MHF4U - Advanced Functions

5.3 Graphs of Rational Function of the Form $f(x)=\frac{ax+b}{cx+d}$

A Characteristics of the Rational Function:

$$f(x)=\frac{ax+b}{cx+d} \quad a,c\neq 0$$

Case 1. $cx+d$ is not a factor of $ax+b$

Domain: $R\setminus\{-d/c\}$

Range: $R\setminus\{a/c\}$

x-intercept: $-b/a$

y-intercept: b/d if $d\neq 0$

Symmetry: neither even nor odd

Vertical asymptote: $x=-d/c$

Horizontal asymptote: $y=a/c$

Continuity: There exists an infinite break at $x=-d/c$.

Ex 1. Find the characteristics of the function

$f(x)=\frac{2x-4}{x+2}$. Then graph it.

$=\frac{2(x-2)}{x+2}$

Zeros: $x=2$

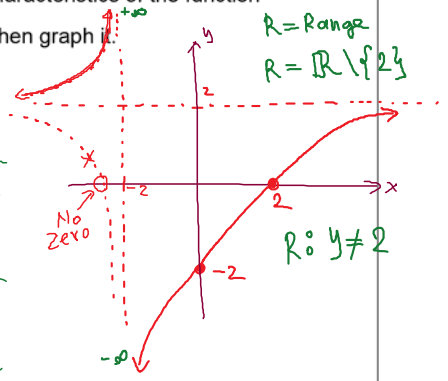
VA: $x=-2$

HA: $y=2$

D: $x\neq -2$

y-int $=-2$

Holes: none



B Characteristics of the Rational Function:

$$f(x)=\frac{ax+b}{cx+d} \quad a,c\neq 0$$

Case 2. $cx+d$ is a factor of $ax+b$

Domain: $R\setminus\{-d/c\}$

Range: $\{a/c\}$

x-intercept: none

y-intercept: b/d if $d\neq 0$

Symmetry: neither even nor odd

Vertical asymptote: none

Horizontal asymptote: $y=a/c$

Continuity: There exists a hole at $x=-d/c$.

Ex 2. Find the characteristics of the function

$f(x)=\frac{3x+6}{2x+4}$. Then graph it.

$=\frac{3(x+2)}{2(x+2)}$

$=\begin{cases}\frac{3}{2}; & x\neq -2\\ \text{und}; & x=-2\end{cases}$

D: $x\neq -2$

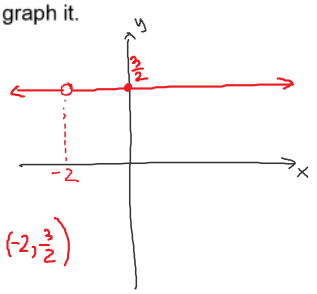
Zeros: no

VA: no

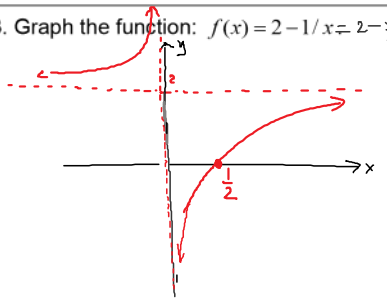
HA: $y=\frac{3}{2}$

y-int $=\frac{6}{4}=\frac{3}{2}$

Hole: $(-2,\frac{3}{2})$



Ex 3. Graph the function: $f(x)=2-1/x=2-\frac{1}{x}=\frac{2x-1}{x}$



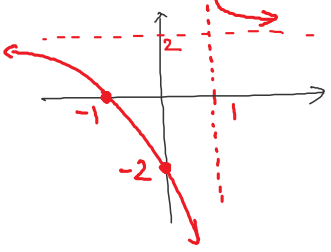
Ex 4. Find a function of the form $f(x)=\frac{ax+b}{cx+d}$ with a horizontal asymptote $y=2$, a vertical asymptote $x=1$ and an x-intercept $x-int=-1$. Then graph it.

$f(x)=2\,\frac{x+1}{x-1}$ $\Rightarrow$ y-int $=f(0)$

$=2\,\frac{0+1}{0-1}$

$=2\,\frac{1}{-1}$

$=-2$



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Ex 5. Graph the function $f(x) = x/(x+1)$ and its reciprocal on the same grid.

Ex 6. Graph the function

$$f(x) = \frac{2x^2+7x+6}{x^2+x-2}$$

Hole: $(-2, \frac{1}{3})$

$$= \frac{(x+2)(2x+3)}{(x+2)(x-1)}$$

$$= \begin{cases} \frac{2x+3}{x-1} & \text{if } x \neq 1, -2 \\ \text{and if } x = 1, -2 \end{cases}$$

Zeros: $x = -\frac{3}{2}$

VA: $x = 1$

HA: $y = 2$

D: $x \neq 1, -2$

y-int $= -3$

$R = \mathbb{R} \setminus \{2, \frac{1}{3}\}$

Ex 7. Graph.

a) $f(x) = \frac{x^2}{4-x^2}$ even

$\frac{1^2}{4-1^2} = \frac{1}{3} > 0$

$f(x) = \frac{x^2}{(2-x)(2+x)}$

asymptote (end behavior) $x \to \pm\infty$

b) $f(x) = \frac{x}{x^2+x-6}$

$= \frac{x}{(x+3)(x-2)}$

$\frac{3}{6(1)} > 0$

c) $f(x) = \frac{x^2-2x}{x+1}$

d) $f(x) = \frac{x^2-9}{4-x^2}$ even

$= \frac{(x-3)(x+3)}{(2-x)(2+x)}$

e) $f(x) = \frac{x^2-4}{x^2+1}$

Ex 8. Graph.

$$f(x) = \frac{(1-x^2)(x-2)}{x^2(x+2)^2(x^2+x-2)}$$

Reading: Nelson Textbook, Pages 263-271
Homework: Nelson Textbook, Page 272: #1, 5c, 6ad, 9, 13, 14

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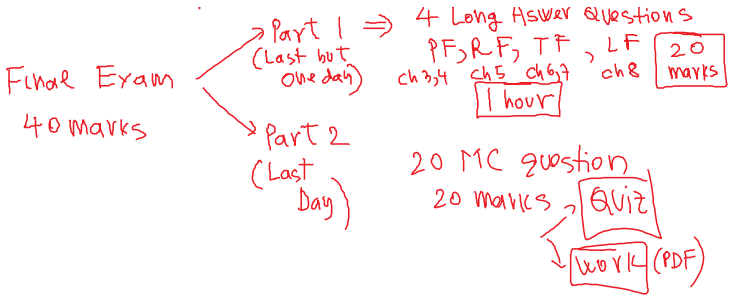
Note. $f(x) = \frac{ax+b}{cx+d}$

Q1 Q2 Q3 Q4 VA HA

Final Exam → Part 1 (Last but one day) ⇒ 4 Long Answer Questions PF, RF, TF, LF ch3,4 ch5 ch6,7 ch8 (1 hour) 20 marks


Final Exam
40 marks
Part 1
(Last but one day)
⇒
4 Long Answer Questions
PF, RF, TF, LF
ch3,4 ch5 ch6,7 ch8
1 hour
20 marks
Part 2
(Last Day)
20 MC question
20 marks
Quiz
work (PDF)