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7.6 Solving Quadratic Trigonometric Equations

A Quadratic Equations
Some trigonometric equations lead to a quadratic equation.

Ex 1. Solve the following trigonometric equation.

$2\sin x = 3 + 2\csc x$ $\quad 0 \le x \le 2\pi$

$2\sin x = 3 + \frac{2}{\sin x}$ $\quad \sin x \neq 0$, $\boxed{s = \sin x}$

$2s = 3 + \frac{2}{s} \quad | \cdot s$

$2s^2 = 3s + 2$

$2s^2 - 3s - 2 = 0$

$s = \frac{3 \pm \sqrt{9 - 4(2)(-2)}}{4} = \frac{3 \pm 5}{4} \nearrow 2, \searrow -\frac{1}{2}$

$\sin x = 2$ (reject) or $\sin x = -\frac{1}{2}$

$\hookrightarrow x = \sin^{-1}\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$

$\pi + \frac{\pi}{6} = \frac{7\pi}{6}$ $\quad 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\boxed{\therefore x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\boxed{x = \frac{3\pi}{2} \pm \frac{\pi}{3} + k(2\pi)}$ (all solutions)

Ex 2. Solve the following trigonometric equation.

$\sin x + 2\cos^2 x = 1$ $\quad 0 \le x \le 2\pi$

$\cos^2 x = 1 - \sin^2 x$

$\sin x + 2(1 - \sin^2 x) = 1$

Let $s = \sin x$

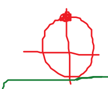
$s + 2(1 - s^2) = 1$

$s + 2 - 2s^2 = 1$

$2s^2 - s - 1 = 0$

$s = \frac{1 \pm \sqrt{1+8}}{4} = \frac{1 \pm 3}{4} \nearrow 1, \searrow -\frac{1}{2}$

$\sin x = 1$ or $\sin x = -\frac{1}{2}$



$\boxed{\therefore x = \frac{\pi}{2}, \frac{7\pi}{6}, \frac{11\pi}{6}}$

$x = \frac{\pi}{2} + k(2\pi)$ or $x = \frac{3\pi}{2} \pm \frac{\pi}{3} + k(2\pi)$

Ex 3. Solve the following trigonometric equation.

$\cos\frac{x}{2} = 1 + \cos x$, $\quad 0 \le x \le 2\pi$

$\cos x = \cos\left(2 \cdot \frac{x}{2}\right) = 2\cos^2\frac{x}{2} - 1$

$\cos\frac{x}{2} = \cancel{1} + 2\cos^2\frac{x}{2} \cancel{-1}$

$\cos\frac{x}{2} = 2\cos^2\frac{x}{2}$

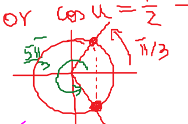
$\left(\cos\frac{x}{2}\right)\left(1 - 2\cos\frac{x}{2}\right) = 0$

$\cos\frac{x}{2} = 0$ or $\cos\frac{x}{2} = \frac{1}{2}$

$u = \frac{x}{2} \Rightarrow$ $0 \le u \le \pi$

$\cos u = 0$ or $\cos u = \frac{1}{2} \rightarrow u = \cos^{-1}\left(\frac{1}{2}\right) = \pi/3$

$2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$



$u = \frac{\pi}{2}, \cancel{\frac{3\pi}{2}}, \frac{\pi}{3}, \cancel{\frac{5\pi}{3}}$

$u = \frac{\pi}{2}, \frac{\pi}{3} \Rightarrow x = 2u \Rightarrow \boxed{\therefore x = \pi, \frac{2\pi}{3}}$

Check: $\cos\frac{\pi}{2} = 1 + \cos\pi$ (OK)

$\cos\frac{\pi}{3} = 1 + \cos\frac{2\pi}{3}$ (OK)

B Cubic Equations
Some trigonometric equations lead to a cubic equation.

Ex 4. Solve the following trigonometric equation.

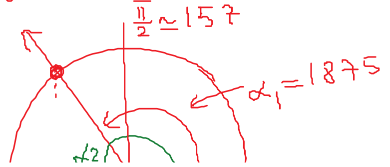
$$4\cos x \sin 2x + 3 = 4\cos^2 x + 6\sin x$$

Reading: Nelson Textbook, Pages 429-435
Homework: Nelson Textbook, Page 436: #5, 7, 8, 10, 18, 19, 20

Review

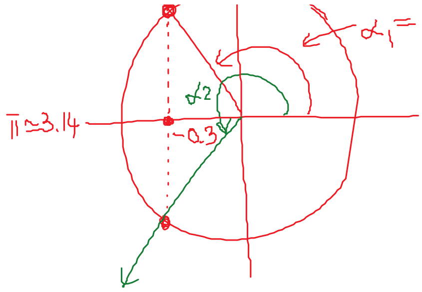
Ex 5 Solve $\cos\alpha = -0.3$, $0 \le x \le 2\pi$

$\alpha = \cos^{-1}(-0.3) \simeq 1.875$



$\alpha_2 = 2\pi - \alpha_1$
$= 2\pi - 1.875$
$\simeq 4.408$

$\therefore \alpha = 1.875, 4.408$



$\therefore \alpha = 1.875, 4.408$

EX 6

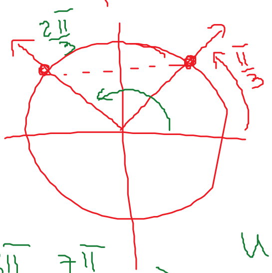
$\sin\left(2x - \frac{\pi}{4}\right) = \frac{\sqrt{3}}{2} \quad ; \quad 0 \le x \le 2\pi$

Let $u = 2x - \frac{\pi}{4} \Rightarrow -\frac{\pi}{4} \le u \le 2(2\pi) - \frac{\pi}{4}$

$\boxed{-\frac{\pi}{4} \le u \le \frac{15\pi}{4}}$ $\simeq 3.75\pi$

$\sin u = \frac{\sqrt{3}}{2}$

$u = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$



$u = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{8\pi}{3}, \frac{7\pi}{3} \Rightarrow$

$+2\pi$

$+2\pi$

$u = 2x - \frac{\pi}{4} \Rightarrow x = \frac{u + \frac{\pi}{4}}{2}$

$\therefore x = \frac{7\pi}{24}, \frac{11\pi}{24}, \frac{35\pi}{24}, \frac{31\pi}{24}$

$\frac{\frac{\pi}{3} + \frac{\pi}{4}}{2} = \frac{7\pi}{24}$

$\frac{\frac{2\pi}{3} + \frac{\pi}{4}}{2} = \frac{11\pi}{24}$

$\frac{\frac{8\pi}{3} + \frac{\pi}{4}}{2} = \frac{35\pi}{24}$

$\frac{\frac{7\pi}{3} + \frac{\pi}{4}}{2} = \frac{31\pi}{24}$