

6.4 Transformations of Trigonometric Functions

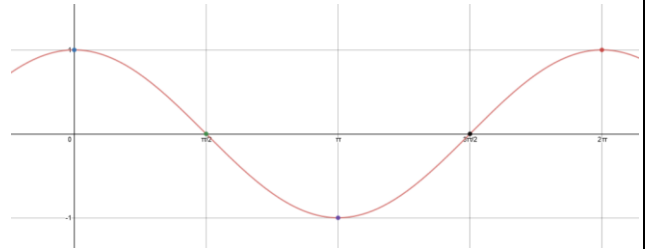
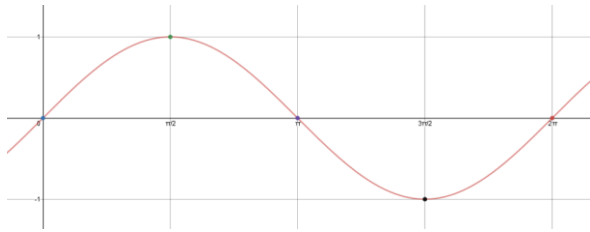
A Transformations

Let analyze the graphs of $y = a \sin[b(x-c)] + d$ or $y = a \cos[b(x-c)] + d$.

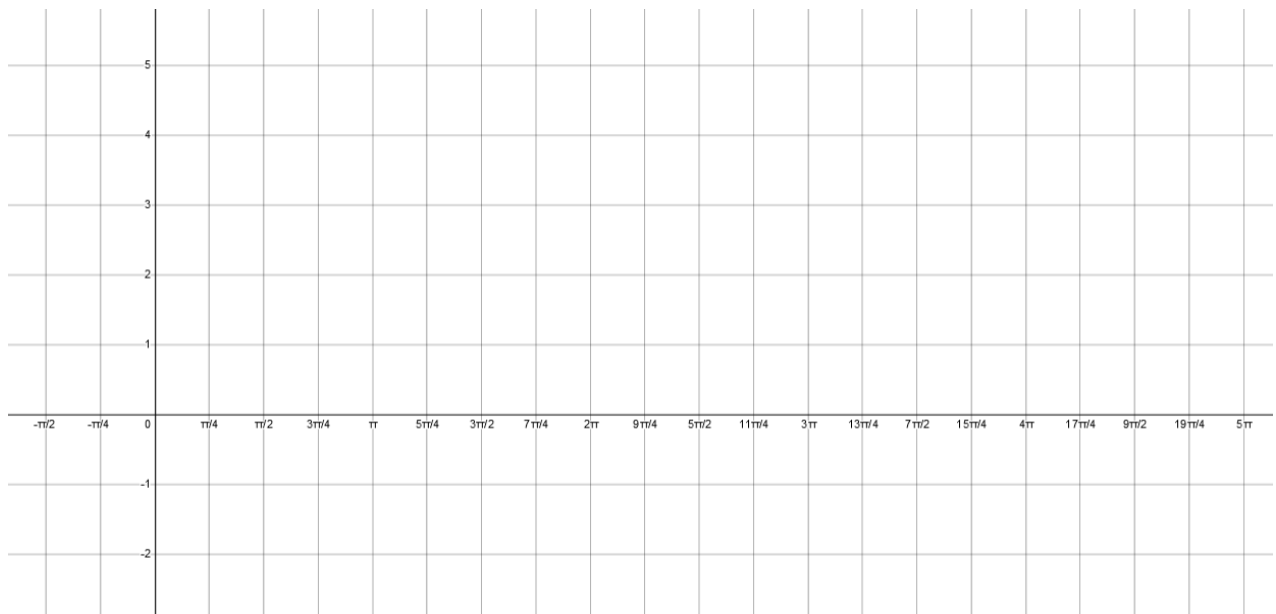
- The graph is scaled vertically by a factor of $|a|$. If $a < 0$ then the graph is also reflected in the x-axis.
- If $|a| > 1$ then the graph is expanded vertically and if $|a| < 1$ then the graph compressed vertically.
- The graph is translated vertically upward if $d > 0$ or downward if $d < 0$ by $|d|$ units.
- The equation of the axis is $y = d$.
- The graph of is scaled horizontally by a factor of $1/|b|$. If $b < 0$ then the graph is also reflected in the y-axis.
- The graph is translated horizontally to the right if $c > 0$ or to the left if $c < 0$ by $|c|$ units.
- The graph is horizontally compressed if $|b| > 1$ and is horizontally expanded if $|b| < 1$.
- The period of the graph is $T = \frac{2\pi}{|b|}$.

