Printout

Friday, November 13, 2020 8:21 AM

7.5 Solving Linear Trigonometric Equations

A Elementary Trigonometric Equations

Use the unit circle to solve elementary trigonometric equations (see Ex 1).

Ex 1. Solve the following trigonometric equations.

a) $\sin x = 0$ $x = ?$ $R=1$

$\sin x = \frac{y_P}{R} = 0 \Rightarrow y_P = 0$

$\boxed{\therefore x = 0, \pi, 2\pi}$ if $0 \le x \le 2\pi$

$\therefore x = k\pi$, $k = 0, \pm1, \pm2, \ldots$ (general solutions)

b) $\sin x = 1 = y_P$ $0 \le x \le 2\pi$ $\pi/2$

$\therefore x = \frac{\pi}{2}$

$\therefore x = \frac{\pi}{2} + k(2\pi)$

c) $\sin x = -1$

$\therefore x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} + k(2\pi)$

d) $\cos x = 0 = x_P$ $\pi/2$ π $\frac{3\pi}{2}$

$\therefore x = \frac{\pi}{2}, \frac{3\pi}{2}$

$x = \frac{\pi}{2} + k\pi$

e) $\cos x = 1$

$x = 0, 2\pi$

$x = k(2\pi)$

f) $\cos = -1 = x_P$ π

$x = \pi$

$x = \pi + k(2\pi)$

g) $\tan x = 0 = \frac{y_P}{x_P}$ $y_P = 0$

$x = 0, \pi, 2\pi$

$x = k\pi$

h) $\tan x = 1 = \frac{y_P}{x_P}$ $x_P = y_P$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{4} + k\pi$

i) $\tan x = -1$

$x = \frac{3\pi}{4}, \frac{7\pi}{4}$

$x = \frac{3\pi}{4} + k\pi$

$-\pi/4$

$\hookrightarrow x = \tan^{-1}(-1) = -\frac{\pi}{4}$ $\xrightarrow{+\pi} 3\pi/4$, $\xrightarrow{+2\pi} 7\pi/4$

B Simple Trigonometric Equations

Use the related angle to find the solutions of simple trigonometric equations (see Ex 2).

Ex 2. Solve the following trigonometric equations.

a) $\sin x = \frac{1}{2} = y_P$ $\frac{5\pi}{6}$ $\pi/6$ $1/2$

$\hookrightarrow x = \sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{5\pi}{6}$

$x = \frac{\pi}{6} + k(2\pi)$ OR $x = \frac{5\pi}{6} + k(2\pi)$

b) $\sin x = -\frac{\sqrt{2}}{2}$ $\pi/4$ $-\frac{\pi}{4}$

$\hookrightarrow x = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

$x = \frac{5\pi}{4}, \frac{7\pi}{4}$

$x = \frac{5\pi}{4} + k(2\pi)$ OR $x = \frac{7\pi}{4} + k(2\pi)$

c) $\cos x = -\frac{1}{2} = x_P$ $\pi/3$ $2\pi/3$ $\frac{\pi}{3}$ $\frac{4\pi}{3}$

$x = \cos^{-1}\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$

$\therefore x = \frac{2\pi}{3}, \frac{4\pi}{3}$

$x = \frac{2\pi}{3} + k(2\pi)$ OR $x = \frac{4\pi}{3} + k(2\pi)$

d) $\cos x = \frac{\sqrt{3}}{2} = x_P$ $\frac{11\pi}{6}$ $\pi/6$ $\pi/6$

$x = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

$\therefore x = \frac{\pi}{6}, \frac{11\pi}{6}$

$x = \pm\frac{\pi}{6} + k(2\pi)$

e) $\tan x = \sqrt{3}$ $\frac{4\pi}{3}$ $\pi/3$

$x = \tan^{-1}\sqrt{3} = \frac{\pi}{3}$

$x = \frac{\pi}{3}, \frac{4\pi}{3}$

$x = \frac{\pi}{3} + k\pi$

f) $\tan x = -\frac{1}{\sqrt{3}}$ $\frac{5\pi}{6}$ $-\frac{\pi}{6}$

$x = \tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\frac{\pi}{6}$

$\therefore x = \frac{5\pi}{6}, \frac{11\pi}{6}$

$x = -\frac{\pi}{6} + k\pi$

g) $\sin x = \cos x$ $\pi/4$

$\frac{\sin x}{\cos x} = 1 = \tan x$

$x = \frac{\pi}{4}, \frac{5\pi}{4}$

$x = \frac{\pi}{4} + k\pi$

C Factoring

Some trigonometric equations can be solved by factoring.

