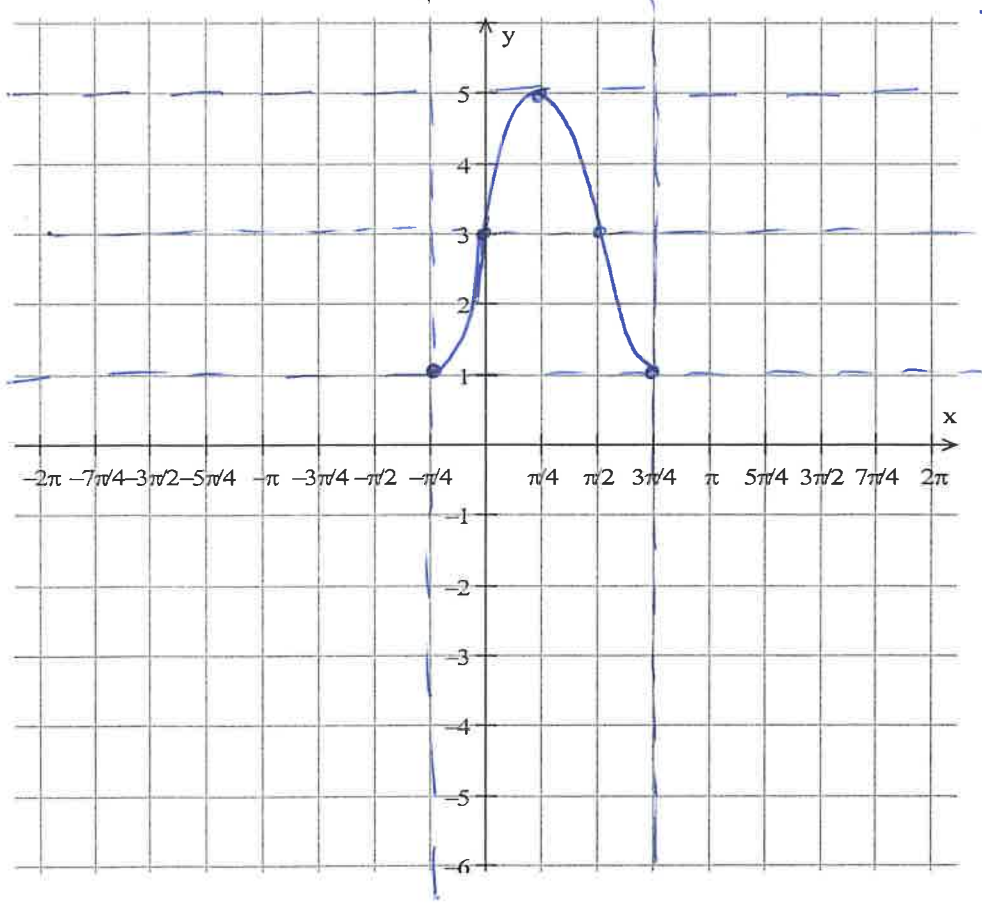
$y = -2\cos\left[2\left(x + \frac{\pi}{4}\right)\right] + 3$

[CA 6 marks]

[C 3] a) Describe the transformations involved.

- reflection in the x-axis
- vertical expansion by 2
- horizontal compression by $\frac{1}{2}$
- horizontal shift left by $\frac{\pi}{4}$
- vertical shift up by 3

[A 3] b) Graph the function on the grid below by using a method at your convenience.



$y_{axis} = 3$

$A = 2$

$y_{max} = 5$

$y_{min} = 1$

$PS = -\frac{\pi}{4}$

$T = \frac{2\pi}{2} = \pi$

10

5. The angle α has the terminal arm in the second quadrant and $\tan\alpha = -3$. Explain how you would find the exact values of $\sin\alpha$ and $\cos\alpha$. [C 4 marks]

$$\tan\alpha = \frac{3}{-1} = \frac{y}{x}$$

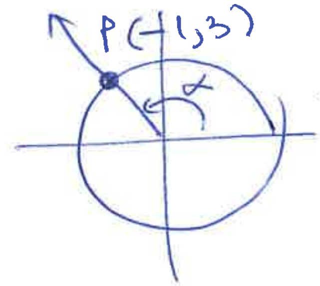
$$R = \sqrt{x^2 + y^2}$$

$$= \sqrt{(-1)^2 + 3^2}$$

$$= \sqrt{10}$$

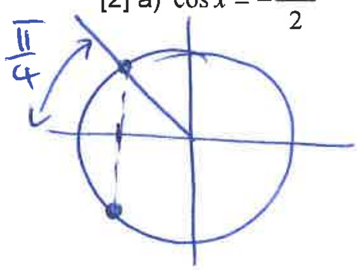
$$\sin\alpha = \frac{y}{R} = \frac{3}{\sqrt{10}}$$

$$\cos\alpha = \frac{x}{R} = \frac{-1}{\sqrt{10}}$$



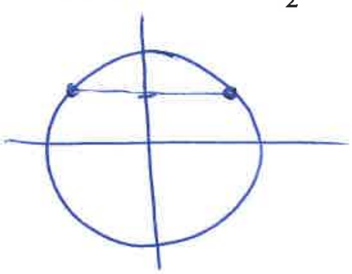
6. Solve the following trigonometric equations: [K/U 5 marks]

