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5.4 Solving Rational Equations

A Rational Equations

To solve a rational equation:

- State restrictions
- Multiply by the LCD (Least Common Denominator)
- Solve the polynomial equation (algebraically or by using technology)
- Verify restrictions
- Verify your solutions by substitution

Ex 1. Solve the following rational equation:

Restrictions: $x \neq 2, -3$

$$\frac{x}{x-2} - \frac{2}{x+3} = \frac{10}{x^2+x-6}$$

$(x+3)(x-2)$

• LCD $= (x+3)(x-2)$

$$\frac{x}{\cancel{x-2}} \cdot (x+3)\cancel{(x-2)} - \frac{2}{\cancel{x+3}} \cdot \cancel{(x+3)}(x-2) = \frac{10}{\cancel{(x+3)}\cancel{(x-2)}} \cdot \cancel{(x+3)}\cancel{(x-2)}$$

$x(x+3) - 2(x-2) = 10$

$x^2 + 3x - 2x + 4 - 10 = 0$

$x^2 + x - 6 = 0$

$(x+3)(x-2) = 0$

$x = -3$ or $x = 2$

$x \neq 2, -3$ (restrictions)

$\therefore$ no solutions

B Cross Multiplication

A rational equation of the form: $\frac{P(x)}{Q(x)} = \frac{R(x)}{S(x)}$, where $P(x)$, $Q(x)$, $R(x)$, and $S(x)$ are polynomial functions, may be solve by cross-multiplication:

$$\frac{P(x)}{Q(x)} = \frac{R(x)}{S(x)} \Leftrightarrow P(x)S(x) = Q(x)R(x)$$

Note. Do not forget to state and verify restrictions.

Ex 2. Use cross-multiplication to solve:

$$\frac{x-1}{2x+3} = \frac{x+2}{3x-2}$$

$x \neq -\frac{3}{2}, \frac{2}{3}$

$(x-1)(3x-2) = (2x+3)(x+2)$

$3x^2 - 3x - 2x + 2 = 2x^2 + 4x + 3x + 6$

$x^2 - 12x - 4 = 0$

$x = \frac{12 \pm \sqrt{144+16}}{2}$

$= \frac{12 \pm 4\sqrt{10}}{2}$

$\therefore x = 6 \pm 2\sqrt{10}$

C Shortcut

A rational equation of the form $\frac{P(x)}{Q(x)} = \frac{0}{1}$ is equivalent (if restrictions are satisfied) to the equation:

$$\boxed{P(x) = 0}$$

Note. Do not forget to state and verify restrictions.

Ex 3. Solve for x.

$x \neq \pm 1$

$$\frac{x^2+x-2}{x^2-1} = \frac{0}{1}$$

$x^2 + x - 2 = 0$

$(x+2)(x-1) = 0$

$x = -2$ or $x = 1$

$x \neq \pm 1$

$\therefore x = -2$

D No solution

A rational equation of the form:

$$\frac{constant}{P(x)} = \frac{0}{1} \Rightarrow const = 0$$

$5 = 0$

false

no solutions

does not have any solution.

Ex 4. Solve for x.

$x \neq 1, \frac{1}{2}$

$$\frac{2}{2x-1} + \frac{1}{1-x} = 0$$

$$\frac{2(1-x)+1(2x-1)}{(2x-1)(1-x)} = 0$$

$$\frac{2-2x+2x-1}{(2x-1)(1-x)} = 0$$

$$\frac{1}{(2x-1)(1-x)} = \frac{0}{1}$$

$1 = 0$

false

$\therefore$ no solutions

$$\frac{2}{2x-1} = \frac{-1}{1-x}$$

$2(1-x) = -(2x-1)$

$2 - 2x = -2x + 1$

$2 = 1$

false

$\therefore$ no sols

Ex 5. Solve for x.

a) $$\frac{1+x}{1} + \frac{4}{x} = \frac{9}{x-1}$$ $x \neq 0, 1$

$(x+1)x(x-1) + 4(x-1) = 9x$

$x^3 - x + 4x - 4 - 9x = 0$

$\underbrace{x^3 - 6x - 4}_{P(x)} = 0$

$\pm 1, \pm 2, \pm 4$

$P(1) \neq 0$

$P(-1) \neq 0$

$P(2) = 8 - 12 - 4 \neq 0$

$P(-2) = -8 + 12 - 4 = 0$

-2	1	0	-6	-4
		-2	4	4
	1	-2	-2	0

$x^2 - 2x - 2 = 0$

$$x = \frac{2 \pm \sqrt{4+8}}{2}$$

$\therefore x = -2, 1 \pm \sqrt{3}$

b) $$\frac{2x^3 - 1}{4 - x^2} = \frac{x}{x+2}$$ $x \neq \pm 2$

$$\frac{2x^3 - 1}{(2-x)\cancel{(2+x)}} = \frac{x}{\cancel{x+2}}$$

$2x^3 - 1 = x(2-x)$

$\underbrace{2x^3 + x^2 - 2x - 1}_{P(x)} = 0$

± 1

$P(1) = 0$

1	2	1	-2	-1
	0	2	3	1
	2	3	1	0

$2x^2 + 3x + 1 = 0$

$$x = \frac{-3 \pm \sqrt{9-8}}{4}$$

$$= \frac{-3 \pm 1}{4} \to -1, -\frac{1}{2}$$

$\therefore x = 1, -1, -\frac{1}{2}$

Ex 6. Solve for x.

a) $$\frac{\frac{x}{x-1} + \frac{x}{x+1}}{\frac{3}{x+1} - \frac{2}{x-1}} = \frac{3x+2}{x-5} \Rightarrow \frac{\frac{x(x+1)+x(x-1)}{\cancel{(x-1)(x+1)}}}{\frac{3(x-1)-2(x+1)}{\cancel{(x-1)(x+1)}}} = \frac{3x+2}{x-5}$$

$x \neq \pm 1, 5$

$$\frac{2x^2}{\cancel{x-5}} = \frac{3x+2}{\cancel{x-5}}$$

$2x^2 - 3x - 2 = 0$

$$x = \frac{3 \pm \sqrt{9+16}}{4} = \frac{3 \pm 5}{4} \to 2, -\frac{1}{2}$$

$\therefore x = 2, -\frac{1}{2}$

b) $$\frac{|x-1|}{x+1} + \frac{x+2}{|x-2|} = 3$$

Reading: Nelson Textbook, Pages 278-285

Homework: Nelson Textbook, Page 285: #1, 2a, 3d, 5c, 6c, 7f, 13, 15