$-\frac{\pi}{3} + \pi$ $-\frac{\pi}{3} + 2\pi$ $-\frac{\pi}{3}$

$x = 0, \pi, \frac{2\pi}{3}, \frac{5\pi}{3}$

$x = k\pi$ OR $x = -\frac{\pi}{3} + k\pi$

Ex 3. Solve the following trigonometric equations.

a) $\sin x \cos x = 0$

$\sin x = 0$ OR $\cos x = 0$

$x = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$x = k\frac{\pi}{2}$

b) $\sqrt{3}\tan x + \tan^2 x = 0$

$(\tan x)(\sqrt{3} + \tan x) = 0$

$\tan x = 0$ OR $\tan x = -\sqrt{3}$

Note Find the solutions in the interval $0 \le x \le 2\pi$

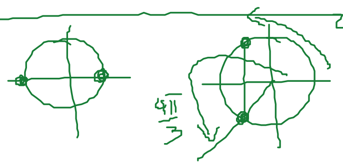
D Trigonometric Identities

Some trigonometric equations can be solved by using trigonometric identities.

$u = \frac{\pi}{4}, \frac{3\pi}{4}$

$x = u - \frac{\pi}{4}$

$\boxed{\therefore\ x = 0, \frac{\pi}{2}}$



$2\pi/3$

$\frac{4\pi}{3}$

$\therefore\ x = 0, \pi, 2\pi, \frac{2\pi}{3}, \frac{4\pi}{3}$

Ex 4. Solve the following trigonometric equations.

a) $\sin x + \cos x = 1$ (not easy !)

$(\sin x)\underbrace{\frac{1}{\sqrt{2}}}_{\cos\frac{\pi}{4}} + \underbrace{\frac{1}{\sqrt{2}}}_{\sin\frac{\pi}{4}}\cos x = \frac{1}{\sqrt{2}}$

$\sin\underbrace{\left(x + \frac{\pi}{4}\right)}_{u} = \frac{1}{\sqrt{2}}$

$u = x + \frac{\pi}{4}$

$0 \le x \le 2\pi$

$\frac{\pi}{4} \le u \le 2\pi + \frac{\pi}{4} = \frac{9\pi}{4}$

$\sin u = \frac{1}{\sqrt{2}}$

b) $\sin 2x + \sin x = 0$

$2\sin x\cos x + \sin x = 0$

$(\sin x)(2\cos x + 1) = 0$

$\sin x = 0$ OR $\cos x = -\frac{1}{2}$

$0 \le x \le 2\pi$

$\hookrightarrow x = \cos^{-1}\left(-\frac{1}{2}\right)$

$= \frac{2\pi}{3}$

E Restricted Solutions

Some trigonometric equations may have solutions restricted to specific intervals.

$\sin u = \frac{1}{2}$

$u = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}, \frac{25\pi}{6}, \frac{29\pi}{6}$

$+2\pi$ $+2\pi$ $+4\pi$ $+4\pi$

$x = \frac{u}{3}$

$\therefore\ x = \frac{\pi}{18}, \frac{5\pi}{18}, \frac{13\pi}{18}, \frac{17\pi}{18}, \frac{25\pi}{18}, \frac{29\pi}{18}$

Check: $2\sin\left(3 \cdot \frac{\pi}{18}\right) - 1 \overset{?}{=} 0$

Ex 5. Solve the following trigonometric equations.

a) $2\sin 3x - 1 = 0, \quad 0 \le x \le 2\pi$

$\sin\underbrace{(3x)}_{u} = \frac{1}{2}$

$u = 3x$

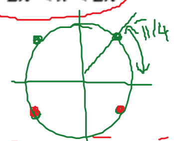
$0 \le x \le 2\pi$

$0 \le u \le 6\pi$

b) $2\sin^2 x - 1 = 0, \quad -2\pi < x < 2\pi$

$\sin^2 x = \frac{1}{2}$

$\sin x = \pm\frac{1}{\sqrt{2}}$



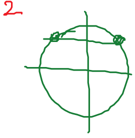
$\therefore\ x = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}, \pm\frac{5\pi}{4}, \pm\frac{7\pi}{4}$

c) $4\sin x\cos x = \sqrt{3}, \quad 0 \le x \le 2\pi$

$2(2\sin x\cos x) = \sqrt{3}$

$\sin(2x) = \frac{\sqrt{3}}{2}$

$\sin u = \frac{\sqrt{3}}{2}$



$u = 2x$

$0 \le u \le 4\pi$

$u = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{7\pi}{3}, \frac{8\pi}{3}$

$+2\pi$ $+2\pi$

$x = \frac{u}{2}$

$\therefore\ x = \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$

Reading: Nelson Textbook, Pages 419-426
Homework: Nelson Textbook, Page 427: #6, 9, 10, 13, 14, 17, 18

© 2011 Iulia & Teodoru Gugoiu - Page 2 of 2