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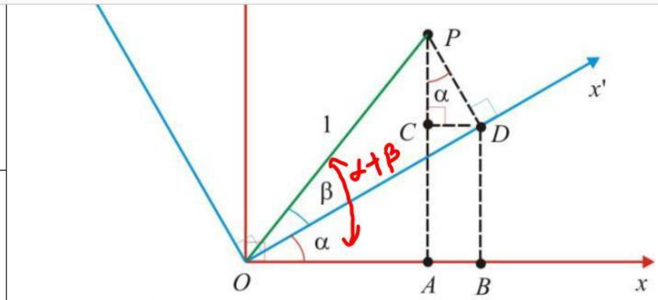
$\ldots$in α cos β + cos α sin β

$\ldots$cos α cos β − sin α sin β

$\ldots\dfrac{\tan\alpha + \tan\beta}{\ldots - \tan\alpha\tan\beta}$

$\ldots$ormulas for $\sin(\alpha+\beta)$ and

$\ldots$ addition formula for $\tan(\alpha+\beta)$.



$\sin(\alpha+\beta) = AP = AC + CP = BD + PD\cos\alpha$

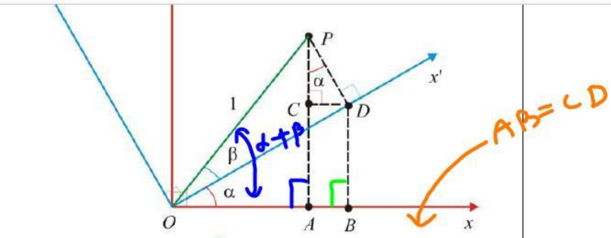
$= OD\sin\alpha + \sin\beta\cos\alpha$

$= \underline{\cos\beta}\,\underline{\sin\alpha} + \underline{\sin\beta}\,\underline{\cos\alpha}$

8.50 x 11.00 in

$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta \quad ✓$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

Ex 1. Use the addition formulas for $\sin(\alpha+\beta)$ and $\cos(\alpha+\beta)$ to prove the addition formula for $\tan(\alpha+\beta)$.



$$\cos(\alpha+\beta)=\frac{OA}{1}=OB-AB$$
$$=OD\cos\alpha-PD\sin\alpha$$
$$=\cos\beta\cos\alpha-\sin\beta\sin\alpha \quad ✓$$

$$\tan(\alpha+\beta)=\frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)}=\frac{\sin\alpha\cos\beta+\cos\alpha\sin\beta}{\cos\alpha\cos\beta-\sin\alpha\sin\beta}$$

$$=\frac{\dfrac{\sin\alpha\cancel{\cos\beta}}{\cos\alpha\cancel{\cos\beta}}+\dfrac{\cancel{\cos\alpha}\sin\beta}{\cancel{\cos\alpha}\cos\beta}}{\dfrac{\cancel{\cos\alpha}\cancel{\cos\beta}}{\cancel{\cos\alpha}\cancel{\cos\beta}}-\dfrac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$

$\beta \to -\beta$

B Subtraction Formulas

The following relations are trigonometric identities called subtraction formulas:

$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta \checkmark$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\tan(\alpha-\beta)=\frac{\tan\alpha-\tan\beta}{1+\tan\alpha\tan\beta}$$

Ex 2. Replace in the addition formulas β by $-\beta$, then use the symmetry of the sine and cosine functions and the addition formula to prove the subtraction formulas.

$$\sin(\alpha-\beta)=\sin[\alpha+(-\beta)]$$
$$=\sin\alpha\cos(-\beta)+\cos\alpha\sin(-\beta)$$
$$=\sin\alpha\cos\beta-\cos\alpha\sin\beta \checkmark$$

Ex 3. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$

Ex 3. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas:

$$\sin\left(\frac{\pi}{2}-\theta\right)=\cos\theta$$
$$\cos\left(\frac{\pi}{2}-\theta\right)=\sin\theta$$
$$\tan\left(\frac{\pi}{2}-\theta\right)=\cot\theta$$

$$\to \sin\left(\frac{\pi}{2}-\theta\right)$$
$$=\sin\frac{\pi}{2}\cos\theta-\cancel{\cos\frac{\pi}{2}}^{\,0}\sin\theta$$
$$=\cos\theta$$