[2] a) $\cos x = -\frac{\sqrt{2}}{2}$



$$\therefore x = \pi \pm \frac{\pi}{4} + k(2\pi)$$

[3] b) $\sin(3x - \pi) = \frac{1}{2}, 0 \le x \le 2\pi$



$$3x - \pi = \frac{\pi}{6} + k(2\pi) \quad \text{or} \quad 3x - \pi = \frac{5\pi}{6} + k(2\pi)$$

$$x = \frac{7\pi}{18} + k\frac{2\pi}{3} \quad \text{or} \quad x = \frac{11\pi}{18} + k\frac{2\pi}{3}$$

$$x \in \left\{\frac{7\pi}{18}, \frac{19\pi}{18}, \frac{31\pi}{18}, \frac{11\pi}{18}, \frac{23\pi}{18}, \frac{35\pi}{18}\right\}$$

7. Prove the following trigonometric identities: [A 4 marks]

[2] a) $\frac{\cos x + \sin x}{\cos x - \sin x} = \tan\left(x + \frac{\pi}{4}\right)$

$$RS = \tan\left(x + \frac{\pi}{4}\right) = \frac{\sin\left(x + \frac{\pi}{4}\right)}{\cos\left(x + \frac{\pi}{4}\right)} = \frac{(\sin x)\frac{\sqrt{2}}{2} + (\cos x)\frac{\sqrt{2}}{2}}{(\cos x)\frac{\sqrt{2}}{2} - (\sin x)\frac{\sqrt{2}}{2}}$$

$$= \frac{\cos x + \sin x}{\cos x - \sin x} = LS$$

[2] b) $\cos(a+b)\cos(a-b) = \cos^2 b - \sin^2 a$

$$LS = \cos(a+b)\cos(a-b) = (\cos a\cos b - \sin a\sin b)(\cos a\cos b + \sin a\sin b)$$

$$= \cos^2 a\cos^2 b - \sin^2 a\sin^2 b = \cos^2 a\cos^2 b - (1 - \cos^2 a)(1 - \cos^2 b)$$

$$= \cancel{\cos^2 a\cos^2 b} - 1 + \cos^2 a + \cos^2 b - \cancel{\cos^2 a\cos^2 b}$$

$$= \cos^2 b - (1 - \cos^2 a) = \cos^2 b - \sin^2 a = RS$$

13

8. Express $\cos(3x)$ in terms of $\cos x$. [A 4 marks]

$\cos 3x = \cos(x+2x) = \cos x \cos 2x - \sin x \sin 2x$

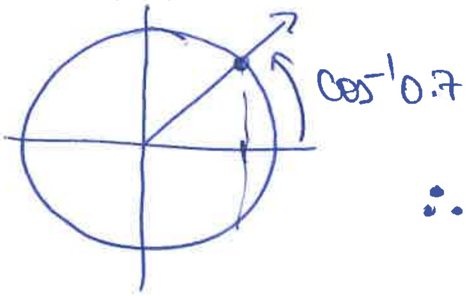
$= \cos x(2\cos^2 x - 1) - 2\sin^2 x \cos x$

$= 2\cos^3 x - \cos x - 2\cos x(1-\cos^2 x)$

$= 4\cos^3 x - 3\cos x$

9. Solve for α. [A 3 marks]

$\cos(\alpha/2) = 0.7 \quad ; \quad 0 \le \alpha \le 2\pi$



$\frac{\alpha}{2} = \pm\cos^{-1} 0.7 + k(2\pi)$

$\alpha = \pm 1.59 + k(4\pi)$

$\therefore \alpha \in \{1.59\}$

10. If $\cos x = -3/7$, find the exact value of $\cos 4x$. Show your work. [T 3 marks]

$\cos 4x = 2\cos^2 2x - 1$

$= 2(2\cos^2 x - 1)^2 - 1$

$= 2\left[2\left(\frac{-3}{7}\right)^2 - 1\right]^2 - 1 = 2\left[\frac{18}{49} - 1\right]^2 - 1$

$= \frac{1922}{2401} - 1 = -\frac{479}{2401}$

11. Solve for θ. [T 4 marks]

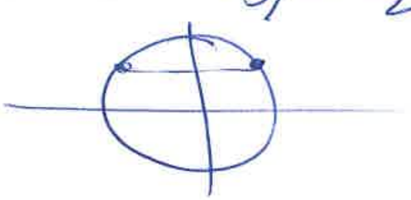
$\sin\theta - \sqrt{3}\cos\theta = \sqrt{2}$

$\cancel{\sqrt{2}} \quad \cancel{\sqrt{2}} \quad \cancel{\sqrt{2}}$

$\cancel{\sin\left(\theta - \frac{\pi}{4}\right) = 1}$

$(\sin\theta)\frac{1}{2} - \frac{\sqrt{3}}{2}\cos\theta = \frac{\sqrt{2}}{2}$

$\sin\left(\theta - \frac{\pi}{3}\right) = \frac{\sqrt{2}}{2}$



$\theta - \frac{\pi}{3} = \frac{\pi}{4} + k(2\pi)$ or

$\theta - \frac{\pi}{3} = \frac{3\pi}{4} + k(2\pi)$

$\therefore \theta = \frac{7\pi}{12} + k(2\pi)$

or

$\theta = \frac{13\pi}{12} + k(2\pi)$

14