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Tuesday, November 3, 2020 8:04 AM

5.5 Solving Rational Inequalities

A Rational Inequalities

In order to solve a rational (nonlinear) inequality:

1. state *restrictions*
2. *move* all the terms to one side
3. find the *LCD* (Least Common Denominator) and simplify the rational expression
4. *factor* both the numerator and the denominator
5. find the *sign* of the rational expression by using a *sign chart*, *graph* or *critical numbers* method
6. *conclude* and verify if *restrictions* are satisfied

Ex 1. Is possible to use cross multiplication to solve the following inequality? Explain. Solve it by using four different methods.

$$\frac{1}{x} \le \frac{2}{x+1}$$

wrong!

$\Rightarrow x+1 \le 2x \Rightarrow 1 \le x \Rightarrow x \ge 1$

! Cross multiplication is NOT allowed for rational inequalities

$x \neq 0, -1$

$\frac{1}{x} - \frac{2}{x+1} \le 0$

$\frac{1(x+1)-2x}{x(x+1)} \le 0$

$\underbrace{\frac{1-x}{x(x+1)}}_{f(x)} \le 0$

Let use the Sign Chart Method

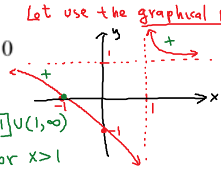
x		-1		0		1	
1−x	+	+	+	+	+	0	−
x	−	−	−	0	+	+	+
x+1	−	0	+	+	+	+	+
f(x)	+	und	−	und	+	0	−

$\therefore x \in (-1, 0) \cup [1, \infty)$

Ex 2. Solve the following inequalities:

a) $\overbrace{\frac{x+1}{x-1}}^{f(x)} \ge 0$

Let use the graphical method

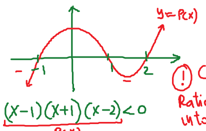


$x \neq 1$

$\therefore x \in (-\infty, -1] \cup (1, \infty)$

$x \le -1$ or $x > 1$

b) $\frac{x^2-1}{x-2} \le 0$



$x \neq 2$

$\frac{(x-1)(x+1)}{x-2} \le 0 \Rightarrow \underbrace{(x-1)(x+1)(x-2)}_{P(x)} < 0$

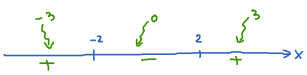
! Change the Rational Inequality into a Polynomial inequality

$\boxed{\frac{P}{Q} > 0 \Rightarrow PQ > 0}$

$\therefore x \in (-\infty, -1] \cup [1, 2)$

c) $\frac{x^2+1}{x^2-4} > 0$

Let use substitution method



$x \neq \pm 2$

$\underbrace{\frac{x^2+1}{(x-2)(x+2)}}_{f(x)} > 0$

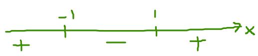
$f(-3) > 0$

$f(0) = -\frac{1}{4} < 0$

$f(3) = \frac{10}{5} > 0$

$\therefore x \in (-\infty, -2) \cup (2, \infty)$

d) $\frac{x^3+1}{x^3-1} < 0$



$\frac{(x+1)\cancel{(x^2-x+1)}}{(x-1)\cancel{(x^2+x+1)}} < 0$

$\frac{x+1}{x-1} < 0$

$\therefore x \in (-1, 1)$

Why not $x = -1$

$\frac{(-1)^3+1}{(-1)^3-1} = \frac{0}{-2} = 0 < 0$ false

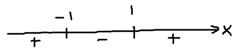
Ex 3. Solve the following inequalities:

a) $\frac{1}{x-1} > \frac{1}{x+1}$

$x \neq \pm 1$

$\frac{1}{x-1} - \frac{1}{x+1} > 0 \Rightarrow \frac{1(\cancel{x}+1) - 1(\cancel{x}-1)}{(x-1)(x+1)} > 0$