Ex 4. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas:

Ex 4. Use $\sin(\pi/2)=1$ and $\cos(\pi/2)=0$ to prove the following formulas: $$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$$ $$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$$ $$\tan\left(\frac{\pi}{2}+\theta\right)=-\cot\theta$$	$\cos\left(\frac{\pi}{2}+\theta\right)=\cancel{\cos\frac{\pi}{2}}^{\,0}\cos\theta-\sin\frac{\pi}{2}^{\,1}\sin\theta$ $=-\sin\theta$
Ex 5. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $$\sin(\pi-\theta)=\sin\theta$$ $$\cos(\pi-\theta)=-\cos\theta$$ $$\tan(\pi-\theta)=-\tan\theta$$	
Ex 6. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas:	

Ex 5. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $$\sin(\pi-\theta)=\sin\theta$$ $$\cos(\pi-\theta)=-\cos\theta$$ $$\tan(\pi-\theta)=-\tan\theta$$	$\tan(\pi-\theta)=\dfrac{\cancel{\tan\pi}^{\,0}-\tan\theta}{1+\cancel{\tan\pi}_{\,0}\tan\theta}$ $=-\tan\theta$
Ex 6. Use $\sin(\pi)=0$ and $\cos(\pi)=-1$ to prove the following formulas: $$\sin(\pi+\theta)=-\sin\theta$$ $$\cos(\pi+\theta)=-\cos\theta$$ $$\tan(\pi+\theta)=\tan\theta$$	
Ex 7. Use $\sin(3\pi/2)=-1$ and $\cos(3\pi/2)=0$ to prove the following formulas: $$\sin\left(\frac{3\pi}{2}-\theta\right)=-\cos\theta$$	

no decimals with radicals

$+ = \frac{\pi}{2}$

Ex 10. Find the exact value of $\sin\frac{5\pi}{12}$.

$$= \sin\left(\frac{\pi}{4}+\frac{\pi}{6}\right)$$

$$= \sin\frac{\pi}{4}\cos\frac{\pi}{6} + \cos\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2} = \frac{\sqrt{2}(\sqrt{3}+1)}{4}$$

Ex 11. Find the exact value of $\cos\frac{\pi}{12}$.

$$= \cos\left(\frac{\pi}{4}-\frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} + \sin\frac{\pi}{4}\sin\frac{\pi}{6}$$

$$= \frac{\sqrt{2}}{2}\,\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\,\frac{1}{2}$$

$$= \frac{\sqrt{2}(\sqrt{3}+1)}{4}$$

Ex 12. Find the exact value of $\tan\frac{7\pi}{12}$.

Ex 13. Graph $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$.

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Ex 12. Find the exact value of $\tan\frac{7\pi}{12}$.

Ex 13. Graph $f(x) = \frac{\sin x + \cos x}{\sin x - \cos x}$.

$$\cdot\frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}}$$

$$= \frac{(\sin x)\overbrace{\frac{1}{\sqrt{2}}}^{\cos\frac{\pi}{4}} + \overbrace{\frac{1}{\sqrt{2}}}^{\sin\frac{\pi}{4}}\cos x}{(\sin x)\underbrace{\frac{1}{\sqrt{2}}}_{\sin\frac{\pi}{4}} - \underbrace{\frac{1}{\sqrt{2}}}_{\cos\frac{\pi}{4}}\cos x} = \frac{\sin\left(x+\frac{\pi}{4}\right)}{-\cos\left(x+\frac{\pi}{4}\right)} = -\tan\left(x+\frac{\pi}{4}\right)$$

Ex 14. Prove that:

$$\cot(\alpha+\beta) = \frac{\cot\alpha\cot\beta - 1}{\cot\alpha + \cot\beta}$$

Ex 15. Prove that:

$$\csc(\alpha+\beta) = \frac{\csc\alpha\csc\beta}{\cot\alpha + \cot\beta}$$

$$= -\tan\left(x+\frac{\pi}{4}\right)$$