B Key Points

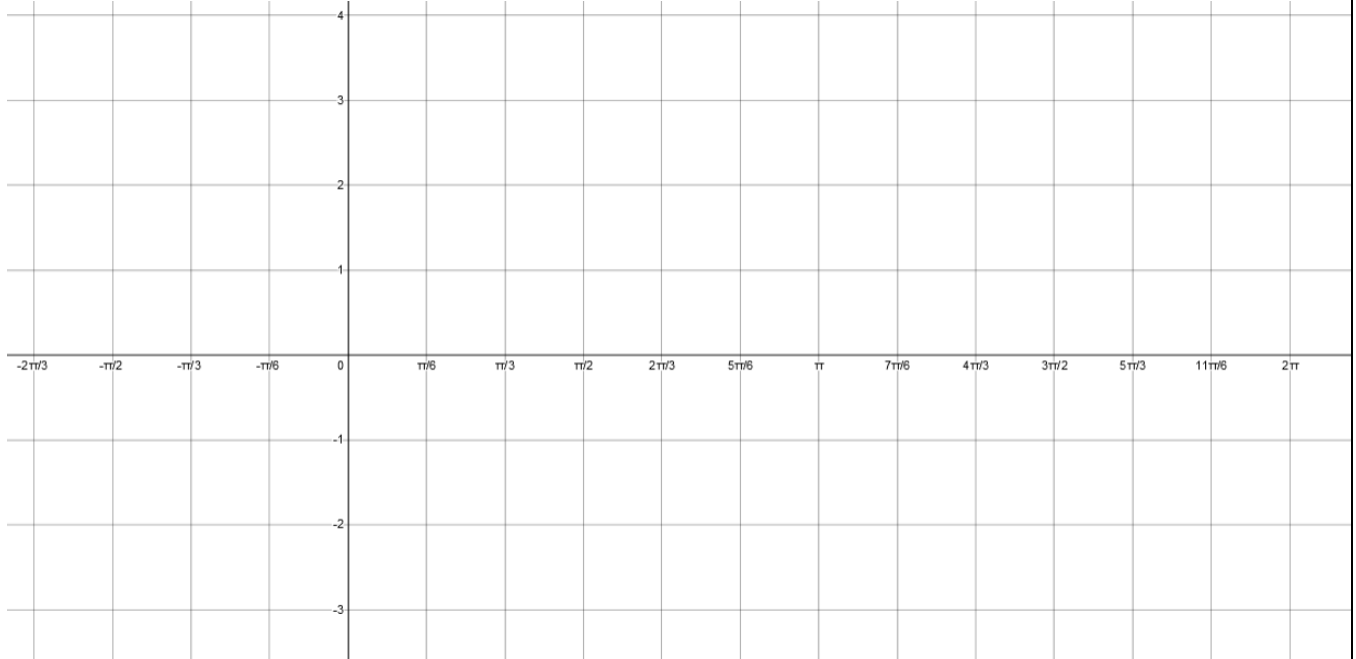
Use the five key points presented in the following diagrams to graph the sine and the cosine functions using transformations:



Ex 1. Use transformations to graph the function $y = -2 \sin(\pi/4 - x) + 3$ on the grid provided below.



Ex 2. Use transformations to graph the function $y = \cos(2x + \pi/3) - 2$ on the grid provided below.



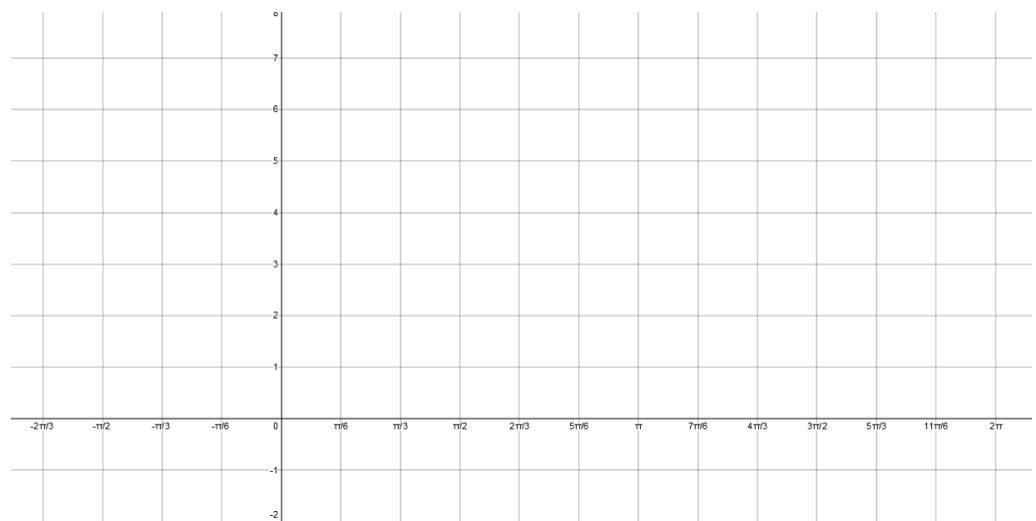
C Shortcut to the graphs of

$$y = a \sin[b(x-c)] + d \text{ or } y = a \cos[b(x-c)] + d$$

- The maximum value is $y_{\max} = d + |a|$
- The minimum value is $y_{\min} = d - |a|$
- The amplitude is $A = \frac{y_{\max} - y_{\min}}{2} = |a|$

- The equation of the axis is $y = \frac{y_{\max} + y_{\min}}{2} = d$
- The period is $T = \frac{2\pi}{|b|}$
- The phase shift is $PS = c$

Ex 3. Use the shortcut method to graph the function $y = -3\sin\left(0.5x + \frac{\pi}{12}\right) + 4$ on the grid provided below.



Reading: Nelson Textbook, Pages 337-343

Homework: Nelson Textbook, Page 344: #4, 6, 8, 9, 11, 14, 16