$\frac{2}{(x-1)(x+1)} > 0$



$\therefore x \in (-\infty, -1) \cup (1, \infty)$

b) $4x - \frac{5}{x-1} \ge 2x - 1$

$4x - 2x + 1 - \frac{5}{x-1} \ge 0$

$x \neq 1$

$2x + 1 - \frac{5}{x-1} \ge 0$

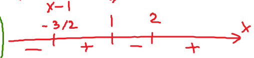
$\frac{(2x+1)(x-1)-5}{x-1} \ge 0$

$\frac{2x^2-2x+x-1-5}{x-1} \ge 0$

$\frac{2x^2-x-6}{x-1} \ge 0$

$\frac{1 \pm \sqrt{1+48}}{4} = \frac{1 \pm 7}{4} < \begin{matrix} 2 \\ -\frac{3}{2} \end{matrix}$

$\frac{(x-2)(2x+3)}{x-1} \ge 0$



$\therefore x \in [-\frac{3}{2}, 1) \cup [2, \infty)$

c) $\frac{4x+5}{x^2} \ge \frac{4}{x+5}$

$x \neq 0, -5$

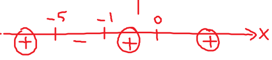
$\frac{4x+5}{x^2} - \frac{4}{x+5} \ge 0$

$\frac{(4x+5)(x+5)-4x^2}{x^2(x+5)} \ge 0$

$\frac{4x^2+20x+5x+25-4x^2}{x^2(x+5)} \ge 0$

$\frac{25(x+1)}{x^2(x+5)} \ge 0$

$\therefore x \in (-\infty, -5) \cup [-1, 0) \cup (0, \infty)$



d) $\frac{x}{2x-4} - \frac{3}{x-6} \le \frac{1}{2}$

$x \neq 2, 6$

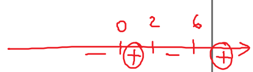
$\frac{x}{2(x-2)} - \frac{3}{x-6} - \frac{1}{2} \le 0$

$x-6$ $\quad 2(x-2)$ $\quad (x-6)(x-2)$

$\frac{x(x-6) - 3(2)(x-2) - 1(x-2)(x-6)}{\cancel{2}(x-2)(x-6)} \le 0$

$\frac{x^2-6x-6x+12-x^2+8x-12}{(x-2)(x-6)} \le 0$

$\frac{-4x}{(x-2)(x-6)} \le 0 \Rightarrow \frac{\cancel{-}4x}{(x-2)(x-6)} \ge 0$



$\therefore x \in [0, 2) \cup (6, \infty)$

$x = 7$

$\frac{7}{(5)(1)} > 0$

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Ex 4. Solve the following inequality:

$$\frac{x}{x-2}+\frac{1}{x-4}\geq\frac{2}{x^2-6x+8}$$

Ex 5. Solve the following inequality:

$$\frac{5}{x}\leq\frac{6}{x-1}<\frac{x}{x-2}$$

$x \neq 0, 1, 2$

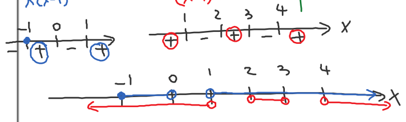
$\frac{5}{x}-\frac{6}{x-1}\leq 0$ and $\frac{6}{x-1}-\frac{x}{x-2}<0$

$\frac{-x-1}{x(x-1)}\leq 0$ and $\frac{6x-12-x^2+x}{(x-1)(x-2)}<0$

$\frac{x+1}{x(x-1)}\geq 0$ and $\frac{x^2-7x+12}{(x-1)(x-2)}>0$

$\therefore x \in [-1, 0) \cup (2, 3) \cup (4, \infty)$

$\frac{x+1}{x(x-1)}\geq 0$ and $\frac{(x-3)(x-4)}{(x-1)(x-2)}>0$



Ex 6. Solve the following inequality:

$$\frac{1}{|x-1|}-\frac{|x+1|}{x}\leq 2$$

Reading: Nelson Textbook, Pages 288-295
Homework: Nelson Textbook, Page 295: #4ab, 5acf, 7, 12, 13

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